Section 5.8 Problems

In Problems 1–40, find the general antiderivative of the given function.

1. $f(x) = 4x^2 - x$
2. $f(x) = 2 - 5x^2$
3. $f(x) = x^2 + 3x - 4$
4. $f(x) = 3x^2 - x^4$
5. $f(x) = x^4 - 3x^2 + 1$
6. $f(x) = 2x^3 + x^2 - 5x$
7. $f(x) = 4x^3 - 2x + 3$
8. $f(x) = x - 2x^2 - 3x^3 - 4x^4$
9. $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$
10. $f(x) = x^2 - \frac{2}{x^2} + \frac{3}{x^3}$
11. $f(x) = 1 - \frac{1}{x^2}$
12. $f(x) = x^3 - \frac{1}{x^3}$
13. $f(x) = \frac{1}{1+x}$
14. $f(x) = \frac{x}{1+x}$
15. $f(x) = 5x^4 + \frac{5}{x^4}$
16. $f(x) = x^7 + \frac{1}{x^7}$
17. $f(x) = \frac{1}{1+2x}$
18. $f(x) = \frac{1}{1+3x}$
19. $f(x) = e^{-3x}$
20. $f(x) = e^{x/2} + e^{-x/2}$
21. $f(x) = 2e^{2x}$
22. $f(x) = -3e^{-4x}$
23. $f(x) = \frac{1}{e^{2x}}$
24. $f(x) = \frac{3}{e^{-x}}$
25. $f(x) = \sin(2x)$
26. $f(x) = \cos(3x)$
27. $f(x) = \sin\left(\frac{x}{3}\right) + \cos\left(\frac{x}{3}\right)$
28. $f(x) = \cos\left(\frac{x}{5}\right) - \sin\left(\frac{x}{5}\right)$
29. $f(x) = 2\sin\left(\frac{\pi}{2}x\right) - 3\cos\left(\frac{\pi}{2}x\right)$
30. $f(x) = -3\sin\left(\frac{\pi}{3}x\right) + 4\cos\left(-\frac{\pi}{4}x\right)$
31. $f(x) = \sec^2(2x)$
32. $f(x) = \sec^2(-4x)$
33. $f(x) = \sec^2\left(\frac{x}{3}\right)$
34. $f(x) = \sec^2\left(-\frac{x}{4}\right)$
35. $f(x) = \frac{\sec x + \cos x}{\cos x}$
36. $f(x) = \sin^2 x + \cos^2 x$
37. $f(x) = x^{-7} + 3x^5 + \sin(2x)$
38. $f(x) = 2e^{-3x} + \sec^2\left(-\frac{x}{2}\right)$
39. $f(x) = \sec^2(3x - 1) + \frac{x^2 - 3}{x}$
40. $f(x) = 5e^{3x} - \sec^2(x - 3)$

In Problems 41–46, assume that a is a positive constant. Find the general antiderivative of the given function.

41. $f(x) = \frac{e^{(a+1)x}}{a}$
42. $f(x) = \sin^2(a^2x + 1)$
43. $f(x) = \frac{1}{ax + 3}$
44. $f(x) = \frac{a}{a + x}$
45. $f(x) = x^{a+2} - a^{x+2}$
46. $f(x) = \frac{e^{-ax} + e^{ax}}{2a}$

In Problems 47–58, find the general solution of the differential equation.

47. $\frac{dy}{dx} = \frac{2}{x} - x, x > 0$
48. $\frac{dy}{dx} = \frac{2}{x^3} - x^3, x > 0$
49. $\frac{dy}{dx} = x(1 + x), x > 0$
50. $\frac{dy}{dx} = e^{-4x}, x > 0$
51. $\frac{dy}{dt} = t(1 - t), t \geq 0$
52. $\frac{dy}{dt} = t^2(1 - t^2), t \geq 0$
53. $\frac{dy}{dt} = e^{-t/2}, t \geq 0$
54. $\frac{dy}{dt} = 1 - e^{-3t}, t \geq 0$
55. $\frac{dy}{ds} = \sin(\pi s), 0 \leq s \leq 1$
56. $\frac{dy}{ds} = \cos(2\pi s), 0 \leq s \leq 1$
57. $\frac{dy}{dx} = \sec^2\left(\frac{x}{2}\right), -1 < x < 1$
58. $\frac{dy}{dx} = 1 + \sec^2\left(\frac{x}{4}\right), -1 < x < 1$

In Problems 59–72, solve the initial-value problem.

59. $\frac{dy}{dx} = 3x^2$, for $x \geq 0$ with $y = 1$ when $x = 0$
60. $\frac{dy}{dx} = \frac{x^2}{3}$, for $x \geq 0$ with $y = 2$ when $x = 0$
61. $\frac{dy}{dx} = 2\sqrt{x}$, for $x \geq 0$ with $y = 2$ when $x = 1$
62. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$, for $x \geq 1$ with $y = 3$ when $x = 4$
63. $\frac{dN}{dt} = \frac{1}{t}$, for $t \geq 1$ with $N(1) = 10$
64. $\frac{dN}{dt} = \frac{t}{t+2}$, for $t \geq 0$ with $N(0) = 2$
65. $\frac{dW}{dt} = e^t$, for $t \geq 0$ with $W(0) = 1$
66. $\frac{dW}{dt} = e^{-3t}$, for $t \geq 0$ with $W(0) = 2$
67. $\frac{dW}{dt} = e^{-3t}$, for $t \geq 0$ with $W(0) = 2/3$
68. $\frac{dW}{dt} = e^{-5t}$, for $t \geq 0$ with $W(0) = 1$
69. $\frac{dT}{dt} = \sin(\pi t)$, for $t \geq 0$ with $T(0) = 3$
70. $\frac{dT}{dt} = \cos(\pi t)$, for $t \geq 0$ with $T(0) = 3$
71. $\frac{dy}{dx} = \frac{e^{-x} + e^x}{2}$, for $x \geq 0$ with $y = 0$ when $x = 0$
72. $\frac{dN}{dt} = t^{-1/3}$, for $t > 0$ with $N(0) = 60$

73. Suppose that the length of a certain organism at age x is given by $L(x)$, which satisfies the differential equation

$$\frac{dL}{dx} = e^{-0.1x}, \quad x \geq 0$$

Find $L(x)$ if the limiting length L_∞ is given by

$$L_\infty = \lim_{x \to \infty} L(x) = 25$$

How big is the organism at age $x = 0$?

74. Fish are indeterminate growers; that is, their length $L(x)$ increases with age x throughout their lifetime. If we plot the growth rate dL/dx versus age x on semilog paper, a straight line with negative slope results. Set up a differential equation that relates growth rate and age. Solve this equation under the assumption that $L(0) = 5$, $L(1) = 10$, and

$$\lim_{x\to\infty} L(x) = 20$$

Graph the solution $L(x)$ as a function of x.

75. An object is dropped from a height of 100 ft. Its acceleration is 32 ft/s^2. When will the object hit the ground, and what will its speed be at impact?

76. Suppose that the growth rate of a population at time t undergoes seasonal fluctuations according to

$$\frac{dN}{dt} = 3\sin(2\pi t)$$

where t is measured in years and $N(t)$ denotes the size of the population at time t. If $N(0) = 10$ (measured in thousands), find an expression for $N(t)$. How are the seasonal fluctuations in the growth rate reflected in the population size?

77. Suppose that the amount of water contained in a plant at time t is denoted by $V(t)$. Due to evaporation, $V(t)$ changes over time. Suppose that the change in volume at time t, measured over a 24-hour period, is proportional to $t(24 - t)$, measured in grams per hour. To offset the water loss, you water the plant at a constant rate of 4 grams of water per hour.

(a) Explain why

$$\frac{dV}{dt} = -at(24 - t) + 4$$

$0 \le t \le 24$, for some positive constant a, describes this situation.

(b) Determine the constant a for which the net water loss over a 24-hour period is equal to 0.

Chapter 5 Key Terms

Discuss the following definitions and concepts:

1. Global or absolute extrema
2. Local or relative extrema: local minimum and local maximum
3. The extreme-value theorem
4. Fermat's theorem
5. Mean-value theorem
6. Rolle's theorem
7. Increasing and decreasing function
8. Monotonicity and the first derivative
9. Concavity: concave up and concave down
10. Concavity and the second derivative
11. Diminishing return
12. Candidates for local extrema
13. Monotonicity and local extrema
14. The second-derivative test for local extrema
15. Inflection points
16. Inflection points and the second derivative
17. Asymptotes: horizontal, vertical, and oblique
18. Using calculus to graph functions
19. L'Hospital's rule
20. Dynamical systems: cobwebbing
21. Stability of equilibria
22. Newton–Raphson method for finding roots
23. Antiderivative

Chapter 5 Review Problems

1. Suppose that

$$f(x) = xe^{-x}, \quad x \ge 0$$

(a) Show that $f(0) = 0$, $f(x) > 0$ for $x > 0$, and

$$\lim_{x\to\infty} f(x) = 0$$

(b) Find local and absolute extrema.

(c) Find inflection points.

(d) Use the foregoing information to graph $f(x)$.

2. Suppose that

$$f(x) = x\ln x, \quad x > 0$$

(a) Define $f(x)$ at $x = 0$ so that $f(x)$ is continuous for all $x \ge 0$.

(b) Find extrema and inflection points.

(c) Graph $f(x)$.

3. In Review Problem 17 of Chapter 2 we introduced the hyperbolic functions

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad x \in \mathbf{R}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad x \in \mathbf{R}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}, \quad x \in \mathbf{R}$$

(a) Show that $f(x) = \tanh x$, $x \in \mathbf{R}$, is a strictly increasing function on $\mathbf{R}$. Evaluate

$$\lim_{x\to-\infty} \tanh x$$

and

$$\lim_{x\to\infty} \tanh x$$

(b) Use your results in (a) to explain why $f(x) = \tanh x$, $x \in \mathbf{R}$, is invertible, and show that its inverse function $f^{-1}(x) = \tanh^{-1} x$ is given by

$$f^{-1}(x) = \frac{1}{2}\ln\frac{1+x}{1-x}$$

What is the domain of $f^{-1}(x)$?

(c) Show that

$$\frac{d}{dx} f^{-1}(x) = \frac{1}{1-x^2}$$

(d) Use your result in (c) and the facts that

$$\tanh x = \frac{\sinh x}{\cosh x}$$

and

$$\cosh^2 x - \sinh^2 x = 1$$

to show that

$$\frac{d}{dx}\tanh x = \frac{1}{\cosh^2 x}$$