Summation Rules

1) \( \sum_{i=1}^{n} c = \underbrace{c + c + \ldots + c}_{n \text{ times}} = nc \)

2) \( \sum_{i=1}^{n} c \cdot f(i) = c \sum_{i=1}^{n} f(i) \)

3) \( \sum_{i=1}^{n} [f(i) \pm g(i)] = \sum_{i=1}^{n} f(i) \pm \sum_{i=1}^{n} g(i) \)

4) \( \sum_{i=1}^{n} i = 1+2+3+\ldots+n = \frac{1}{2} n(n+1) \)

5) \( \sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\ldots+n^2 = \frac{1}{6} n(n+1)(2n+1) \)

6) \( \sum_{i=1}^{n} i^3 = 1^3+2^3+3^3+\ldots+n^3 = \left[\frac{1}{2} n(n+1)\right]^2 \)

7) \( \sum_{i=0}^{n} r^i = 1 + r + r^2 + \ldots + r^n = \frac{1-r^{n+1}}{1-r} \)

Notes:

- a) Rules 1) and 4-6) must start at \( i = 1 \), and Rule 7 must start at \( i = 0 \).
- b) These rules are essential in evaluating Riemann sums, which we will discuss next lecture.