Procedure for Solving Related Rates Problems

1) Draw a picture (if one is not provided) and define the variables. Assign ALL numbers to variables. Remember, rates ALWAYS correspond to a derivative.

2) Determine and CLEARLY STATE goal of the problem, which is ALWAYS finding a rate/derivative.

3) Build your related rates equation. Usually, this involves the implicit differentiation of an equation from geometry. Note: If there's only ONE rate/derivative in your related rates equation, you did something wrong!

4) Isolate the goal rate in 2) and make sure you have the numbers for all the other variables and rates/derivatives. If not, you have to find the missing numbers, usually by solving another equation.

5) Plug in numbers and solve, making sure to INCLUDE UNITS. Think about the solution and its plausibility!
Consider the following triangle:
Assume edge $y$ is decreasing at a rate of 2 ft/min. At what rate is the area of triangle changing when $y = 8$ ft?

Let $A = \text{Area of Triangle}$

Goal: Find $\frac{dA}{dt}$ when $y = 8$ ft and $\frac{dy}{dt} = -2$ ft/min

Assume $x(t), y(t), A(t)$

$A = \frac{1}{2}xy$ $\Rightarrow$ $\frac{d}{dt}[A = \frac{1}{2}xy] = \frac{dA}{dt} = \frac{1}{2}x \frac{dy}{dt} + \frac{1}{2} \frac{dy}{dt} \frac{dx}{dt}$ (*)

Problem: Need to find $x$ & $\frac{dx}{dt}$

Recall: $x^2 + y^2 = 10^2$ $\Rightarrow$ $x^2 = 100 - 64 = 36$ $\Rightarrow$ $x = 6$ ft
Also, $\frac{d}{dt}[x^2 + y^2 = 10^2] = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$\Rightarrow$ \[ \frac{dx}{dt} = -\frac{y \frac{dy}{dt}}{x} = -\frac{8(-2)}{6} = \frac{8}{3} \text{ ft/min} \]

Hence, (*) becomes $\frac{dA}{dt} = \frac{1}{2}(6)(-2) + \frac{1}{2}(\frac{8}{3}) 8 = \frac{14}{3} \text{ ft/min}$