Math 12
Vogler

Exponential Functions

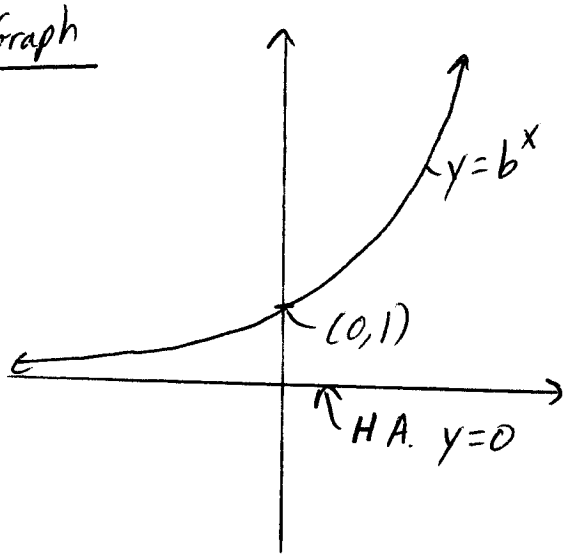
Defn An **exponential function** is any function of the form $f(x) = b^x$ where b is a constant not equal to 1.

Notes: 1) Domain: All real numbers

2) Range: $y > 0$

3) b^x has a horizontal asymptote at $y = 0$.

Graph



Rules

1) $b^m b^n = b^{m+n}$

2) $\frac{b^m}{b^n} = b^{m-n}$

3) $(b^m)^n = b^{mn}$

4) $\frac{1}{b^m} = b^{-m}$

The number e (L. Euler, 1748)

Let $f(x) = (1+\frac{1}{x})^x$ If the number x gets larger and larger, what happens to $f(x)$?

Using a calculator

x	$f(x) = (1+\frac{1}{x})^x$
1	2.000000000
10	2.593742460
100	2.704813829
1,000	2.716923932
10,000	2.718145929
⋮	⋮
1,000,000	2.718280469

which leads to the following:

Defn Let $e = \lim_{x\to\infty} (1+\frac{1}{x})^x \Rightarrow \boxed{e \approx 2.71828}$

↖ (learned in Calculus 1)

Note: The function $y = e^x$ has many 'nice' properties which you will learn about in Calculus.

Ex Assume a deposit of \$1,000 earns 7.5% annual interest compounded continuously (approximately 'daily'). The amount in the account after t years is

$$A(t) = 1{,}000\, e^{0.075t}$$

so if $t = 10$ years the amount becomes

$$A(10) = 1{,}000\, e^{0.075(10)} \approx \$2116.84$$