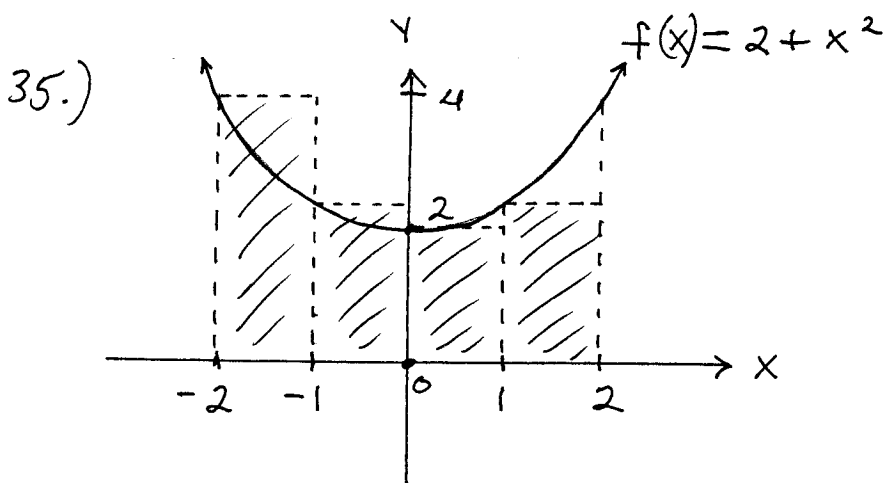
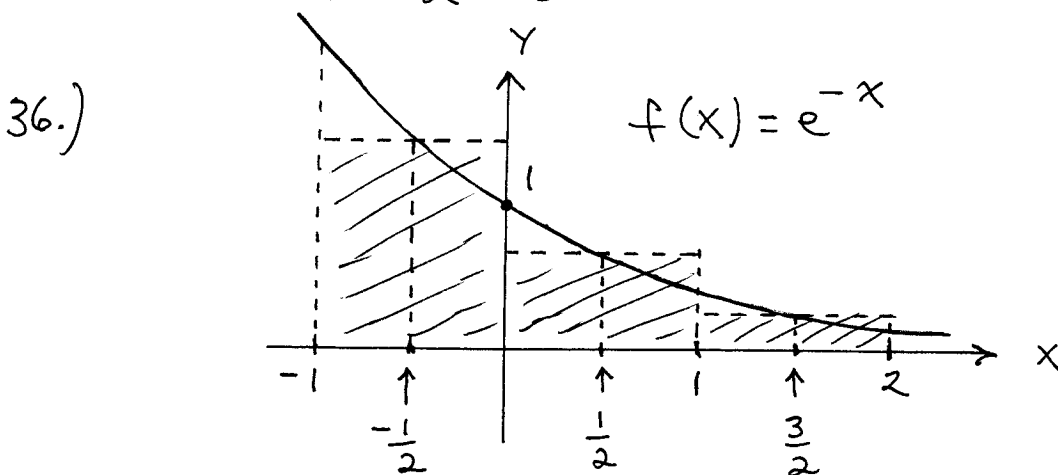


# Section 6.1



$$\int_{-2}^2 (2 + x^2) dx \approx f(-2) \cdot (1) + f(-1) \cdot (1) + f(0) \cdot (1) + f(1) \cdot (1)$$

$$= 6 + 3 + 2 + 3 = 14$$

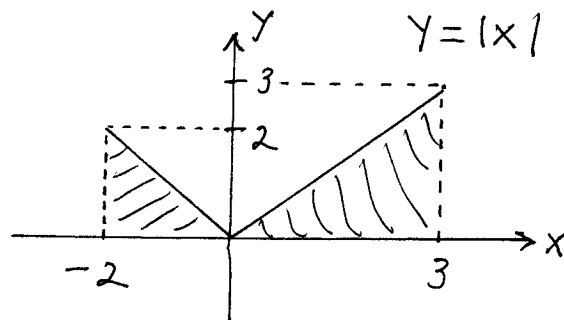


$$\int_{-1}^2 e^{-x} dx \approx f(-\frac{1}{2}) \cdot (1) + f(\frac{1}{2}) \cdot (1) + f(\frac{3}{2}) \cdot (1)$$

$$= e^{\frac{1}{2}} + e^{-\frac{1}{2}} + e^{-\frac{3}{2}} \approx 2.478$$

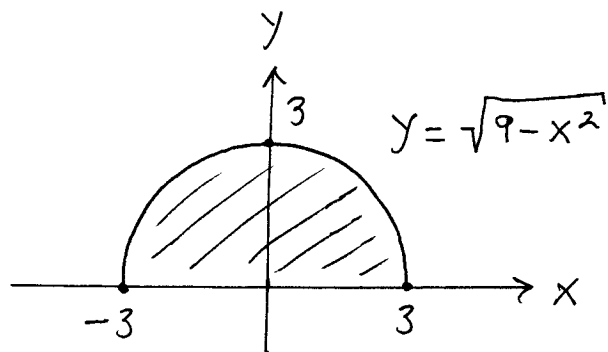
61.)  $\int_{-2}^3 |x| dx$

$$= \frac{1}{2}(2)(2) + \frac{1}{2}(3)(3) = \frac{13}{2}$$



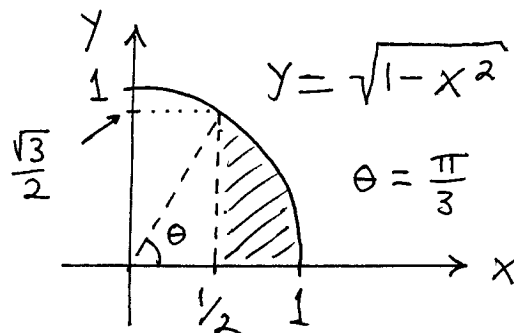
$$62.) \int_{-3}^3 \sqrt{9-x^2} dx$$

$$= \frac{1}{2} \pi (3)^2 = \frac{9}{2} \pi$$



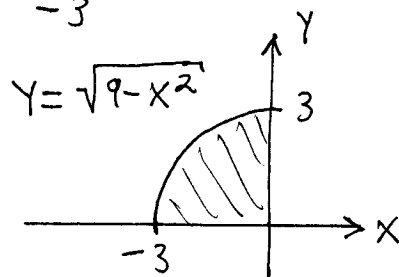
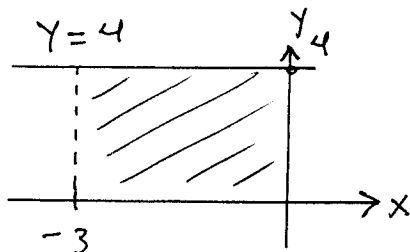
$$64.) \int_{\frac{1}{2}}^1 \sqrt{1-x^2} dx$$

$$= \text{Area } \triangle - \text{Area } \Delta$$



$$= \frac{\frac{\pi}{3}}{2\pi} \pi (1)^2 - \frac{1}{2} \left(\frac{1}{2}\right) \left(\frac{\sqrt{3}}{2}\right) = \frac{1}{6} \pi - \frac{\sqrt{3}}{8}$$

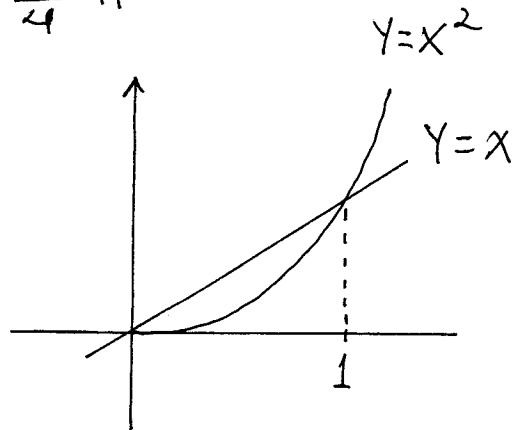
$$67.) \int_{-3}^0 (4 - \sqrt{9-x^2}) dx = \int_{-3}^0 4 dx - \int_{-3}^0 \sqrt{9-x^2} dx$$

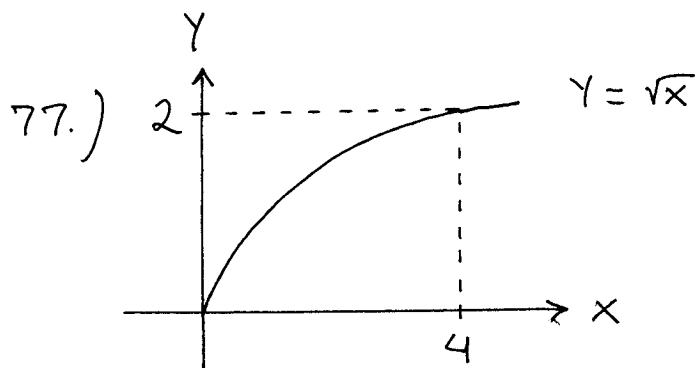


$$= (4)(3) - \frac{1}{4} \pi (3)^2 = 12 - \frac{9}{4} \pi$$

$$75.) x \geq x^2 \text{ for } 0 \leq x \leq 1$$

$$\text{so } \int_0^1 x dx \geq \int_0^1 x^2 dx$$

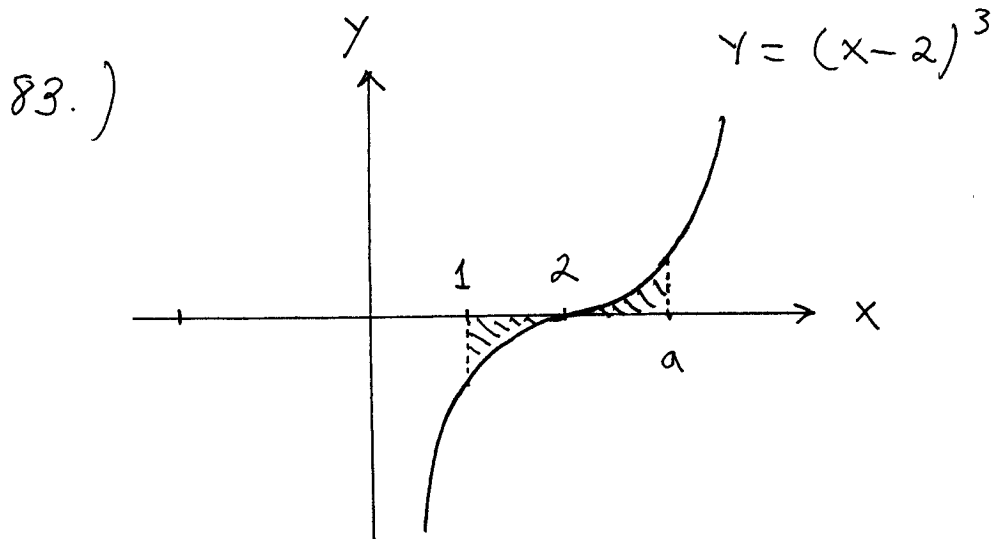




$0 \leq \sqrt{x} \leq 2$   
for  $0 \leq x \leq 4$   
so that

$$\int_0^4 0 \, dx \leq \int_0^4 \sqrt{x} \, dx \leq \int_0^4 2 \, dx \rightarrow$$

$$0 \leq \int_0^4 \sqrt{x} \, dx \leq 4(2) = 8$$



If  $\int_1^a (x-2)^3 \, dx = 0$ , then  $a = 3$   
(by symmetry)

# Worksheet 1

1.) a.)  $\begin{array}{ccccccccccc} & -1 & x_1 & x_2 & x_3 & \dots & x_i & \dots & 2 = x_n \\ & | & | & | & | & & | & & | \\ \hline & & & \underbrace{\hspace{2cm}} & & & & & \end{array}$

$$\Delta x_i = \frac{2 - (-1)}{n} = \frac{3}{n} \text{ for } i=1, 2, 3, \dots$$

and  $x_i = -1 + \frac{3}{n}i$ ;  $f(x) = 5$ ; then

$$\begin{aligned} \int_{-1}^2 5 \, dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x_i \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n 5 \cdot \frac{3}{n} = \lim_{n \rightarrow \infty} \frac{15}{n} \left( \sum_{i=1}^n 1 \right) \\ &= \frac{15}{n} \cdot n = 15 \end{aligned}$$

b.)  $\begin{array}{ccccccccccc} & 0 & x_1 & x_2 & x_3 & \dots & x_i & \dots & 2 = x_n \\ & | & | & | & | & & | & & | \\ \hline & & & \underbrace{\hspace{2cm}} & & & & & \end{array}$

$$\Delta x_i = \frac{2-0}{n} = \frac{2}{n} \text{ for } i=1, 2, 3, \dots$$

and  $x_i = 0 + \frac{2}{n}i = \frac{2}{n}i$  for  $i=1, 2, 3, \dots$ ;  
 $f(x) = x + 3$ ; then

$$\begin{aligned} \int_0^2 (x+3) \, dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x_i \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(\frac{2}{n}i\right) \cdot \frac{2}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{2}{n}i + 3\right) \frac{2}{n} \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{4}{n^2}i + \frac{6}{n}\right) = \lim_{n \rightarrow \infty} \left[ \frac{4}{n^2} \left(\sum_{i=1}^n i\right) + \frac{6}{n} \cdot \left(\sum_{i=1}^n 1\right) \right] \\ &= \lim_{n \rightarrow \infty} \left[ \frac{4}{n^2} \cdot \frac{n(n+1)}{2} + \frac{6}{n} \cdot n \right] \\ &= \lim_{n \rightarrow \infty} \left[ 2 \cdot \left(1 + \frac{1}{n}\right) + 6 \right] = 2(1) + 6 = 8 \end{aligned}$$

c.)  $\begin{array}{ccccccccccc} & -3 & x_1 & x_2 & x_3 & \dots & x_i & \dots & 0 = x_n \\ & | & | & | & | & & | & & | \\ \hline & & & \underbrace{\hspace{2cm}} & & & & & \end{array}$

$$\Delta x_i = \frac{0 - (-3)}{n} = \frac{3}{n} \text{ for } i=1, 2, 3, \dots$$

and  $x_i = -3 + \frac{3}{n}i$  for  $i=1, 2, 3, \dots, n$ ;

$f(x) = x^2 + 2x$ ; then

$$\int_{-3}^0 (x^2 + 2x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x_i$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n (x_i^2 + 2x_i) \cdot \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \left(-3 + \frac{3}{n}i\right)^2 + 2\left(-3 + \frac{3}{n}i\right) \right] \cdot \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ 9 - \frac{18}{n}i + \frac{9}{n^2}i^2 - 6 + \frac{6}{n}i \right] \cdot \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ 3 - \frac{12}{n}i + \frac{9}{n^2}i^2 \right] \cdot \frac{3}{n}$$

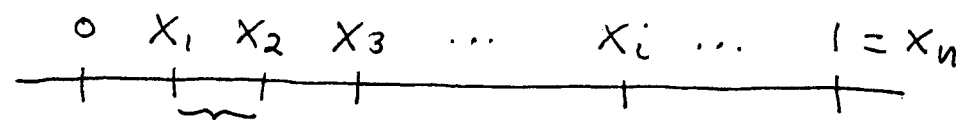
$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \frac{9}{n} - \frac{36}{n^2}i + \frac{27}{n^3}i^2 \right]$$

$$= \lim_{n \rightarrow \infty} \left\{ \frac{9}{n} \cdot \left(\sum_{i=1}^n 1\right) - \frac{36}{n^2} \left(\sum_{i=1}^n i\right) + \frac{27}{n^3} \left(\sum_{i=1}^n i^2\right) \right\}$$

$$= \lim_{n \rightarrow \infty} \left\{ \frac{9}{n} \cdot (n) - \frac{36}{n^2} \cdot \frac{n(n+1)}{2} + \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \right\}$$

$$= \lim_{n \rightarrow \infty} \left\{ 9 - 18 \cdot \left(1 + \frac{1}{n}\right) + \frac{9}{2} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) \right\}$$

$$= 9 - 18(1) + \frac{9}{2}(1)(2) = 0$$

d.) 

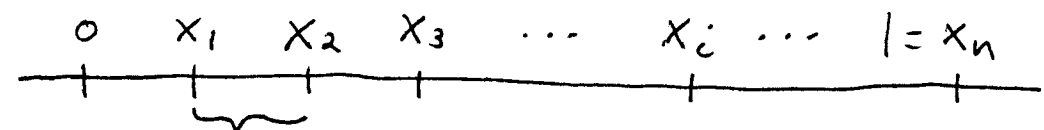
$$\Delta x_i = \frac{1-0}{n} = \frac{1}{n} \text{ for } i=1, 2, 3, \dots$$

and  $x_i = 0 + \frac{1}{n}i = \frac{i}{n}$  for  $i=1, 2, 3, \dots, n$ ;

$f(x) = x^3$ ; then

$$\int_0^1 x^3 dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x_i$$

$$\begin{aligned}
&= \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(\frac{i}{n}\right) \cdot \frac{1}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i}{n}\right)^3 \cdot \frac{1}{n} \\
&= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^3}{n^3} \cdot \frac{1}{n} = \lim_{n \rightarrow \infty} \frac{1}{n^4} \left( \sum_{i=1}^n i^3 \right) \\
&= \lim_{n \rightarrow \infty} \frac{1}{n^4} \cdot \left( \frac{n(n+1)}{2} \right)^2 = \lim_{n \rightarrow \infty} \frac{1}{n^4} \cdot \frac{n^2(n+1)^2}{4} \\
&= \lim_{n \rightarrow \infty} \frac{1}{4} \cdot \left( \frac{n+1}{n} \right)^2 = \lim_{n \rightarrow \infty} \frac{1}{4} \left( 1 + \frac{1}{n} \right)^2 = \frac{1}{4} (1)^2 = \frac{1}{4}
\end{aligned}$$

e.) 

$$\Delta x_i = \frac{1-0}{n} = \frac{1}{n} \text{ for } i=1, 2, 3, \dots$$

and  $x_i = 0 + \frac{1}{n}i = \frac{i}{n}$  for  $i=1, 2, 3, \dots$  ;

$f(x) = 2^x$  ; then

$$\begin{aligned}
\int_0^1 2^x dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x_i \\
&= \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(\frac{i}{n}\right) \cdot \frac{1}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n 2^{i/n} \cdot \frac{1}{n} \\
&= \lim_{n \rightarrow \infty} \left\{ \frac{1}{n} \cdot \left( 2^{1/n} + 2^{2/n} + 2^{3/n} + \dots + 2^{n/n} \right) \right\} \\
&= \lim_{n \rightarrow \infty} \left\{ \frac{1}{n} \left( (2^{1/n}) + (2^{1/n})^2 + (2^{1/n})^3 + \dots + (2^{1/n})^n \right) \right\} \\
&= \lim_{n \rightarrow \infty} \left\{ \frac{1}{n} \cdot 2^{1/n} \cdot \left( 1 + (2^{1/n}) + (2^{1/n})^2 + (2^{1/n})^3 + \dots + (2^{1/n})^{n-1} \right) \right\} \\
&= \lim_{n \rightarrow \infty} \frac{1}{n} \cdot 2^{1/n} \cdot \frac{1 - (2^{1/n})^{(n-1)+1}}{1 - 2^{1/n}} \\
&= \lim_{n \rightarrow \infty} \frac{1}{n} \cdot 2^{1/n} \cdot \frac{1 - 2}{1 - 2^{1/n}}
\end{aligned}$$

$$= \lim_{n \rightarrow \infty} \frac{-1}{n} \cdot \frac{2^{1/n}}{1 - 2^{1/n}} \cdot \frac{2^{-1/n}}{2^{-1/n}}$$

$$= \lim_{n \rightarrow \infty} \frac{-1}{n} \cdot \frac{2^0}{2^{-1/n} - 2^0}$$

$$= \lim_{n \rightarrow \infty} \frac{-\frac{1}{n}}{2^{-1/n} - 1}$$

"0/0"

$$\stackrel{L'H}{=} \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2}}{2^{-1/n} \cdot \frac{1}{n^2} \cdot \ln 2}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{2^{-1/n} \cdot \ln 2}$$

$$= \frac{1}{2^0 \cdot \ln 2}$$

$$= \frac{1}{\ln 2}$$