Section 8.1

1.) $\frac{dY}{dx} = x + \sin x \to Y = \int (x + \sin x)\,dx \to$

$Y = \frac{x^2}{2} - \cos x + c$ and $x = 0, Y = 0 \to$

$0 = 0 - \cos 0 + c = -1 + c \to c = 1 \to$

$Y = \frac{x^2}{2} - \cos x + 1$

2.) $\frac{dY}{dx} = e^{-3x} \to Y = \int e^{-3x}\,dx \to Y = \frac{-1}{3} e^{-3x} + c$

and $x = 0, Y = 10 \to 10 = \frac{-1}{3} e^{0} + c = \frac{-1}{3} + c \to$

$c = \frac{31}{3} \to$ $Y = \frac{-1}{3} e^{-3x} + \frac{31}{3}$

4.) $\frac{dY}{dx} = \frac{1}{1+x^2} \to Y = \int \frac{1}{1+x^2}\,dx \to$

$Y = \arctan x + c$ and $x = 0, Y = 1 \to$

$1 = \arctan 0 + c = 0 + c \to c = 1 \to$

$Y = \arctan x + 1$

7.) $\frac{ds}{dt} = \sqrt{3t+1} \to s = \int \sqrt{3t+1}\,dt \to$

$s = \frac{1}{3} \cdot \frac{2}{3}(3t+1)^{3/2} + c$ and $t = 0, s = 1 \to$

$1 = \frac{2}{9}(1)^{3/2} + c = \frac{2}{9} + c \to c = \frac{7}{9} \to$

$S = \frac{2}{9}(3t+1)^{3/2} + c$

9.) $\frac{dV}{dt} = 1 + \cos t \to V = \int (1 + \cos t)\,dt \to$

$V = t + \sin t + c$ and $t = 0, V = 5 \to$

$5 = 0 + \sin 0 + c = 0 + c \rightarrow \quad c = 5 \rightarrow$

$$V = t + \sin t + 5$$

10.) $\dfrac{dP}{dt} = 3t + 1 \rightarrow \quad P = \dfrac{3}{2}t^2 + t + c$

and $t = 0,\ P = 0 \rightarrow \quad 0 = 0 + 0 + c \rightarrow c = 0$

$\rightarrow \quad P = \dfrac{3}{2}t^2 + t$

12.) $\dfrac{dY}{dX} = 2(1 - Y) \rightarrow \quad \dfrac{1}{1-Y}\,dy = 2\,dx \rightarrow$

$\displaystyle\int \frac{1}{1-Y}\,dy = \int 2\,dx \rightarrow -\ln|1 - Y| = 2X + c$

and $X = 0,\ Y = 2 \rightarrow \quad -\ln 1 = 0 + c \rightarrow c = 0 \rightarrow$

$$-\ln|1 - Y| = 2X$$

13.) $\dfrac{dX}{dt} = -2X \rightarrow \quad \dfrac{1}{X}\,dX = -2\,dt \rightarrow$

$\displaystyle\int \frac{1}{X}\,dx = \int -2\,dt \rightarrow \ln|X| = -2t + c$

and $t = 1,\ X = 5 \rightarrow \ln 5 = -2 + c \rightarrow$

$c = 2 + \ln 5 \rightarrow \quad \ln|X| = -2t + 2 + \ln 5$

17.) $\dfrac{dN}{dt} = 0.3N \rightarrow \displaystyle\int \frac{1}{N}\,dN = \int 0.3\,dt \rightarrow$

$\ln N = 0.3t + c$ and $t = 0,\ N = 20 \rightarrow$

$\ln 20 = 0 + c \rightarrow \quad c = \ln 20 \rightarrow$

$\ln N = 0.3t + \ln 20 \quad \rightarrow$

$$e^{\ln N} = e^{0.3t + \ln 20} = e^{\ln 20} \cdot e^{0.3t} \rightarrow$$

$$\boxed{N = 20e^{0.3t}} \text{ ; if } t = 5 \rightarrow$$

$$N = 20e^{1.5} \approx 89.6 \text{ (or } 90)$$

19.) a.) $\frac{1}{N} \cdot \frac{dN}{dt} = r \rightarrow \int \frac{1}{N}\, dN = \int r\, dt \rightarrow$

$\ln N = rt + c_1 \rightarrow e^{\ln N} = e^{rt + c_1} = e^{c_1} e^{rt}$

$\rightarrow \boxed{N = ce^{rt}}$

b.) apply log: $\log N = \log ce^{rt} \rightarrow$

$\log N = \log c + \log e^{rt} \rightarrow$

$\log N = \log c + rt \cdot \log e \rightarrow$

$\boxed{\log N = \log c + (r \log e)\, t}$ (a line);

$Y = B + M\, t$ ;

to find $r$ from the graph, find slope $M = r \log e$ and divide that number by $\log e \rightarrow$

$$r = \frac{M}{\log e} .$$

c.) To determine $r$: Plot data points for $t$ and $N$ on semi-log paper; assume these data form a line;

find the slope of this line and divide this number by log e.

20.) a.) $\frac{dw}{dt} = -\lambda w(t) \rightarrow \int \frac{1}{w(t)}\,dw = \int -\lambda\,dt$

$\rightarrow \ln w(t) = -\lambda t + c_1 \rightarrow e^{\ln w(t)} = e^{-\lambda t + c_1} \rightarrow$

$w(t) = e^{c_1} e^{-\lambda t} = c e^{-\lambda t} \rightarrow w(t) = c e^{-\lambda t}$;

and $t=0$, $w = w_0 \rightarrow w_0 = c e^0 = c(1) = c \rightarrow$

$\boxed{w(t) = w_0 e^{-\lambda t}}$

b.) $t=0$, $w = 123$ gr. $\rightarrow 123 = w_0 \cdot e^0 = w_0 \rightarrow$

$w(t) = 123\, e^{-\lambda t}$; and $t=5$, $w = 20$ gr. $\rightarrow$

$20 = 123\, e^{-5\lambda} \rightarrow \frac{20}{123} = e^{-5\lambda} \rightarrow$

$\ln\left(\frac{20}{123}\right) = \ln e^{-5\lambda} = -5\lambda \rightarrow$

$\lambda = \frac{-1}{5} \ln\left(\frac{20}{123}\right) \rightarrow \boxed{w(t) = 123\, e^{\frac{1}{5}\ln\left(\frac{20}{123}\right)t}}$;

123 is initial amount, so if

$w = \frac{1}{2}(123) \rightarrow \frac{1}{2}(\cancel{123}) = \cancel{123}\, e^{\frac{1}{5}\ln\left(\frac{20}{123}\right)t}$

$\rightarrow \ln(1/2) = \ln e^{\frac{1}{5}\ln\left(\frac{20}{123}\right)t} \rightarrow$

$\ln(1/2) = \frac{1}{5}\ln\left(\frac{20}{123}\right)t \rightarrow$ half-life is

$$t = \frac{5\ln(1/2)}{\ln\left(\frac{20}{123}\right)} \approx 1.91 \text{ minutes}$$

22.) a.) $\frac{dL}{dt} = k(34 - L) \rightarrow$

$$\int \frac{1}{34-L}\,dL = \int k\,dt \rightarrow -\ln(34-L) = kt + c_1 \rightarrow$$

$$\ln(34 - L) = -kt + c_2 \rightarrow$$

$$34 - L = e^{-kt+c_2} = e^{c_2}e^{-kt} = ce^{-kt} \rightarrow$$

$$L = 34 - ce^{-kt}; \text{ and } t=0,\ L=2 \rightarrow$$

$$2 = 34 - ce^{0} = 34 - c \rightarrow c = 32 \rightarrow$$

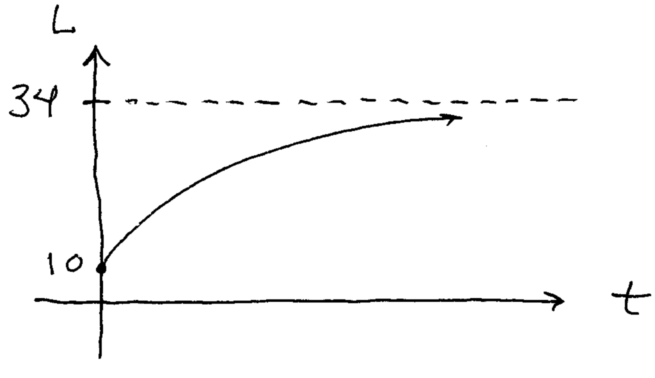
$$\boxed{L = 34 - 32e^{-kt}}$$

b.) If $t=4$, $L=10 \rightarrow 10 = 34 - 32e^{-4k} \rightarrow$

$$32e^{-4k} = 24 \rightarrow e^{-4k} = \frac{24}{32} = \frac{3}{4} \rightarrow$$

$$\ln e^{-4k} = \ln\left(\frac{3}{4}\right) \rightarrow -4k = \ln(3/4) \rightarrow$$

$$\underline{k = \frac{-1}{4}\ln(3/4)} \rightarrow \boxed{L = 34 - 32e^{\frac{1}{4}\ln\left(\frac{3}{4}\right)t}}$$



c.) $t = 10 \rightarrow L \approx 18.4$

d.) $\lim\limits_{t\to\infty} L = \lim\limits_{t\to\infty}\left[34 - 32e^{\overbrace{\frac{1}{4}\ln\left(\frac{3}{4}\right)}^{\approx -0.0719}t}\right]$

$= 34 - 32\cancel{e}^{-\infty \to 0} = 34$

28.) $\frac{dY}{dx} = (Y-1)(Y-2) \rightarrow \int \frac{1}{(Y-1)(Y-2)}\,dY = \int dx$

$\rightarrow \int\left[\frac{A}{Y-1} + \frac{B}{Y-2}\right]dY = x + c \rightarrow$

$\Big( A(Y-2) + B(Y-1) = 1$ ,

let $Y=1$: $-A = 1 \rightarrow A = -1$

let $Y=2$: $B = 1$ $\Big)$

$\rightarrow \int\left[\frac{-1}{Y-1} + \frac{1}{Y-2}\right]dy = x + c \rightarrow$

$-\ln|Y-1| + \ln|Y-2| = x + c \rightarrow$

$\ln\left|\frac{Y-2}{Y-1}\right| = x + c$; and $x=0, Y=0 \rightarrow$

$\ln 2 = 0 + c \rightarrow c = \ln 2 \rightarrow$

$\boxed{\ln\left|\frac{Y-2}{Y-1}\right| = x + \ln 2}$

32.) $\frac{dY}{dx} = (1+Y)^2 \rightarrow \int \frac{1}{(1+Y)^2}\,dY = \int dx \rightarrow$

$\boxed{\frac{-1}{1+Y} = x + c}$ .

36.) $\frac{dY}{dx} = Y^2 + 4 \rightarrow \int \frac{1}{Y^2+2^2}\, dY = \int dx \rightarrow$

$$\boxed{\frac{1}{2}\arctan\left(\frac{Y}{2}\right) = x + C}$$

37.) $\frac{dN}{dt} = 0.34N\left(1 - \frac{N}{200}\right) \rightarrow$

$$\int \frac{1}{N\left(1-\frac{N}{200}\right)} \cdot \frac{200}{200}\, dN = \int 0.34\, dt \rightarrow$$

$$\int \frac{200}{N(200-N)}\, dN = 0.34t + c_1 \rightarrow$$

$$\int \left[\frac{A}{N} + \frac{B}{200-N}\right] dN = 0.34t + c_1 \rightarrow$$

$\big($ $A(200-N) + BN = 200$ ,
let $N=0$: $200A = 200 \rightarrow A = 1$
let $N = 200$: $200B = 200 \rightarrow B = 1$ $\big) \rightarrow$

$$\int \left[\frac{1}{N} + \frac{1}{200-N}\right] dN = 0.34t + c_1 \rightarrow$$

$$\ln N + {}^{-}\ln(200-N) = 0.34t + c_1 \rightarrow$$

$$\ln \frac{N}{200-N} = 0.34t + c_1 \rightarrow$$

$$\frac{N}{200-N} = e^{0.34t + c_1} = e^{c_1} e^{0.34t} = ce^{0.34t} \rightarrow$$

$$\frac{N}{200-N} = ce^{0.34t} \text{ ; and } t=0,\ N=50 \rightarrow$$

$$\frac{50}{150} = ce^0 = c(1) = c \rightarrow c = 1/3 \rightarrow$$

$$\frac{N}{200-N} = \frac{1}{3}e^{0.34t} \; ; \; (\text{SOLVE for } N.) \rightarrow$$

$$N = \frac{200}{3}e^{0.34t} - \frac{1}{3}e^{0.34t}N \longrightarrow$$

$$N + \frac{1}{3}e^{0.34t}N = \frac{200}{3}e^{0.34t} \longrightarrow$$

$$\left(1 + \frac{1}{3}e^{0.34t}\right)N = \frac{200}{3}e^{0.34t} \longrightarrow$$

$$N = \frac{\frac{200}{3}e^{0.34t}}{1+\frac{1}{3}e^{0.34t}} \cdot \frac{3/e^{0.34t}}{3/e^{0.34t}} \longrightarrow$$

$$\boxed{N = \frac{200}{3e^{-0.34t}+1}} \; ;$$

$$\lim_{t\to\infty} N = \lim_{t\to\infty} \frac{200}{3e^{-0.34t}+1} = \frac{200}{3(0)+1} = 200$$

44.) $\frac{dY}{dx} = 2 \cdot \frac{Y}{x} \rightarrow \int \frac{1}{Y}\,dY = \int \frac{2}{x}\,dx \longrightarrow$

$\ln|Y| = 2\ln|x| + c$ and $x=1,\ Y=1 \rightarrow$

$\ln 1 = 2\ln 1 + c \rightarrow 0 = 0 + c \rightarrow c = 0 \rightarrow$

$$\boxed{\ln|Y| = 2\ln|x|}$$

45.) $\frac{dY}{dX} = \frac{X+1}{Y} \rightarrow \int Y\,dY = \int (X+1)\,dx \rightarrow$

$\frac{1}{2}Y^2 = \frac{1}{2}X^2 + X + C$ and $X=0,\ Y=2 \rightarrow$

$2 = 0 + 0 + C \rightarrow C = 2 \rightarrow \boxed{\frac{1}{2}Y^2 = \frac{1}{2}X^2 + X + 2}$

47.) $\frac{dY}{dX} = (Y+1)e^{-X} \rightarrow \int \frac{1}{Y+1}\,dY = \int e^{-X}\,dx \rightarrow$

$\ln|Y+1| = -e^{-X} + c$ and $X=0,\ Y=2 \rightarrow$

$\ln 3 = -1 + c \rightarrow c = 1 + \ln 3 \rightarrow$

$\boxed{\ln|Y+1| = -e^{-X} + 1 + \ln 3}$

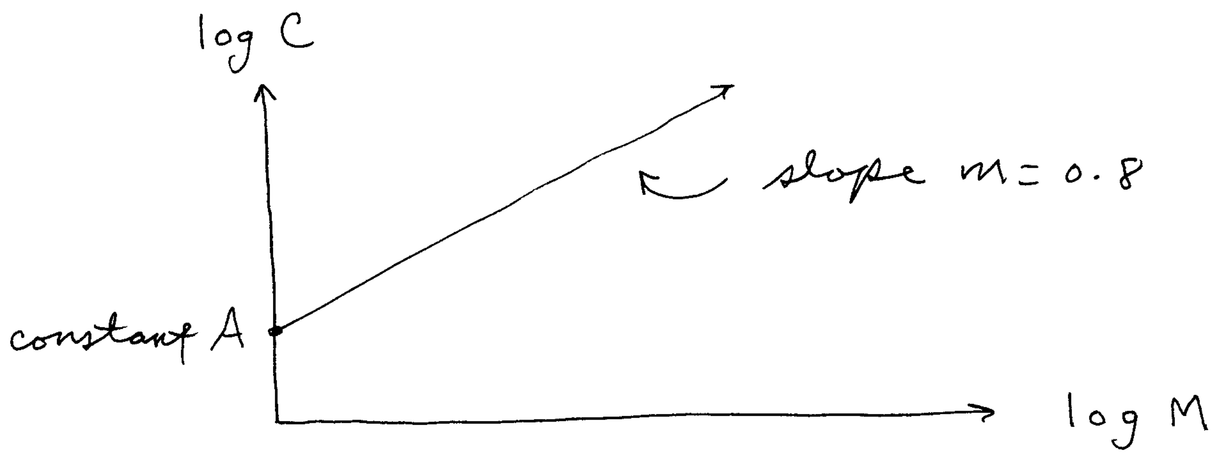
48.) $\frac{dY}{dX} = X^2Y^2 \rightarrow \int \frac{1}{Y^2}\,dY = \int X^2\,dx \rightarrow$

$\frac{-1}{Y} = \frac{1}{3}X^3 + c$ and $X=1,\ Y=1 \rightarrow$

$-1 = \frac{1}{3} + c \rightarrow c = -\frac{4}{3} \rightarrow$

$\boxed{\frac{-1}{Y} = \frac{1}{3}X^3 - \frac{4}{3}}$

53.) Let $C$ be maximal rate of oxygen consumption, let $M$ be body mass:




Line Equation : $\log C = \log A + (0.8) \log M$

$$\rightarrow \quad 10^{\log C} = 10^{\log A + (0.8) \log M}$$

$$\rightarrow \quad C = 10^{\log A} \cdot 10^{\log M^{0.8}}$$

$$\rightarrow \quad C = A \cdot M^{0.8} \xrightarrow{D}$$

$$\frac{dC}{dM} = A \cdot (0.8) M^{-0.2} = (0.8) A M^{-0.2} \cdot \frac{M}{M}$$

$$= (0.8) \underbrace{A M^{0.8}}_{C} \cdot \frac{1}{M} = (0.8) \cdot \frac{C}{M} \longrightarrow$$

$$\boxed{\frac{dC}{dM} = (0.8) \frac{C}{M}}$$