Section 2.5

16.) $y = \dfrac{x+3}{x^2-3x-10}$; $y = x+3$ and $y = x^2-3x-10$ are continuous for all values of x since they are polynomials; therefore, since $y = \dfrac{x+3}{x^2-3x-10}$ is the <u>quotient</u> of these functions, it is <u>continuous for all values of x</u> except where $x^2-3x-10 = (x-5)(x+2) = 0$, i.e., <u>except for $x = 5$ and $x = -2$</u>.

20.) $y = \dfrac{x+2}{\cos x}$; $y = x+2$ is continuous for all values of x since it is a polynomial; $y = \cos x$ is continuous for all values of x since it is a well-known trig function; therefore, since $y = \dfrac{x+2}{\cos x}$ is the <u>quotient</u> of these functions, it is <u>continuous for all values of x</u> except where $\cos x = 0$, i.e., <u>except for $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$</u>

26.) $y = (3x-1)^{1/4}$; let $f(x) = x^{1/4}$, which is continuous for $x \geq 0$, and let $g(x) = 3x-1$, which is continuous for all values of x since it is a polynomial; since $y = (3x-1)^{1/4} = f(3x-1) = f(g(x))$ is <u>functional</u> <u>composition</u>, it is continuous for all x-values for which $3x - 1 \geq 0$, i.e., for $x \geq 1/3$.

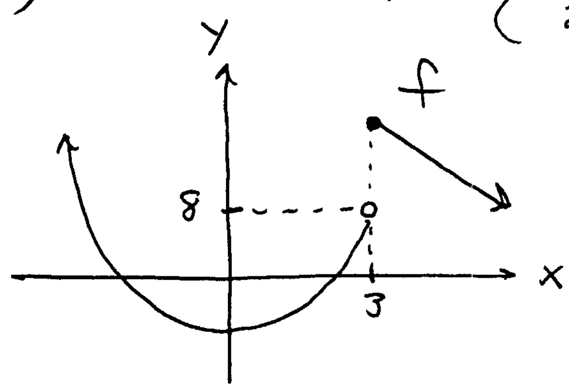
42.) $g(x) = \dfrac{x^2 - 16}{x^2 - 3x - 4}$ then

$$\lim_{x \to 4} g(x) = \lim_{x \to 4} \frac{x^2 - 16}{x^2 - 3x - 4}$$

$$\text{"}\tfrac{0}{0}\text{"} \lim_{x \to 4} \frac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}(x+1)} = \frac{8}{5} ,$$ so

define $g(4) = 8/5$ and g will be continuous at $x = 4$.

43.) Let $f(x) = \begin{cases} x^2 - 1 , & \text{if } x < 3 \\ 2ax , & \text{if } x \geq 3 \end{cases}$



$y = x^2 - 1$ is continuous for $x < 3$ (polynomial); $y = 2ax$ is continuous for $x > 3$ (line); make f continuous at $x = 3$ by forcing limits to be equal:

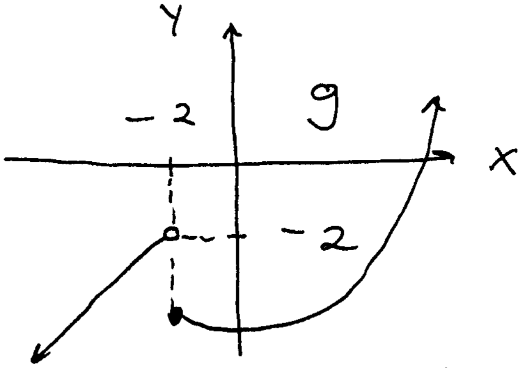
$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (x^2 - 1) = 9 - 1 = 8 ,$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (2ax) = 6a ,$$ so

$6a = 8 \longrightarrow$ $a = 4/3$.

44.) Let $g(x) = \begin{cases} x, & \text{if } x < -2 \\ bx^2, & \text{if } x \geq -2 \end{cases}$

$Y = x$ is continuous for $x < -2$ (line), $Y = bx^2$ is continuous for $x > -2$ (parabola); make g continuous at $x = -2$ by forcing limits to be equal:



$\lim_{x \to -2^-} g(x) = \lim_{x \to -2^-} x = -2,$

$\lim_{x \to -2^+} g(x) = \lim_{x \to -2^+} bx^2 = 4b,$

so $4b = -2 \rightarrow b = -1/2$.

I.) Prove $x^3 = x + 2$ is solvable:

$x^3 = x + 2 \rightarrow x^3 - x - 2 = 0$, so let $\underline{f(x) = x^3 - x - 2}$ and $\underline{m = 0}$; note that $f(1) = -2 < 0$ and $f(2) = 4 > 0$ so $m = 0$ is <u>between</u> $f(1)$ and $f(2)$; use the interval $\underline{[1, 2]}$; f is a <u>continuous</u> function on $[1, 2]$ since it is a polynomial. By the IMVT it follows that there is a number c, $1 \leq c \leq 2$, so that $f(c) = m$, i.e.,

$c^3 - c - 2 = 0$, and the

original equation is solvable.

II.) Prove $2 + \sin x = x$ is solvable:

$2 + \sin x = x \rightarrow 2 - x + \sin x = 0$ so let $\underline{f(x) = 2 - x + \sin x}$ and $\underline{m = 0}$; f is <u>continuous</u> for all values of x since it is the sum of continuous functions ($y = 2 - x$, a line, and $y = \sin x$, a well-known trig function); note that $f(0) = 2 > 0$ and $f(\pi) = 2 - \pi - \sin \pi = 2 - \pi < 0$, so $m = 0$ is <u>between</u> $f(0)$ and $f(\pi)$; use the interval $\underline{[0, \pi]}$. By the IMVT it follows that there is a number c, $0 \leq c \leq \pi$, so that $f(c) = m$, i.e.,

$$2 - c + \sin c = 0,$$

and the original equation is solvable.

Worksheet 1

1.) a.) $f(x) = \begin{cases} x^2+3 & \text{if } x \neq -1 \\ 2 & \text{if } x = -1 \end{cases}$; at $x=-1$:

i.) $f(-1) = 2$

ii.) $\lim\limits_{x \to -1} f(x) = \lim\limits_{x \to -1} (x^2+3) = 1+3 = 4$

iii.) $\lim\limits_{x \to -1} f(x) = 4 \neq 2 = f(-1)$ so

f is NOT continuous at $x=-1$

b.) $g(x) = \begin{cases} x+1 & \text{if } x \geq 0 \\ 2-x^2 & \text{if } x < 0 \end{cases}$; at $x=0$:

i.) $g(0) = 0+1 = 1$

ii.) $\lim\limits_{x \to 0^+} g(x) = \lim\limits_{x \to 0^+} (x+1) = 1$;

$\lim\limits_{x \to 0^-} g(x) = \lim\limits_{x \to 0^-} (2-x^2) = 2$; so

$\lim\limits_{x \to 0} g(x)$ DNE ; thus

g is NOT continuous at $x=0$

c.) $f(x) = \begin{cases} x-2 & \text{if } x > 1 \\ 0 & \text{if } x = 1 \\ -x & \text{if } x < 1 \end{cases}$; at $x=1$:

i.) $f(1) = 0$

ii.) $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} (x-2) = -1$;

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (-x) = -1 \text{ ; so}$$

$$\lim_{x\to 1} f(x) = -1$$

iii.) $\lim_{x\to 1} f(x) = -1 \neq 0 = f(1)$, so

f is NOT continuous at $x=1$.

2.) a.) $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$ is continuous for all values of x since it is a polynomial.

b.) $g(x) = \dfrac{\sin x}{x^2+4}$; $Y = \sin x$ (well known) and $Y = x^2+4$ (polynomial) are continuous for all values of x ; so the QUOTIENT $g(x) = \dfrac{\sin x}{x^2+4}$ is continuous for all values of x since $x^2+4 \neq 0$.

c.) $f(x) = \dfrac{x+3}{x^2-4}$; $Y = x+3$ and $Y = x^2-4$ (polynomials) are continuous for all values of x ; the QUOTIENT $f(x) = \dfrac{x+3}{x^2-4}$ is continuous for all values of x except where $x^2-4=0$, i.e., except when $x = \pm 2$.

d.) $g(x) = \cos(x^3 - x)$; $f(x) = \cos x$ (well known) and $k(x) = x^3 - x$ (polynomial) are continuous for all values of x ; so the COMPOSITION

$f(k(x)) = f(x^3 - x) = \cos(x^3 - x)$ is

continuous for all values of x.

3.) a.) $$f(x) = \begin{cases} \dfrac{x^2 - 7x + 6}{x - 6} & \text{if } x \neq 6 \\ A & \text{if } x = 6 \end{cases} ;$$

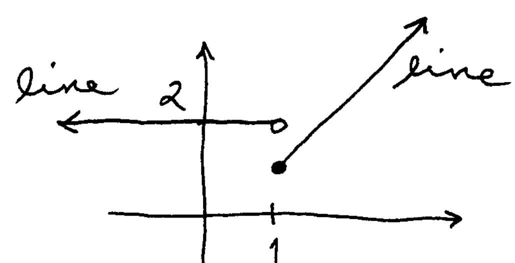
$$\lim_{x \to 6} f(x) = \lim_{x \to 6} \frac{x^2 - 7x + 6}{x - 6} = \lim_{x \to 6} \frac{\cancel{(x-6)}(x-1)}{\cancel{x-6}}$$

$= 6 - 1 = 5$ and $f(6) = A$ so choose $A = 5$

b.) $$f(x) = \begin{cases} A^2 x - A & \text{if } x \geq 1 \\ 2 & \text{if } x < 1 \end{cases} ;$$

Require that

$\lim_{x \to 1^+} f(x) = 2 \rightarrow$

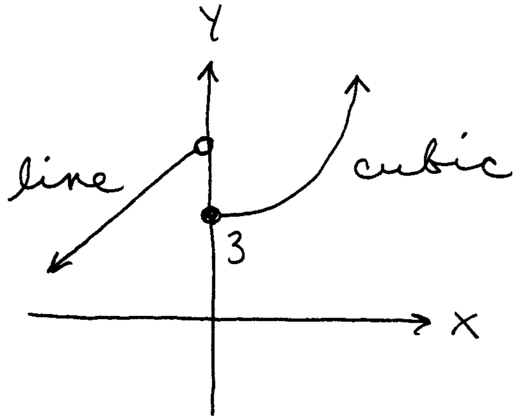


$\lim_{x \to 1^+} (A^2 x - A) = 2 \rightarrow$

$A^2 - A = 2 \rightarrow A^2 - A - 2 = 0 \rightarrow$

$(A-2)(A+1) = 0 \rightarrow$ $A = 2$ or $A = -1$.

c.) $$f(x) = \begin{cases} \dfrac{A + x}{A + 1} & \text{if } x < 0 \\ Ax^3 + 3 & \text{if } x \geq 0 \end{cases} ;$$



Require that $\lim_{x \to 0^-} f(x) = 3 \rightarrow$

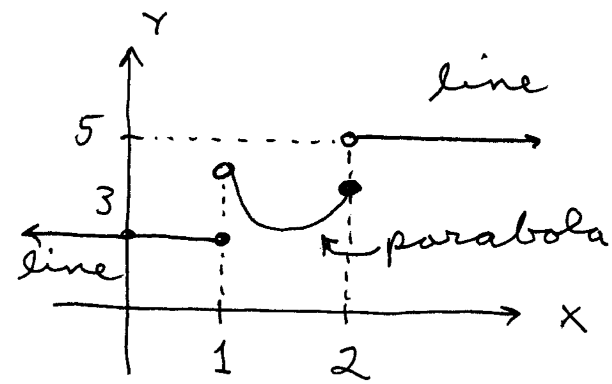
$$\lim_{x \to 0^-} \frac{A+x}{A+1} = 3 \rightarrow$$

$$\frac{A+0}{A+1} = 3 \rightarrow A = 3A + 3 \rightarrow -3 = 2A \rightarrow$$

$$A = -3/2$$

d.) $f(x) = \begin{cases} 3 & \text{if } x \leq 1 \\ Ax^2 + B & \text{if } 1 < x \leq 2 \\ 5 & \text{if } x > 2 \end{cases}$

Require <u>TWO</u> conditions:



i.) $\lim_{x \to 1^+} f(x) = 3 \rightarrow$

$\lim_{x \to 1^+} (Ax^2 + B) = 3 \rightarrow$

$\boxed{A + B = 3}$ AND

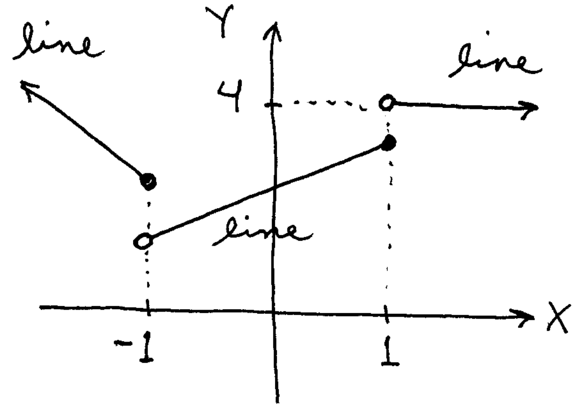
ii.) $\lim_{x \to 2^-} f(x) = 5 \rightarrow \lim_{x \to 2^-} (Ax^2 + B) = 5 \rightarrow$

$\boxed{4A + B = 5}$; solve system:

$B = 3 - A \rightarrow (\text{SUB}) \rightarrow 4A + (3 - A) = 5 \rightarrow$

$3A = 2 \rightarrow$ $A = 2/3 ,\ B = 7/3$

e.) $f(x) = \begin{cases} Ax - B & \text{if } x \leq -1 \\ 2x + 3A + B & \text{if } -1 < x \leq 1 \\ 4 & \text{if } x > 1 \end{cases}$



Require $\underline{\underline{\text{Two}}}$ conditions :

i.) $\lim\limits_{x \to 1^-} f(x) = 4 \rightarrow$

$\lim\limits_{x \to 1^-} (2x + 3A + B) = 4 \rightarrow$

$2 + 3A + B = 4 \rightarrow \boxed{3A + B = 2}$ AND

ii.) $\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^+} f(x) \rightarrow$

$\lim\limits_{x \to -1^-} (Ax - B) = \lim\limits_{x \to -1^+} (2x + 3A + B) \rightarrow$

$-A - B = -2 + 3A + B \rightarrow 2 = 4A + 2B \rightarrow$

$\boxed{2A + B = 1}$; solve system :

$3A + B = 2 \rightarrow B = 2 - 3A \rightarrow$ (SUB) $\rightarrow$

$2A + (2 - 3A) = 1 \rightarrow -A = -1 \rightarrow$

$A = 1, \ B = -1$