

Section 5.3

- 9) a.) $9 = 3^2$ means $\log_3 9 = 2$
 b.) $1000 = 10^3$ means $\log_{10} 1000 = 3$
 c.) $7^3 = 343$ means $\log_7 343 = 3$
 d.) $\sqrt{2} = 2^{1/2}$ means $\log_2 \sqrt{2} = 1/2$

- 10) a.) $\frac{1}{125} = 5^{-3}$ means $\log_5 \frac{1}{125} = -3$
 b.) $e^0 = 1$ means $\log_e 1 = 0$
 c.) $5^x = 6$ means $\log_5 6 = x$
 d.) $e^{3t} = 8$ means $\log_e 8 = 3t$

- 11) a.) $\log_2 32 = 5$ means $2^5 = 32$
 b.) $\log_{10} 1 = 0$ means $10^0 = 1$
 c.) $\log_e \sqrt{e} = 1/2$ means $e^{1/2} = \sqrt{e}$
 d.) $\log_3 (1/81) = -4$ means $3^{-4} = 1/81$
 e.) $\log_t u = v$ means $t^v = u$

15a) $\log_5 30 = z$ means $5^z = 30$ and
 $5^2 = 25$ so $z > 2$; $\log_8 60 = w$ means
 $8^w = 60$ and $8^2 = 64$ so $w < 2$; thus
 $w < 2 < z$ so $\log_8 60 < \log_5 30$

- 17) a.) $\log_9 27 = \log_9 3^3 = \log_9 (3^2)^{3/2}$
 $= \log_9 9^{3/2} = 3/2$
 b.) $\log_4 1/32 = \log_4 2^{-5} = \log_4 (2^2)^{-5/2}$
 $= \log_4 4^{-5/2} = -5/2$

$$c.) \log_5 5\sqrt{5} = \log_5 5^{1.5^{1/2}} = \log_5 5^{1+1/2} \\ = \log_5 5^{3/2} = 3/2$$

$$18) a.) \log_{25} (1/625) = \log_{25} 25^{-2} = -2$$

$$b.) \log_{16} (1/64) = \log_{16} 64^{-1} = \log_{16} (4^3)^{-1} \\ \log_{16} 4^{-3} = \log_{16} (4^2)^{-3/2} = \log_{16} 16^{-3/2} = -3/2$$

$$c.) \log_{10} 10 = 1$$

$$d.) \log_2 8\sqrt{2} = \log_2 2^3 \cdot 2^{1/2} = \log_2 2^{3+1/2} = \log_2 2^{7/2} = 7/2$$

$$19) a.) \log_4 X = -2 \text{ means } 4^{-2} = X \rightarrow X = 1/16$$

$$b.) \ln X = -2 \text{ means } \log_e X = -2 \text{ means } \\ e^{-2} = X \rightarrow X = 1/e^2 \approx 0.14$$

$$20) a.) \log_5 X = e \text{ means } 5^e = X \approx 79.43$$

$$b.) \ln X = -e \text{ means } \log_e X = -e \text{ means } \\ e^{-e} = X \rightarrow X = 1/e^e \approx 0.07$$

$$21) a.) Y = \log_4 5X \text{ so } 5X > 0 \rightarrow X > 0/5 \rightarrow \\ X > 0 \text{ so Domain: all } X > 0$$

$$b.) Y = \log_{10} (3-4X) \text{ so } 3-4X > 0 \rightarrow -4X > -3 \rightarrow \\ X < -3/-4 \rightarrow X < 3/4 \text{ so Domain: all } X < 3/4$$

$$c.) Y = \ln (X^2 - 25) \text{ so } X^2 - 25 > 0 \rightarrow$$

$$(X-5)(X+5) > 0$$

so

$$\begin{array}{ccccccc} & + & 0 & - & 0 & + & \\ & | & & & | & & \\ X & = & -5 & & & & X = 5 \end{array}$$

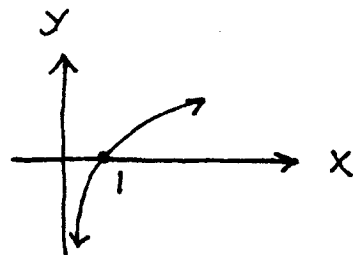
$$\text{Domain: } X > 5 \text{ or } X < -5$$

22) a.) $y = \ln(2-x-x^2)$ so $2-x-x^2 > 0 \rightarrow$
 $x^2+x-2 < 0 \rightarrow (x-1)(x+2) < 0$ $\begin{array}{ccccccc} & + & 0 & - & 0 & + & \\ & & | & & | & & \\ & & x=-2 & & x=1 & & \end{array}$
 so Domain: $-2 < x < 1$

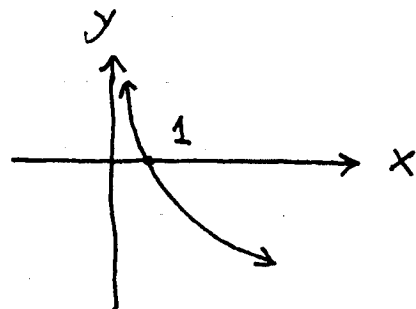
b.) $y = \log_{10} \frac{2x+3}{x-5}$ so $\frac{2x+3}{x-5} > 0 \rightarrow$
 $\begin{array}{ccccccc} & + & 0 & - & \text{No} & + & \\ & & | & & | & & \\ & & x=-3/2 & & x=5 & & \end{array}$ Domain: $x > 5$ or $x < -3/2$

23) $A = (0, 1)$, $B = (1, 0)$, $C = (-1, e^{-1})$,
 $D = (e^{-1}, -1)$

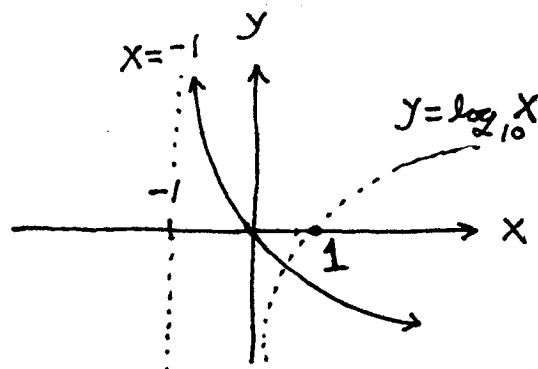
26) a.) $y = \ln x$;
 Domain: all $x > 0$;
 Range: all y -values ;
 $x=0$: impossible ; $y=0$: $x=1$;
 asymptote: $x=0$



b.) $y = -\ln x$;
 Domain: all $x > 0$;
 Range: all y -values ;
 $x=0$: impossible ; $y=0$: $x=1$;
 asymptote: $x=0$



28) $y = -\log_{10}(x+1)$;
 start with $y = \log_{10} x$;
 translate left 1 ;
 reflect in the x -axis ;



Domain : all $x > -1$; Range : all y -values;
 $x=0 : y=0$; $y=0 : x=0$;
asymptote : $x=-1$

35) a.) $\ln e^4 = \log_e e^4 = 4$
b.) $\ln(1/e) = \log_e e^{-1} = -1$
c.) $\ln \sqrt{e} = \ln e^{1/2} = 1/2$

36) a.) $\ln e = \log_e e = 1$
b.) $\ln e^{-2} = \log_e e^{-2} = -2$
c.) $(\ln e)^{-2} = (1)^{-2} = \frac{1}{1^2} = \frac{1}{1} = 1$

(Extra) $10^x = 145 \rightarrow \log_{10} 10^x = \log_{10} 145 \rightarrow$
 $x = \log_{10} 145 \approx 2.16$

40) $(10^x)^2 = 40 \rightarrow 10^{2x} = 40 \rightarrow$
 $\log_{10} 10^{2x} = \log_{10} 40 \rightarrow 2x = \log_{10} 40 \rightarrow$
 $x = \frac{1}{2} \log_{10} 40 \approx 0.80$

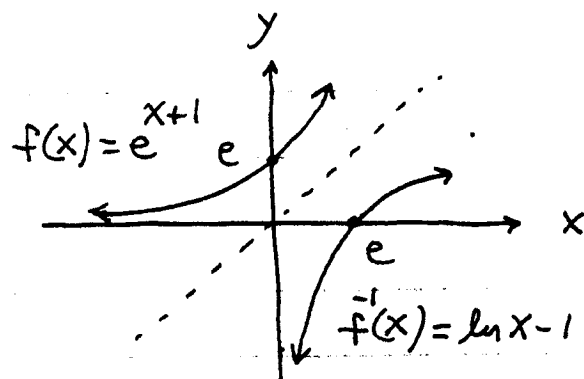
42) $e^{t-1} = 16 \rightarrow \ln e^{t-1} = \ln 16 \rightarrow$
 $t-1 = \ln 16 \rightarrow t = 1 + \ln 16 \approx 3.77$

43) $e^{1-4t} = 12.405 \rightarrow \ln e^{1-4t} = \ln 12.405 \rightarrow$
 $1-4t = \ln 12.405 \rightarrow -4t = \ln 12.405 - 1 \rightarrow$
 $t = -\frac{1}{4} (\ln 12.405 - 1) \approx -0.38$

$$\begin{aligned}
 53) \quad a.) \quad pH &= -\log [H^+] = -\log [3 \times 10^{-4}] \\
 &= -\{\log 3 + \log 10^{-4}\} = -\{0.48 - 4\} \\
 &= -\{-3.52\} = 3.52 < 7 \text{ so acid}
 \end{aligned}$$

$$b.) \quad pH = -\log [H^+] = -\log [1] = -(0) = 0$$

$$\begin{aligned}
 65) \quad y &= e^{x+1} \\
 (\text{switch variables} \\
 \text{and solve for } y.) &\rightarrow \\
 x = e^{y+1} &\rightarrow \ln x = \ln e^{y+1} \\
 \rightarrow \ln x = y+1 &\rightarrow y = \ln x - 1 \\
 \rightarrow f^{-1}(x) &= \ln x - 1
 \end{aligned}$$



$$\begin{aligned}
 &\rightarrow f^{-1}(x) = \ln x - 1 \\
 x=0 : f^{-1}(0) &\text{ impossible ; } y=0 : \ln x - 1 = 0 \rightarrow \\
 \ln x = 1 &\rightarrow x = e ; \text{ asymptote : } x=0
 \end{aligned}$$

$$\begin{aligned}
 73) \quad a.) \quad f(x) &= \ln(\ln x) \rightarrow \\
 \ln x > 0 &\rightarrow e^{\ln x} > e^0 \rightarrow x > 1 \text{ so} \\
 \text{Domain : all } x &> 1
 \end{aligned}$$

$$\begin{aligned}
 b.) \quad y &= \ln(\ln x) \rightarrow x = \ln(\ln y) \rightarrow \\
 e^x &= e^{\ln(\ln y)} \rightarrow e^x = \ln y \rightarrow \\
 e^{e^x} &= e^{\ln y} \rightarrow e^{e^x} = y \rightarrow f^{-1}(x) = e^{e^x}
 \end{aligned}$$

$$\begin{aligned}
 74) \quad g(x) &= \ln(\ln(\ln x)) \rightarrow \ln(\ln x) > 0 \rightarrow \\
 e^{\ln(\ln x)} &> e^0 \rightarrow \ln x > 1 \rightarrow \\
 e^{\ln x} &> e^1 \rightarrow x > e \text{ so}
 \end{aligned}$$

$$\text{Domain : all } x > e$$