Math 21A
Vogler

The Mean Value Theorem

Defn

1) A function f takes on its minimum value at $x=c$ if $f(x) > f(c)$ for $x \neq c$.

2) A function f takes on its maximum value at $x=c$ if $f(x) < f(c)$ for $x \neq c$.

Thm Assume f is differentiable and takes on its maximum of minimum value at $x=c$. Then $f'(c)=0$.

Extreme Value Thm

If f is continuous on a closed interval $[a,b]$, then f obtains both a maximum & minimum value in $[a,b]$.

Rolle's Thm

Assume f is continuous on closed interval $[a,b]$ and differentiable on open interval (a,b) with $f(a)=f(b)$. Then there is at least one number c, $a<c<b$ such that

$$f'(c)=0$$

Mean Value Theorem (MVT)

Assume f is continuous on closed interval $[a,b]$ and differentiable on open interval (a,b). Then there is at least one number c, $a<c<b$, such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Thm Assume $f'(x)=0$ for all values of x in closed interval $[a,b]$. Then $f(x)=k$, a constant function on $[a,b]$.

Math 21A
Vogler

Mean Value Theorem-- An Application

Let s(t) be the total miles you have driven after t hours on Interstate 80 from Davis to San Francisco. Assume also that $s(1/3) = 10$ miles and $s(1.2) = 82$ miles. At a checkpoint along the way you admit these facts to a highway patrol officer, who, while a student at UC Davis, took Math 21A. The officer makes a few quick calculations, smiles, mumbles something about the Mean Value Theorem, then politely prepares to write you a traffic ticket. Explain why.