

Section 9.1

$$\begin{aligned} 2) \quad & \sin \frac{\pi}{6} \cos \frac{\pi}{3} + \cos \frac{\pi}{6} \sin \frac{\pi}{3} = \sin \left(\frac{\pi}{6} + \frac{\pi}{3} \right) \\ & = \sin \left(\frac{\pi}{6} + \frac{2\pi}{6} \right) = \sin \left(\frac{3\pi}{6} \right) = \sin \left(\frac{\pi}{2} \right) = 1 \end{aligned}$$

$$\begin{aligned} 4) \quad & \sin 110^\circ \cos 20^\circ - \cos 110^\circ \sin 20^\circ = \sin (110^\circ - 20^\circ) \\ & = \sin 90^\circ = 1 \end{aligned}$$

$$\begin{aligned} 7) \quad & \cos \frac{2\pi}{9} \cos \frac{\pi}{18} + \sin \frac{2\pi}{9} \sin \frac{\pi}{18} = \cos \left(\frac{2\pi}{9} - \frac{\pi}{18} \right) \\ & = \cos \left(\frac{4\pi}{18} - \frac{\pi}{18} \right) = \cos \left(\frac{3\pi}{18} \right) = \cos \left(\frac{\pi}{6} \right) = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} 8) \quad & \cos \frac{3\pi}{10} \cos \frac{\pi}{5} - \sin \frac{3\pi}{10} \sin \frac{\pi}{5} = \cos \left(\frac{3\pi}{10} + \frac{\pi}{5} \right) \\ & = \cos \left(\frac{3\pi}{10} + \frac{2\pi}{10} \right) = \cos \left(\frac{5\pi}{10} \right) = \cos \left(\frac{\pi}{2} \right) = 0 \end{aligned}$$

$$\begin{aligned} 12) \quad & \cos \left(\frac{3\pi}{2} + \theta \right) = \cos \left(\frac{3\pi}{2} \right) \cos \theta - \sin \left(\frac{3\pi}{2} \right) \sin \theta \\ & = (0) \cdot \cos \theta - (-1) \cdot \sin \theta = \sin \theta \end{aligned}$$

$$\begin{aligned} 14) \quad & \sin (\theta - \pi) = \sin \theta \cos \pi - \cos \theta \sin \pi \\ & = \sin \theta \cdot (-1) - \cos \theta \cdot (0) = -\sin \theta \end{aligned}$$

$$\begin{aligned} 17) \quad & \cos 75^\circ = \cos (45^\circ + 30^\circ) \\ & = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \\ & = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} 18) \quad & \sin \frac{\pi}{12} = \sin \left(\frac{4\pi}{12} - \frac{3\pi}{12} \right) = \sin \left(\frac{\pi}{3} - \frac{\pi}{4} \right) \\ & = \sin \left(\frac{\pi}{3} \right) \cos \left(\frac{\pi}{4} \right) - \cos \left(\frac{\pi}{3} \right) \sin \left(\frac{\pi}{4} \right) \end{aligned}$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\begin{aligned} 19) \quad \sin \frac{7\pi}{12} &= \sin \left(\frac{4\pi}{12} + \frac{3\pi}{12} \right) = \sin \left(\frac{\pi}{3} + \frac{\pi}{4} \right) \\ &= \sin \left(\frac{\pi}{3} \right) \cos \left(\frac{\pi}{4} \right) + \cos \left(\frac{\pi}{3} \right) \sin \left(\frac{\pi}{4} \right) \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} 20) \quad a.) \quad \sin 105^\circ &= \sin (60^\circ + 45^\circ) \\ &= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} b.) \quad \cos 105^\circ &= \cos (60^\circ + 45^\circ) \\ &= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

$$\begin{aligned} 22) \quad \sin \left(t + \frac{\pi}{6} \right) - \sin \left(t - \frac{\pi}{6} \right) \\ &= \cancel{\sin t} \cos \frac{\pi}{6} + \cos t \sin \frac{\pi}{6} \\ &\quad - \left(\cancel{\sin t} \cos \frac{\pi}{6} - \cos t \sin \frac{\pi}{6} \right) \\ &= \cos t \cdot \left(\frac{1}{2} \right) + \cos t \cdot \left(\frac{1}{2} \right) = \cos t \cdot \left(\frac{1}{2} + \frac{1}{2} \right) \\ &= \cos t \cdot (1) = \cos t \end{aligned}$$

$$\begin{aligned} 24) \quad \cos \left(\theta - \frac{\pi}{4} \right) + \cos \left(\theta + \frac{\pi}{4} \right) \\ &= \cos \theta \cos \frac{\pi}{4} + \cancel{\sin \theta} \sin \frac{\pi}{4} \\ &\quad + \left(\cos \theta \cos \frac{\pi}{4} - \cancel{\sin \theta} \sin \frac{\pi}{4} \right) \\ &= \cos \theta \cdot \frac{\sqrt{2}}{2} + \cos \theta \cdot \frac{\sqrt{2}}{2} = \cos \theta \cdot \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) \\ &= \cos \theta \cdot \frac{2\sqrt{2}}{2} = \sqrt{2} \cdot \cos \theta \end{aligned}$$

25)

$$x^2 + 12^2 = 13^2 \rightarrow x^2 = 169 - 144 = 25 \rightarrow$$

$$x = -5$$

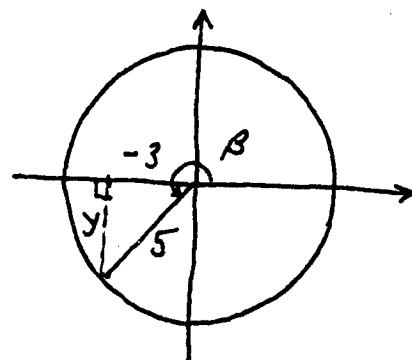
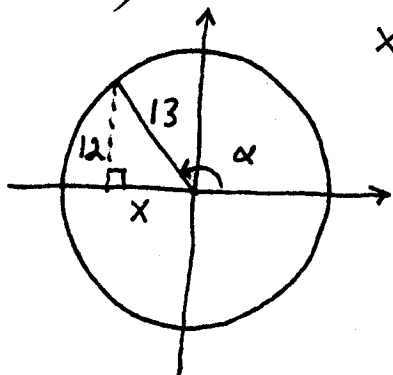
and

$$3^2 + y^2 = 5^2 \rightarrow$$

$$y^2 = 25 - 9 \rightarrow$$

$$y^2 = 16 \rightarrow$$

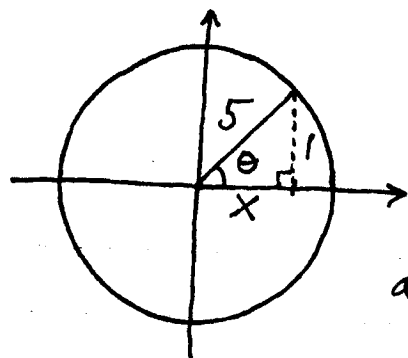
$$y = -4$$



$$\begin{aligned} \text{a.) } \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ &= \frac{12}{13} \cdot \frac{-3}{5} + \frac{-5}{13} \cdot \frac{-4}{5} = \frac{-36 - 20}{65} = \frac{-56}{65} \end{aligned}$$

$$\begin{aligned} \text{b.) } \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ &= \frac{-5}{13} \cdot \frac{-3}{5} - \frac{12}{13} \cdot \frac{-4}{5} = \frac{15 + 48}{65} = \frac{63}{65} \end{aligned}$$

29)



$$1^2 + x^2 = 5^2 \rightarrow$$

$$x^2 = 25 - 1 = 24 \rightarrow$$

$$x = \sqrt{24}$$

$$\text{a.) } \cos 2\theta = \frac{x}{5} = \frac{\sqrt{24}}{5}$$

$$\text{b.) } \sin 2\theta = 2 \sin \theta \cos \theta = 2 \cdot \frac{1}{5} \cdot \frac{\sqrt{24}}{5} = \frac{2\sqrt{24}}{25}$$

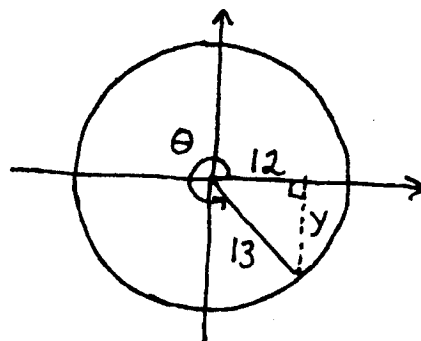
30)

$$12^2 + y^2 = 13^2 \rightarrow$$

$$y^2 = 169 - 144 = 25 \rightarrow y = -5$$

$$\text{a.) } \sin \theta = \frac{y}{12} = \frac{-5}{12}$$

$$\text{b.) } \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$



$$= \left(\frac{12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2 = \frac{144 - 25}{169} = \frac{119}{169}$$

$$\begin{aligned} 36) \quad \text{Show } \sec(\alpha + \beta) &= \frac{\sec \alpha \sec \beta}{1 - \tan \alpha \tan \beta} : \\ \frac{\sec \alpha \sec \beta}{1 - \tan \alpha \tan \beta} &= \frac{\frac{1}{\cos \alpha} \cdot \frac{1}{\cos \beta}}{1 - \frac{\sin \alpha}{\cos \alpha} \cdot \frac{\sin \beta}{\cos \beta}} \cdot \frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} \\ &= \frac{1}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} = \frac{1}{\cos(\alpha + \beta)} = \sec(\alpha + \beta) \end{aligned}$$

$$\begin{aligned} 37) \quad \tan(s + t) &= \frac{\tan s + \tan t}{1 - \tan s \tan t} \\ &= \frac{2 + 3}{1 - (2)(3)} = \frac{5}{-5} = -1 ; \\ \tan(s - t) &= \frac{\tan s - \tan t}{1 + \tan s \tan t} = \frac{2 - 3}{1 + (2)(3)} = \frac{-1}{7} \end{aligned}$$

$$\begin{aligned} 43) \quad \frac{\tan 70^\circ - \tan 10^\circ}{1 + \tan 70^\circ \tan 10^\circ} &= \tan(70^\circ - 10^\circ) \\ &= \tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} \end{aligned}$$

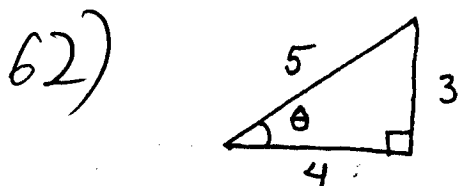
$$\begin{aligned} 44) \quad \frac{2 \tan \frac{\pi}{12}}{1 - \tan^2 \frac{\pi}{12}} &= \tan\left(\frac{\pi}{12} + \frac{\pi}{12}\right) = \tan\left(\frac{2\pi}{12}\right) \\ &= \tan\left(\frac{\pi}{6}\right) = \frac{\sin(\pi/6)}{\cos(\pi/6)} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} \end{aligned}$$

$$48) \quad \tan 15^\circ = \tan(45^\circ - 30^\circ)$$

$$\begin{aligned}
 &= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + (1) \cdot \left(\frac{1}{\sqrt{3}}\right)} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\
 &= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}
 \end{aligned}$$

50) Show $\frac{\cos(s-t)}{\cos s \sin t} = \cot t + \tan s$:

$$\begin{aligned}
 \frac{\cos(s-t)}{\cos s \sin t} &= \frac{\cos s \cos t + \sin s \sin t}{\cos s \sin t} \\
 &= \frac{\cos s \cos t}{\cos s \sin t} + \frac{\sin s \sin t}{\cos s \sin t} = \cot t + \tan s
 \end{aligned}$$



Show $5 \sin(x+\theta) = 4 \sin x + 3 \cos x$:

$$\begin{aligned}
 5 \sin(x+\theta) &= 5 [\sin x \cos \theta + \cos x \sin \theta] \\
 &= 5 \left[\sin x \cdot \frac{4}{5} + \cos x \cdot \frac{3}{5} \right] \\
 &= 4 \sin x + 3 \cos x
 \end{aligned}$$