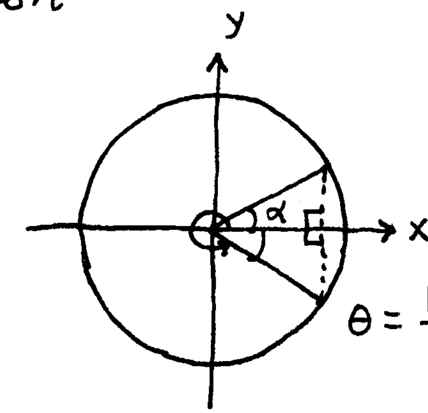
Section 8.1

1)



$\theta = \frac{11\pi}{6}$

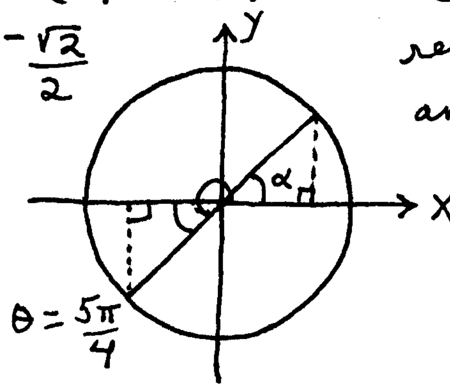
reference angle $\alpha = \frac{\pi}{6}$:

a.) $\cos\left(\frac{11\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

c.) $\sin\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = \frac{-1}{2}$

4) a.) $\cos\left(\frac{13\pi}{4}\right) = \cos\left(\frac{5\pi}{4} + \frac{8\pi}{4}\right) = \cos\left(\frac{5\pi}{4} + 2\pi\right)$

$= \cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = \frac{-\sqrt{2}}{2}$



$\theta = \frac{5\pi}{4}$

reference angle $\alpha = \frac{\pi}{4}$

c.) $\sin\left(\frac{13\pi}{4}\right) = \sin\left(\frac{5\pi}{4} + \frac{8\pi}{4}\right)$

$= \sin\left(\frac{5\pi}{4} + 2\pi\right)$

$= \sin\left(\frac{5\pi}{4}\right)$

$= -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

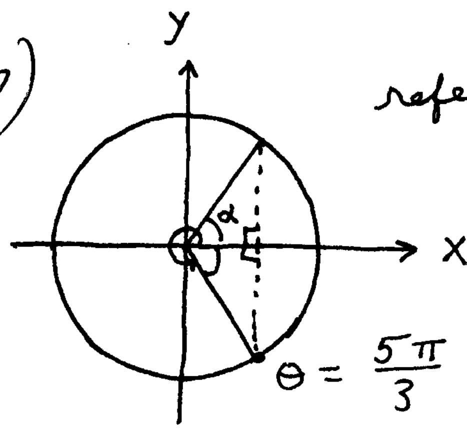
6) a.) $\cos\left(\frac{9\pi}{4}\right) = \cos\left(\frac{\pi}{4} + \frac{8\pi}{4}\right) = \cos\left(\frac{\pi}{4} + 2\pi\right)$

$= \cos\left(\frac{\pi}{4}\right) = \sqrt{2}/2$

c.) $\sin\left(\frac{9\pi}{4}\right) = \sin\left(\frac{\pi}{4} + \frac{8\pi}{4}\right) = \sin\left(\frac{\pi}{4} + 2\pi\right)$

$= \sin\left(\frac{\pi}{4}\right) = \sqrt{2}/2$

7)



$\theta = \frac{5\pi}{3}$

reference angle $\alpha = \pi/3$:

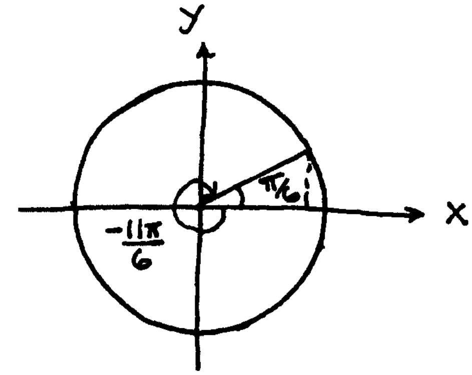
a.) $\sec\left(\frac{5\pi}{3}\right) = \frac{1}{\cos\left(\frac{5\pi}{3}\right)}$

$= \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{1}{\frac{1}{2}} = 2$

c.) $\tan\left(5\pi/3\right) = \frac{\sin\left(5\pi/3\right)}{\cos\left(5\pi/3\right)} = \frac{-\sin\left(\frac{\pi}{3}\right)}{\cos\left(\pi/3\right)} = \frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

9) a.) If $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ then $\cos t = 0$

10) c.) If $t = -\frac{11\pi}{6}, -\frac{23\pi}{6}, -\frac{35\pi}{6}, -\frac{47\pi}{6}$ then $\sin t = \frac{1}{2}$



11) a.) $\cos 2.06 \approx -0.47$, $\sin 2.06 \approx 0.88$, $\tan 2.06 \approx -1.88$, $\sec 2.06 \approx -2.13$, $\csc 2.06 \approx 1.13$, $\cot 2.06 \approx -0.53$

14) a.) $\cos 1000 \approx 0.56$, $\sin 1000 \approx 0.83$, $\tan 1000 \approx 1.47$, $\sec 1000 \approx 1.78$, $\csc 1000 \approx 1.21$, $\cot 1000 \approx 0.68$

15) a.) $\sin^2 t + \cos^2 t = 1 \rightarrow$ (let $t = \pi/3$)

$\sin^2 \frac{\pi}{3} + \cos^2 \frac{\pi}{3} = 1 \rightarrow \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1 \rightarrow$

$\frac{3}{4} + \frac{1}{4} = 1$ (TRUE)

16) a.) $\tan^2 t + 1 = \sec^2 t \rightarrow$ (let $t = \frac{3\pi}{4}$)

$\tan^2\left(\frac{3\pi}{4}\right) + 1 = \sec^2\left(\frac{3\pi}{4}\right) \rightarrow \frac{\sin^2\left(\frac{3\pi}{4}\right)}{\cos^2\left(\frac{3\pi}{4}\right)} + 1 = \frac{1}{\cos^2\left(\frac{3\pi}{4}\right)} \rightarrow$

$\frac{\sin^2\left(\frac{\pi}{4}\right)}{\left(-\cos\left(\frac{\pi}{4}\right)\right)^2} + 1 = \frac{1}{\left(-\cos\left(\frac{\pi}{4}\right)\right)^2} \rightarrow \frac{\left(\sqrt{2}/2\right)^2}{\left(-\sqrt{2}/2\right)^2} + 1 = \frac{1}{\left(-\frac{\sqrt{2}}{2}\right)^2} \rightarrow$

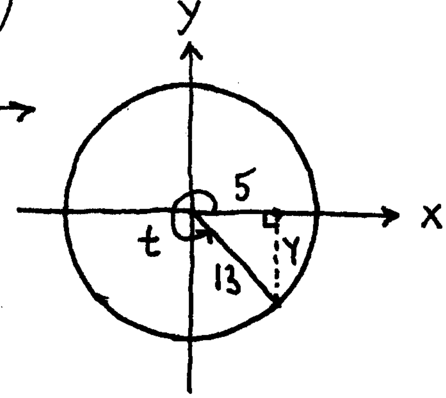
$\frac{\frac{1}{2}}{\frac{1}{2}} + 1 = \frac{1}{\frac{1}{2}} \rightarrow 1 + 1 = 2$ (TRUE)

24) $\sin 2t = 2\sin t \rightarrow$ (let $t = \frac{\pi}{2}$)

$\sin 2\left(\frac{\pi}{2}\right) = 2\sin\left(\frac{\pi}{2}\right) \rightarrow \sin\pi = 2\sin\left(\frac{\pi}{2}\right) \rightarrow$

$0 = 2\cdot(1) \rightarrow 0 = 2$ (FALSE)

26) $\cos t = 5/13 = \text{adj.}/\text{hyp.} \rightarrow$



$5^2 + y^2 = 13^2 \rightarrow y^2 = 169 - 25 \rightarrow$

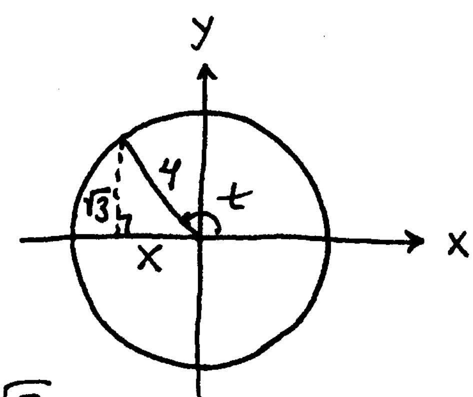
$y^2 = 144 \rightarrow y = \pm 12 \rightarrow$

$y = -12$ then

$\sin t = \text{opp.}/\text{hyp.} = -12/13$ and

$\cot t = \dfrac{\text{adj.}}{\text{opp.}} = \dfrac{5}{-12} = \dfrac{-5}{12}$

27) $\sin t = \sqrt{3}/4 = \text{opp.}/\text{hyp} \rightarrow$

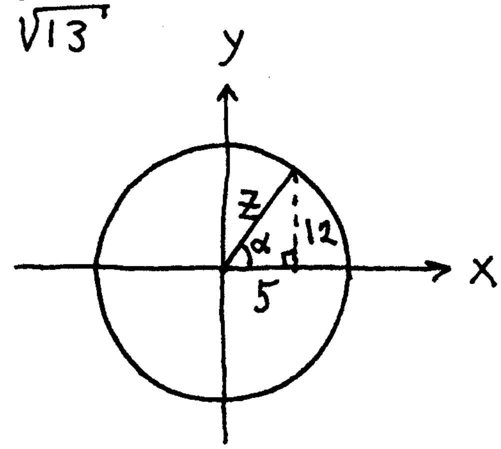


$x^2 + (\sqrt{3})^2 = 4^2 \rightarrow x^2 = 16 - 3 \rightarrow$

$x^2 = 13 \rightarrow x = \pm\sqrt{13} \rightarrow x = -\sqrt{13}$

then $\tan t = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{\sqrt{3}}{-\sqrt{13}} = \dfrac{-\sqrt{3}}{\sqrt{13}}$

29) $\tan\alpha = \dfrac{12}{5} = \dfrac{\text{opp.}}{\text{adj.}} \longrightarrow$



$5^2 + 12^2 = z^2 \rightarrow 25 + 144 = z^2$

$\rightarrow z^2 = 169 \rightarrow z = \pm\sqrt{169} \rightarrow$

$z = \pm 13 \rightarrow z = 13$ then

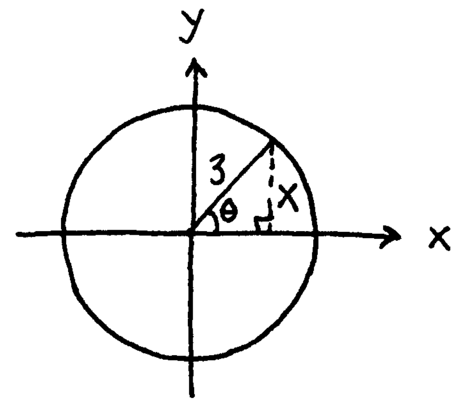
$\cos\alpha = \text{adj.}/\text{hyp.} = 5/13$

$\sec\alpha = \dfrac{1}{\cos\alpha} = \dfrac{1}{5/13} = 13/5$, and

$\sin\alpha = \dfrac{\text{opp.}}{\text{hyp.}} = \dfrac{12}{13}$

31) $x = 3\sin\theta \rightarrow$

$\sin\theta = \frac{x}{3} = \frac{\text{opp.}}{\text{hyp.}} \rightarrow$

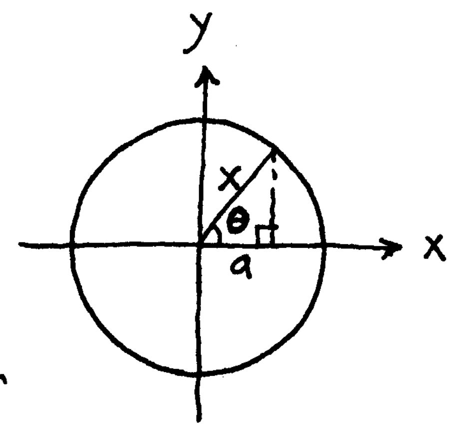


$$\sqrt{9-x^2} = \sqrt{9-(3\sin\theta)^2}$$
$$= \sqrt{9 - 9\sin^2\theta}$$
$$= \sqrt{9(1-\sin^2\theta)} = \sqrt{9\cdot\cos^2\theta}$$
$$= 3\cdot\sqrt{\cos^2\theta} = 3\cdot|\cos\theta| = 3\cos\theta$$

(since $\cos\theta > 0$ in 1st quadrant)

36) $x = a\sec\theta \rightarrow$

$\sec\theta = \frac{x}{a} = \text{hyp.}/\text{adj.} \rightarrow$



$$\frac{\sqrt{x^2-a^2}}{x} = \frac{\sqrt{(a\sec\theta)^2 - a^2}}{a\sec\theta}$$
$$= \frac{\sqrt{a^2\sec^2\theta - a^2}}{a\sec\theta} = \frac{\sqrt{a^2(\sec^2\theta-1)}}{a\sec\theta}$$
$$= \frac{\sqrt{a^2\tan^2\theta}}{a\sec\theta} = \frac{\cancel{a}\sqrt{\tan^2\theta}}{\cancel{a}\sec\theta} = \frac{|\tan\theta|}{\sec\theta} = \frac{\tan\theta}{\sec\theta}$$

(since $\tan\theta > 0$ in 1st quadrant)

$$= \frac{\sin\theta/\cos\theta}{1/\cos\theta} = \frac{\sin\theta}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{1} = \sin\theta$$

38) a.) $\sin(-t) = -\sin t = -(0.35)$

c.) $\cos(-\alpha) = \cos\alpha = 0.21$

48) Show $\sin^2 t - \cos^2 t = \dfrac{1-\cot^2 t}{1+\cot^2 t}$:

$$\frac{1-\cot^2 t}{1+\cot^2 t} = \frac{1 - \dfrac{\cos^2 t}{\sin^2 t}}{1 + \dfrac{\cos^2 t}{\sin^2 t}} = \frac{\dfrac{\sin^2 t}{\sin^2 t} - \dfrac{\cos^2 t}{\sin^2 t}}{\dfrac{\sin^2 t}{\sin^2 t} + \dfrac{\cos^2 t}{\sin^2 t}}$$

$$= \frac{\dfrac{\sin^2 t - \cos^2 t}{\sin^2 t}}{\dfrac{\sin^2 t + \cos^2 t}{\sin^2 t}} = \frac{\sin^2 t - \cos^2 t}{\cancel{\sin^2} t} \cdot \frac{\cancel{\sin^2 t}}{1}$$

$$= \sin^2 t - \cos^2 t$$

49) $\dfrac{1}{1+\sec s} + \dfrac{1}{1-\sec s} = -2\cot^2 s$:

$$\frac{1}{1+\sec s} + \frac{1}{1-\sec s} = \frac{1}{1+\sec s} \cdot \frac{1-\sec s}{1-\sec s} + \frac{1}{1-\sec s} \cdot \frac{1+\sec s}{1+\sec s}$$

$$= \frac{1-\sec s}{1-\sec^2 s} + \frac{1+\sec s}{1-\sec^2 s} = \frac{1-\cancel{\sec s}+1+\cancel{\sec s}}{-\tan^2 s}$$

$$= \frac{-2}{\dfrac{\sin^2 s}{\cos^2 s}} = -2\,\frac{\cos^2 s}{\sin^2 s} = -2\cot^2 s$$