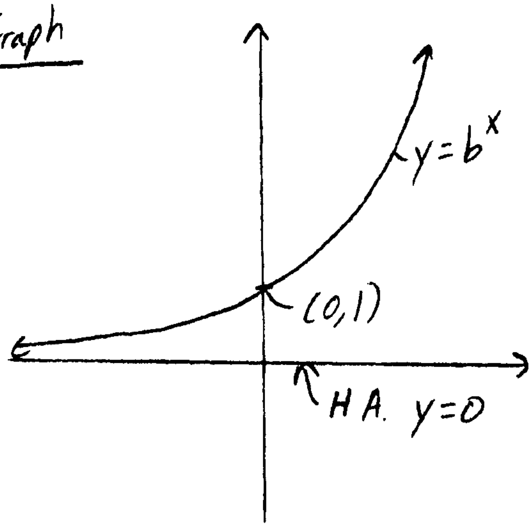
Math 21A
Vogler

Exponential Functions

Defn An *exponential function* is any function of the form $f(x) = b^x$ where b is a constant not equal to 1.

Notes: 1) Domain: All real numbers
2) Range: $y > 0$
3) b^x has a horizontal asymptote at $y=0$.

Graph



Rules

1) $b^m b^n = b^{m+n}$
2) $\frac{b^m}{b^n} = b^{m-n}$
3) $(b^m)^n = b^{mn}$
4) $\frac{1}{b^m} = b^{-m}$

Notes: a) Rule (1) can be viewed as 'outside multiplication' is 'inside addition.'
b) Rule (2) can be viewed as 'outside division' is 'inside subtraction.'
c) Rule (3) can be viewed as 'outside powers' is 'inside multiplication.'
d) Rule (4) can be viewed as negative powers is just division in 'disguise'.

The number e (L. Euler, 1748)

Let $f(x) = (1+\frac{1}{x})^x$ If the number x gets larger and larger, what happens to $f(x)$?

Using a calculator

x	$f(x) = (1+\frac{1}{x})^x$
1	2.000000000
10	2.593742460
100	2.704813829
1,000	2.716923932
10,000	2.718145927
⋮	⋮
1,000,000	2.718280469

which leads to the following:

Defn Let $e = \lim_{x\to\infty} (1+\frac{1}{x})^x \Rightarrow \boxed{e \doteq 2.71828}$

- Note that given b^x, we can rewrite it (using log rules) as $b^x = e^{\ln b^x} = e^{x\ln b} = e^{(\ln b)x}$.

So all exponential functions can be rewritten using the exponential with base e, which is why we will focus more attention in this class to exponential functions of the form $f(x)=e^{rt}$ for some number r.