

Asymptotes for Rational Functions

Defn A line is an asymptote for a graph if the distance between the line and the graph approaches zero as we move out farther and farther along the line.

Facts 1) A Vertical Asymptote (V.A.) @ $x=a$ occurs when you divide by zero when $x=a$ & the numerator is non-zero.

2) A Horizontal Asymptote (H.A.) @ $y=L$ occurs when the degree of the numerator is less than or equal to degree of the denominator.

Two cases: a) degrees are equal $\Rightarrow L$ is ratio of coefficients

Ex $f(x) = \frac{5x^2 - 4x + 3}{10x^2 - 5x - 7} \Rightarrow y = L = \frac{5}{10} = \frac{1}{2}$ is H.A.

b) degree of numerator is smaller $\Rightarrow L = 0$

Ex $f(x) = \frac{3x - 4}{5x^2 + 3x} \Rightarrow y = L = 0$ is H.A.

3) A Tilted Asymptote (T.A.) occurs when degree of numerator is exactly one greater than degree of denominator.

Ex $f(x) = \frac{x^2 - 3}{x - 2} \nearrow \begin{matrix} = x + 2 + \frac{1}{x - 2} \\ \text{Through polynomial division.} \end{matrix} \Rightarrow \text{T.A. is } y = x + 2$

Note: These facts require Calculus 1 to prove.