Section 5.4

1) $\log_{10} 70 - \log_{10} 7 = \log_{10}\left(\frac{70}{7}\right) = \log_{10} 10 = 1$

8) $e^{\ln 3} + e^{\ln 2} - e^{\ln e} = 3 + 2 - e^1 = 5 - e$

9) $2^{\log_2 5} - 3\log_5 5^{1/3} = 5 - 3\left(\frac{1}{3}\right) = 5 - 1 = 4$

13) $\log_5 6 + \log_5\left(\frac{1}{3}\right) + \log_5 10$

$= \log_5(6 \cdot \frac{1}{3}) + \log_5 10 = \log_5 2 + \log_5 10$

$= \log_5(2 \cdot 10) = \log_5 20$

15) a.) $\ln 3 - 2\ln 4 + \ln 32 = \ln 3 - \ln 4^2 + \ln 32$

$= \ln 3 - \ln 16 + \ln 32 = \ln\left(\frac{3}{16}\right) + \ln 32$

$= \ln\left(\frac{3}{16} \cdot 32\right) = \ln 6$

b.) $\ln 3 - 2(\ln 4 + \ln 32) = \ln 3 - 2\ln(4 \cdot 32)$

$= \ln 3 - \ln 128^2 = \ln 3 - \ln 16,384 = \ln\left(\frac{3}{16,384}\right)$

16a) $\log_{10}(x^2-16) - 3\log_{10}(x+4) + 2\log_{10} x$

$= \log_{10}(x^2-16) - \log_{10}(x+4)^3 + \log_{10} x^2$

$= \log_{10} \frac{x^2-16}{(x+4)^3} + \log_{10} x^2 = \log_{10} \frac{(x-4)(x+4)}{(x+4)^3 2} + \log_{10} x^2$

$= \log_{10} \frac{x-4}{(x+4)^2} + \log_{10} x^2 = \log_{10} \frac{(x-4)\cdot x^2}{(x+4)^2}$

21a) $\log_{10} \frac{x^2}{1+x^2} = \log_{10} x^2 - \log_{10}(1+x^2)$

$= 2\log_{10} x - \log_{10}(1+x^2)$

22b) $\ln \frac{x(4x+1)^{1/3}}{(2x-1)^{1/2}} = \ln x \cdot (4x+1)^{1/3} - \ln(2x-1)^{1/2}$

$= \ln x + \ln(4x+1)^{1/3} - \frac{1}{2}\ln(2x-1)$

$= \ln x + \frac{1}{2}\ln(4x+1) - \frac{1}{2}\ln(2x-1)$

27a) $\log_b 6 = \log_b 2\cdot 3 = \log_b 2 + \log_b 3 = A+B$

28d) $\log_b 0.3 = \log_b \frac{3}{10} = \log_b 3 - \log_b 10$

$= \log_b 3 - \log_b 2\cdot 5 = \log_b 3 - (\log_b 2 + \log_b 5)$

$= B - A - C$

29c) $\log_b (5/9) = \log_b 5 - \log_b 3^2$

$= \log_b 5 - 2\log_b 3 = C - 2B$

31a) $\log_3 b = \frac{\log_b b}{\log_b 3} = \frac{1}{B}$

32a) $\log_{b^2} 5 = \dfrac{\log_b 5}{\log_b b^2} = \dfrac{c}{2}$

34a) $\log_{5b} 1.2 = \dfrac{\log_b (12/10)}{\log_b (5b)} = \dfrac{\log_b (6/5)}{\log_b (5b)}$

$= \dfrac{\log_b (2\cdot 3) - \log_b 5}{\log_b 5 + \log_b b} = \dfrac{\log_b 2 + \log_b 3 - \log_b 5}{\log_b 5 + \log_b b}$

$= \dfrac{A + B - C}{C + 1}$

43) $5 = 2\cdot e^{2x-1} \rightarrow \ln 5 = \ln 2\cdot e^{2x-1} \rightarrow$

$\ln 5 = \ln 2 + \ln e^{2x-1} \rightarrow \ln 5 - \ln 2 = 2x - 1 \rightarrow$

$\ln (5/2) + 1 = 2x \rightarrow x = \frac{1}{2}(1 + \ln (5/2))$

46) $5^{3x-1} = 27 \rightarrow \ln 5^{3x-1} = \ln 27 \rightarrow$

$(3x-1)\ln 5 = \ln 27 \rightarrow (3\ln 5)x - \ln 5 = \ln 27 \rightarrow$

$(3\ln 5) = \ln 27 + \ln 5 \rightarrow x = \dfrac{\ln (27\cdot 5)}{\ln 5^3} \rightarrow$

$x = \dfrac{\ln 135}{\ln 125}$

48) $10^{2x+3} = 280 \rightarrow \ln 10^{2x+3} = \ln 280 \rightarrow$

$(2x+3)\ln 10 = \ln 280 \rightarrow (2\ln 10)x + 3\ln 10 = \ln 280 \rightarrow$

$(\ln 10^2)x = \ln 280 - \ln 10^3 \rightarrow x = \dfrac{\ln (280/1000)}{\ln 100} \rightarrow$

$x = \dfrac{\ln (7/25)}{\ln 100}$

62) a.) $\log(10+1) = \log 10 + \log 1 \rightarrow$

$\log 11 = 1+0 \rightarrow \log 11 = 1 \rightarrow 1.04 = 1$ (FALSE)

b.) $\frac{\log 10}{\log 10} = \log 10 - \log 10 \rightarrow \frac{1}{1} = 1-1 \rightarrow 1 = 0$ (FALSE)

c.) $(\log 10)(\log 10) = \log 10 + \log 10 \rightarrow (1)(1) = 1+1 \rightarrow 1 = 2$ (FALSE)

d.) $(\log 10)^2 = 2(\log 10) \rightarrow (1)^2 = 2(1) \rightarrow 1 = 2$ (FALSE)

66) $\left[\frac{1}{\pi}\ln(640320^3+744)\right]^2 = 163$?

78) $(\log_2 3)(\log_3 4)(\log_4 5) = \frac{\ln 3}{\ln 2}\cdot\frac{\ln 4}{\ln 3}\cdot\frac{\ln 5}{\ln 4} = \frac{\ln 5}{\ln 2}$

$= \log_2 5$