Section 5.5

1) $5^{x} = 3^{2x-1} \rightarrow \ln 5^{x} = \ln 3^{2x-1} \rightarrow$
$x \cdot \ln 5 = (2x-1) \cdot \ln 3 \rightarrow x \ln 5 = 2x \ln 3 - \ln 3 \rightarrow$
$x \ln 5 - 2x \ln 3 = -\ln 3 \rightarrow x(\ln 5 - 2\ln 3) = -\ln 3 \rightarrow$
$x = \frac{-\ln 3}{\ln 5 - \ln 3^2} = \frac{-\ln 3}{\ln 5 - \ln 9} = \frac{\ln 3}{\ln 9 - \ln 5} = \frac{\ln 3}{\ln(9/5)} \approx 1.869$

6) $\log_2(2x^2-4) = 5 \rightarrow 2x^2 - 4 = 2^5 \rightarrow$
$2x^2 = 4 + 32 \rightarrow x^2 = 18 \rightarrow x = \pm\sqrt{18} \approx \pm 4.243$

10) $3(2^{2x}) - 11(2^x) - 4 = 0 \rightarrow$
$3(2^x)^2 - 11(2^x) - 4 = 0 \rightarrow (3 \cdot 2^x + 1)(2^x - 4) = 0 \rightarrow$
$3 \cdot 2^x + 1 = 0$ OR $2^x - 4 = 0 \rightarrow 3 \cdot 2^x = -1 \rightarrow$
$2^x = -1/3 \rightarrow \log_2 2^x = \log_2(-1/3)$ (impossible) OR
$2^x = 4 \rightarrow \boxed{x = 2}$.

13b) $\log_3 6x = 6 \log_3 x \rightarrow \log_3 6x = \log_3 x^6 \rightarrow$
$3^{\log_3 6x} = 3^{\log_3 x^6} \rightarrow 6x = x^6 \rightarrow 0 = x^6 - 6x \rightarrow$
$0 = x(x^5 - 1) \rightarrow \cancel{x = 0}$ (impossible) OR
$\boxed{x = 1}$

17) $\log_2(\log_3 x) = -1 \rightarrow 2^{\log_2(\log_3 x)} = 2^{-1} \rightarrow$
$\log_3 x = \frac{1}{2} \rightarrow 3^{\log_3 x} = 3^{1/2} \rightarrow x = \sqrt{3}$.

19) $\ln 4 - \ln x = \frac{\ln 4}{\ln x} \rightarrow$ (mult. by $\ln x$)

$(\ln 4)(\ln x) - (\ln x)^2 = \ln 4 \rightarrow$

$0 = (\ln x)^2 - (\ln 4)(\ln x) + \ln 4 \rightarrow$

(use quadratic equation)

$$\ln x = \frac{-(-\ln 4) \pm \sqrt{(-\ln 4)^2 - 4(1)\ln 4}}{2(1)}$$

(NO solution since negative number inside square root)

23) $\log_{16} \frac{x+3}{x-1} = \frac{1}{2} \rightarrow \frac{x+3}{x-1} = 16^{1/2} = 4 \rightarrow$

$x+3 = 4(x-1) \rightarrow x+3 = 4x-4 \rightarrow 0 = 3x-7 \rightarrow$

$3x = 7 \rightarrow x = 7/3 \approx 2.333$

25c) $e^{2x} - 2e^{x} - 3 = 0 \rightarrow (e^{x})^2 - 2(e^{x}) - 3 = 0 \rightarrow$

$(e^{x}-3)(e^{x}+1) = 0 \rightarrow e^{x} = 3$ or $e^{x} = -1 \rightarrow$

$\ln e^{x} = \ln 3 \rightarrow \boxed{x = \ln 3 \approx 1.099}$ OR $\ln e^{x} = \ln(-1)$

(impossible)

26a) $4e^{6x} - 12e^{3x} + 9 = 0 \rightarrow$

$4(e^{3x})^2 - 12(e^{3x}) + 9 = 0 \rightarrow (2 \cdot e^{3x} - 3)(2 \cdot e^{3x} - 3) = 0 \rightarrow$

$2e^{3x} = 3 \rightarrow e^{3x} = 3/2 \rightarrow \ln e^{3x} = \ln(3/2) \rightarrow$

$3x = \ln(3/2) \rightarrow x = \frac{1}{3}\ln(3/2) \approx 0.135$

27) $e^{x} - e^{-x} = 1 \rightarrow e^{x}e^{x} - e^{-x}e^{x} = e^{x} \rightarrow$

$e^{x+x} - e^{-x+x} - e^{x} = 0 \rightarrow e^{2x} - e^{x} - e^{0} = 0 \rightarrow$

$(e^x)^2-(e^x)-1=0 \rightarrow$ (Use quadratic formula.)

$e^x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-1)}}{2(1)}=\frac{1\pm\sqrt{5}}{2} \rightarrow$

$e^x=\frac{1+\sqrt{5}}{2}$ OR $e^x=\frac{1-\sqrt{5}}{2} \rightarrow$

$\ln e^x=\ln\frac{1+\sqrt{5}}{2}$ OR $\ln e^x=\ln\frac{1-\sqrt{5}}{2} \rightarrow$

$\boxed{x=\ln\frac{1+\sqrt{5}}{2}\approx 0.481}$ OR $x=\ln\frac{1-\sqrt{5}}{2}\approx\ln(-0.618)$
(impossible)

28) $e^x+e^{-x}=2 \rightarrow e^x e^x+e^{-x}e^x=2e^x \rightarrow$
$(e^x)^2-2(e^x)+1=0 \rightarrow (e^x-1)(e^x-1)=0 \rightarrow e^x=1 \rightarrow x=0$

29) $2^{5x}=3^x(5^{x+3}) \rightarrow \ln 2^{5x}=\ln 3^x\cdot(5^{x+3}) \rightarrow$
$5x\cdot\ln 2=\ln 3^x+\ln 5^{x+3} \rightarrow 5\ln 2\cdot x=x\cdot\ln 3+(x+3)\ln 5 \rightarrow$
$\ln 2^5\cdot x=\ln 3\cdot x+\ln 5\cdot x+3\cdot\ln 5 \rightarrow$
$\ln 32\cdot x-\ln 3\cdot x-\ln 5\cdot x=\ln 5^3 \rightarrow x(\ln 32-\ln 3-\ln 5)=\ln 125 \rightarrow$
$x\left(\ln(32/3)-\ln 5\right)=\ln 125 \rightarrow x\cdot\ln\left(32/3\div 5\right)=\ln 125 \rightarrow$
$x\cdot\ln\left(32/3\cdot 1/5\right)=\ln 125 \rightarrow \boxed{x=\frac{\ln 125}{\ln(32/15)}\approx 6.372}$

31) $\log_6 x+\log_6(x+1)=1 \rightarrow \log_6 x(x+1)=1 \rightarrow$
$x^2+x=6^1 \rightarrow x^2+x-6=0 \rightarrow (x-2)(x+3)=0 \rightarrow$
$\boxed{x=2}$ OR $x\cancel{=}-3$ (impossible)

36) $\ln x+\ln(x+1)=\ln 12 \rightarrow \ln x(x+1)=\ln 12 \rightarrow$
$e^{\ln(x^2+x)}=e^{\ln 12} \rightarrow x^2+x=12 \rightarrow$

$x^2+x-12=0 \rightarrow (x-3)(x+4)=0 \rightarrow \boxed{x=3}$ OR
$\cancel{x=}-4$ (impossible)

38) $\ln(x+1)=2+\ln(x-1) \rightarrow$
$\ln(x+1)-\ln(x-1)=2 \rightarrow \ln\left(\frac{x+1}{x-1}\right)=2 \rightarrow$
$\frac{x+1}{x-1}=e^2 \rightarrow x+1=e^2(x-1) \rightarrow x+1=e^2x-e^2 \rightarrow$
$1+e^2=e^2x-x \rightarrow 1+e^2=x(e^2-1) \rightarrow x=\frac{1+e^2}{e^2-1}$

41) $\log_{10}(x-6)+\log_{10}(x+3)=1 \rightarrow$
$\log_{10}(x-6)(x+3)=1 \rightarrow x^2-3x-18=10^1 \rightarrow$
$x^2-3x-28=0 \rightarrow (x-7)(x+4)=0 \rightarrow \boxed{x=7}$ or
$\cancel{x=}-4$ $(\log_{10}(-1)!!!)$

53) $4(10-e^x) \le -3 \rightarrow 10-e^x \le -3/4 \rightarrow$
$-e^x \le -3/4-10 \rightarrow -e^x \le -\frac{47}{4} \rightarrow e^x \ge 47/4 \rightarrow$
$\ln e^x \ge \ln(47/4) \rightarrow x \ge \ln(47/4)$

55) $\ln(2-5x)>2 \rightarrow e^{\ln(2-5x)}>e^2 \rightarrow$
$2-5x>e^2 \rightarrow -5x>e^2-2 \rightarrow x<-\frac{1}{5}(e^2-2)$

58) $4^{5-x}>15 \rightarrow \ln 4^{5-x}>\ln 15 \rightarrow$
$(5-x)\ln 4>\ln 15 \rightarrow 5-x>\ln 15/\ln 4 \rightarrow$
$-x>\frac{\ln 15}{\ln 4}-5 \rightarrow x<-\left(\frac{\ln 15}{\ln 4}-5\right) \rightarrow x<5-\frac{\ln 15}{\ln 4}$

61) $\log_2 \frac{2x-1}{x-2}<0 \rightarrow 2^{\log_2 \frac{2x-1}{x-2}}<2^0 \rightarrow$
$\frac{2x-1}{x-2}<1 \rightarrow 2x-1<x-2 \rightarrow x<-1$

62) $\ln \frac{3X-2}{4X+1} > \ln 4 \rightarrow e^{\ln \frac{3X-2}{4X+1}} > e^{\ln 4} \rightarrow$

$\frac{3X-2}{4X+1} > 4 \rightarrow 3X-2 > 16X+4 \rightarrow -13X > 6 \rightarrow$

$X < -6/13$

63) $e^{X^2-4X} \geq e^5 \rightarrow \ln e^{X^2-4X} \geq \ln e^5 \rightarrow$

$X^2-4X \geq 5 \rightarrow X^2-4X-5 \geq 0 \rightarrow (X-5)(X+1) \geq 0 \rightarrow$

$$\begin{array}{ccccc} + & 0 & - & 0 & + \\ \hline & X=-1 & & X=5 & \end{array} \rightarrow X \geq 5 \text{ or } X \leq -1$$

68) a.) $y = \ln X + \ln(X+2)$ then $X > 0$ and

$X+2 > 0 \rightarrow X > 0$ and $X > -2$ so Domain: all $X > 0$

b.) $\ln X + \ln(X+2) \leq \ln 35 \rightarrow \ln X(X+2) \leq \ln 35 \rightarrow$

$e^{\ln(X^2+2X)} \leq e^{\ln 35} \rightarrow X^2+2X \leq 35 \rightarrow$

$X^2+2X-35 \leq 0 \rightarrow (X-5)(X+7) \leq 0 \rightarrow$ $$\begin{array}{ccccc} + & 0 & - & 0 & + \\ \hline & X=-7 & & X=5 & \end{array}$$

$\rightarrow -7 \leq X \leq 5$

69) $\log_2 X + \log_2(X+1) - \log_2(2X+6) < 0 \rightarrow$

$\log_2 X(X+1) < \log_2(2X+6) \rightarrow 2^{\log_2(X^2+X)} < 2^{\log_2(2X+6)}$

$\rightarrow X^2+X < 2X+6 \rightarrow X^2-X-6 < 0 \rightarrow (X-3)(X+2) < 0$

$$\rightarrow \begin{array}{ccccc} + & 0 & - & 0 & + \\ \hline & X=-2 & & X=3 & \end{array} \rightarrow -2 < X < 3$$

70) $\log_{10}(X^2-6X-6) > 0 \rightarrow 10^{\log_{10}(X^2-6X-6)} > 10^0 \rightarrow$

$X^2-6X-6 > 1 \rightarrow X^2-6X-7 > 0 \rightarrow (X-7)(X+1) > 0 \rightarrow$

$$\begin{array}{ccccc} + & 0 & - & 0 & + \\ \hline & X=-1 & & X=7 & \end{array} \rightarrow X < -1 \text{ or } X > 7$$

75) $3(\ln x)^2 - \ln x^2 - 8 = 0 \rightarrow$

$3(\ln x)^2 - 2(\ln x) - 8 = 0 \rightarrow (3\ln x + 4)(\ln x - 2) = 0 \rightarrow$

$3\ln x + 4 = 0$ or $\ln x - 2 = 0 \rightarrow 3\ln x = -4 \rightarrow \ln x = \frac{-4}{3}$

$\rightarrow \boxed{x = e^{-4/3}}$ or $\ln x = 2 \rightarrow \boxed{x = e^2}$

78) $\frac{\ln(\sqrt{x+4} + 2)}{\ln\sqrt{x}} = 2 \rightarrow \ln(\sqrt{x+4} + 2) = 2 \cdot \ln x^{1/2}$

$\rightarrow \ln(\sqrt{x+4} + 2) = \cancel{2} \cdot \frac{1}{\cancel{2}} \ln x \rightarrow e^{\ln(\sqrt{x+4}+2)} = e^{\ln x} \rightarrow$

$\sqrt{x+4} + 2 = x \rightarrow \sqrt{x+4} = x - 2 \rightarrow x + 4 = (x-2)^2 \rightarrow$

$x + 4 = x^2 - 4x + 4 \rightarrow 0 = x^2 - 5x \rightarrow 0 = x(x-5) \rightarrow$

$x \neq 0$ ($\ln 0$!!) or $\boxed{x = 5}$

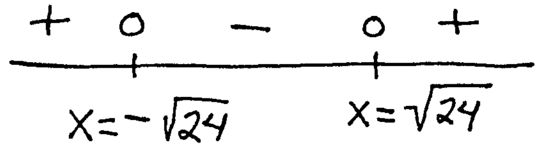
92) $\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x} > 2 \rightarrow$

$\frac{\log_2 2}{\log_2 x} + \frac{\log_3 3}{\log_3 x} + \frac{\log_4 4}{\log_4 x} > 2 \rightarrow$

$\log_x 2 + \log_x 3 + \log_x 4 > 2 \rightarrow \log_x(2 \cdot 3 \cdot 4) > 2 \rightarrow$

$x^{\log_x 24} > x^2 \rightarrow 24 > x^2 \rightarrow 0 > x^2 - 24 \rightarrow$

$0 > (x - \sqrt{24})(x + \sqrt{24}) \rightarrow$



$\rightarrow$

$-\sqrt{24} < x < \sqrt{24}$

93) $x^{(x^x)} = (x^x)^x \rightarrow x^{(x^x)} = x^{(x^2)} \rightarrow$

$\ln x^{(x^x)} = \ln x^{(x^2)} \rightarrow x^x \cdot \ln x = x^2 \cdot \ln x \rightarrow$

$(x^x - x^2) \cdot \ln x = 0 \rightarrow \ln x = 0 \rightarrow \boxed{x = 1}$ or

$x^x - x^2 = 0 \rightarrow x^x = x^2 \rightarrow \ln x^x = \ln x^2 \rightarrow$

$x \cdot \ln x = 2 \cdot \ln x \rightarrow (x-2) \cdot \ln x = 0 \rightarrow \ln x = 0 \rightarrow$

$x=1$ or $\boxed{x=2}$.

96) $f(x)=\ln(x+\sqrt{x^2+1}) \rightarrow y=\ln(x+\sqrt{x^2+1}) \rightarrow$
(Switch variables and solve for y.) $\rightarrow$
$x=\ln(y+\sqrt{y^2+1}) \rightarrow y+\sqrt{y^2+1}=e^x \rightarrow$
$\sqrt{y^2+1}=e^x-y \rightarrow y^2+1=(e^x-y)^2 \rightarrow$
$\cancel{y^2}+1=e^{2x}-2e^xy+\cancel{y^2} \rightarrow 2e^xy=1+e^{2x} \rightarrow$
$y=\dfrac{1+e^{2x}}{2e^{2x}} \rightarrow f^{-1}(x)=\dfrac{1+e^{2x}}{2e^{2x}}$