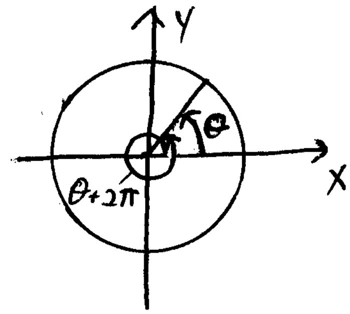
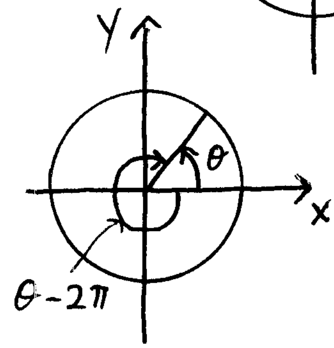
Math 12
Vogler

Periodicity & Shift Trig. Identities

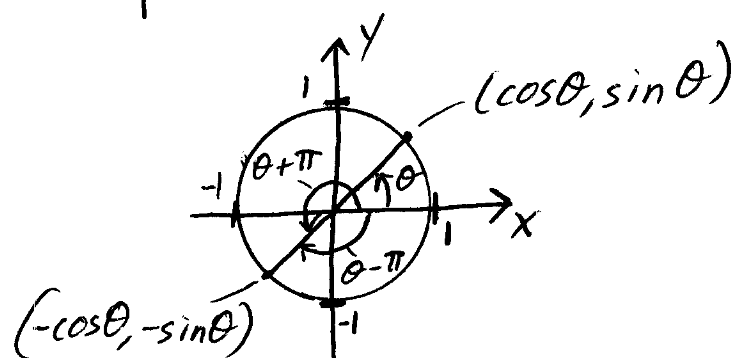
1) $\cos(\theta + 2\pi) = \cos\theta$
2) $\sin(\theta + 2\pi) = \sin\theta$



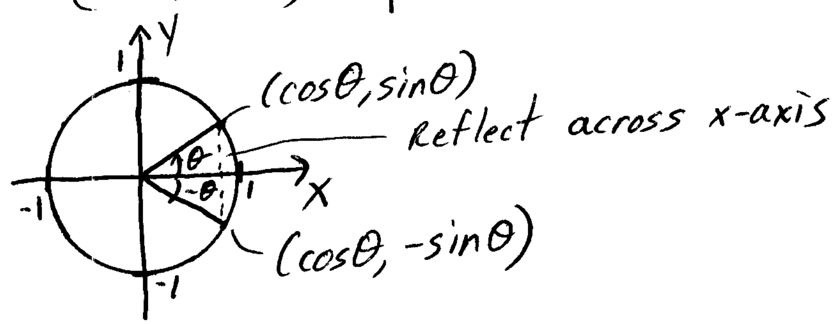
3) $\cos(\theta - 2\pi) = \cos\theta$
4) $\sin(\theta - 2\pi) = \sin\theta$



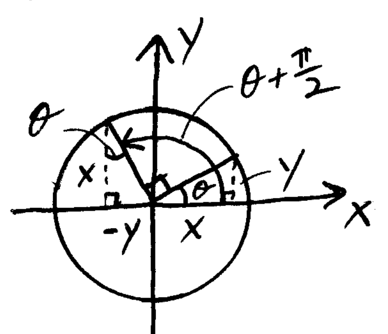
5) $\sin(\theta + \pi) = -\sin\theta$
6) $\cos(\theta + \pi) = -\cos\theta$
7) $\sin(\theta - \pi) = -\sin\theta$
8) $\cos(\theta - \pi) = -\cos\theta$



9) $\sin(-\theta) = -\sin\theta$
10) $\cos(-\theta) = +\cos\theta$

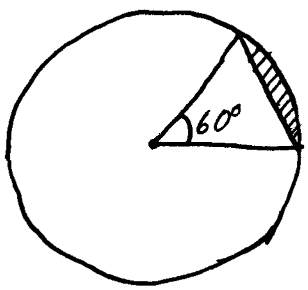


11) $\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$
12) $\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$



Recall: $x = \cos\theta$
$y = \sin\theta$

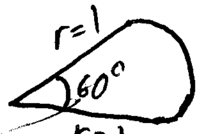
Ex Consider the following unit circle. Find the area of shaded region.



Notice that

Shaded Area =  − 

(1) (2)

(1) 

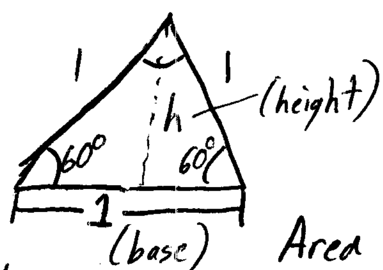
Recall: Area of Sector $= \frac{1}{2}\theta r^2$

In radians

$\Rightarrow$ Area $= \frac{1}{2} \frac{\pi}{3} 1^2 = \frac{\pi}{6}$

$\left(60^\circ = 60^\circ \cdot \frac{\pi \text{ rads}}{180^\circ} = \frac{\pi}{3} \text{ rads}\right)$

(2) 

$\Rightarrow$ 

To find h:

$\sin 60^\circ = \frac{h}{1} \Rightarrow h = \frac{\sqrt{3}}{2}$

Area of Triangle $= \frac{1}{2} b \cdot h$

$= \frac{1}{2} \cdot 1 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

(By properties of triangles, the other angles must equal because their corresponding sides equal. Hence, this is an equalateral triangle)

Hence, $\boxed{\text{Shaded Area} = \frac{\pi}{6} - \frac{\sqrt{3}}{4}} \simeq 0.091$