Math 21A
Vogler

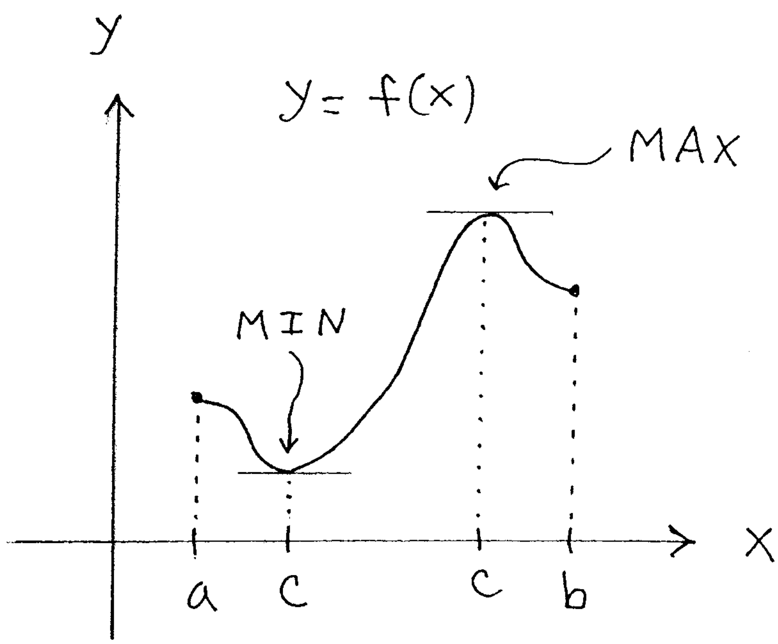
The Intermediate Value Theorem (IMVT) and Other Theorems

Maximum Value Theorem : Let f be a continuous function on the closed interval $[a, b]$. Then there is at least one number c in $[a, b]$ at which f takes on its maximum value, i.e.,

$$f(c) \geq f(x) \quad \text{for all } x \text{ in } [a, b] \ .$$

Minimum Value Theorem : Let f be a continuous function on the closed interval $[a, b]$. Then there is at least one number c in $[a, b]$ at which f takes on its minimum value, i.e.,

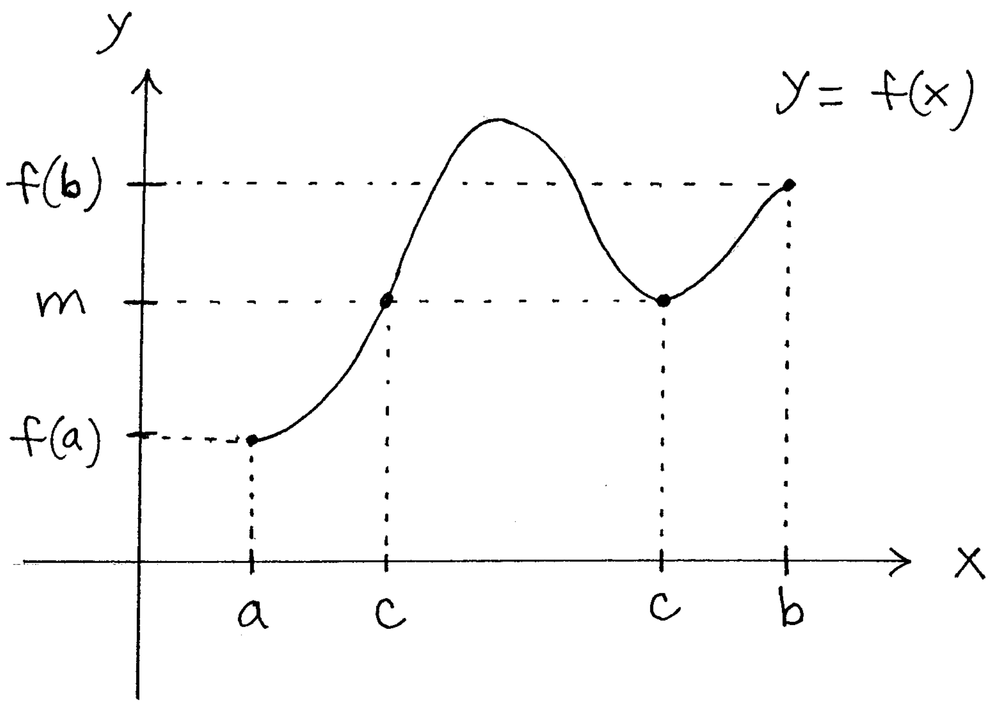
$$f(c) \leq f(x) \quad \text{for all } x \text{ in } [a, b] \ .$$



REMARK : If f is not continuous or if the interval is not closed, then the conclusions of the previous theorems are not guaranteed, but may sometimes be true.

Intermediate Value Theorem (IMVT) : Let f be a continuous function on the closed interval $[a, b]$. Let m be any number between $f(a)$ and $f(b)$. Then there is at least one number c in $[a, b]$ which satisfies

$$f(c) = m .$$



When applying the IMVT to a problem, the following five steps must be clearly established:

1. Define a function f.
2. Define a number m.
3. Establish that f is continuous.
4. Choose an interval $[a, b]$.
5. Indicate that m is between $f(a)$ and $f(b)$.

Once these five steps have been established, the conclusion of the IMVT can be invoked.