Section 9.1

2) $\sin\frac{\pi}{6}\cos\frac{\pi}{3}+\cos\frac{\pi}{6}\sin\frac{\pi}{3}=\sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right)$
$=\sin\left(\frac{\pi}{6}+\frac{2\pi}{6}\right)=\sin\left(\frac{3\pi}{6}\right)=\sin\left(\frac{\pi}{2}\right)=1$

4) $\sin 110^\circ\cos 20^\circ-\cos 110^\circ\sin 20^\circ=\sin(110^\circ-20^\circ)$
$=\sin 90^\circ=1$

7) $\cos\frac{2\pi}{9}\cos\frac{\pi}{18}+\sin\frac{2\pi}{9}\sin\frac{\pi}{18}=\cos\left(\frac{2\pi}{9}-\frac{\pi}{18}\right)$
$=\cos\left(\frac{4\pi}{18}-\frac{\pi}{18}\right)=\cos\left(\frac{3\pi}{18}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

8) $\cos\frac{3\pi}{10}\cos\frac{\pi}{5}-\sin\frac{3\pi}{10}\sin\frac{\pi}{5}=\cos\left(\frac{3\pi}{10}+\frac{\pi}{5}\right)$
$=\cos\left(\frac{3\pi}{10}+\frac{2\pi}{10}\right)=\cos\left(\frac{5\pi}{10}\right)=\cos\left(\frac{\pi}{2}\right)=0$

12) $\cos\left(\frac{3\pi}{2}+\theta\right)=\cos\left(\frac{3\pi}{2}\right)\cos\theta-\sin\left(\frac{3\pi}{2}\right)\sin\theta$
$=(0)\cdot\cos\theta-(-1)\cdot\sin\theta=\sin\theta$

14) $\sin(\theta-\pi)=\sin\theta\cos\pi-\cos\theta\sin\pi$
$=\sin\theta\cdot(-1)-\cos\theta\cdot(0)=-\sin\theta$

17) $\cos 75^\circ=\cos(45^\circ+30^\circ)$
$=\cos 45^\circ\cos 30^\circ-\sin 45^\circ\sin 30^\circ$
$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}=\frac{\sqrt{6}-\sqrt{2}}{4}$

18) $\sin\frac{\pi}{12}=\sin\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right)=\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$
$=\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)-\cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

19) $\sin\frac{7\pi}{12} = \sin\left(\frac{4\pi}{12}+\frac{3\pi}{12}\right) = \sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

$$= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

20) a.) $\sin 105° = \sin(60° + 45°)$

$$= \sin 60° \cos 45° + \cos 60° \sin 45°$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

b.) $\cos 105° = \cos(60° + 45°)$

$$= \cos 60° \cos 45° - \sin 60° \sin 45°$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

22) $\sin\left(t+\frac{\pi}{6}\right) - \sin\left(t-\frac{\pi}{6}\right)$

$$= \cancel{\sin t \cos\tfrac{\pi}{6}} + \cos t \sin\frac{\pi}{6}$$
$$- \left(\cancel{\sin t \cos\tfrac{\pi}{6}} - \cos t \sin\frac{\pi}{6}\right)$$

$$= \cos t\cdot\left(\tfrac{1}{2}\right) + \cos t\cdot\left(\tfrac{1}{2}\right) = \cos t\cdot\left(\tfrac{1}{2}+\tfrac{1}{2}\right)$$

$$= \cos t\cdot(1) = \cos t$$

24) $\cos\left(\theta-\frac{\pi}{4}\right) + \cos\left(\theta+\frac{\pi}{4}\right)$

$$= \cos\theta\cos\frac{\pi}{4} + \cancel{\sin\theta\sin\tfrac{\pi}{4}}$$
$$+ \left(\cos\theta\cos\frac{\pi}{4} - \cancel{\sin\theta\sin\tfrac{\pi}{4}}\right)$$

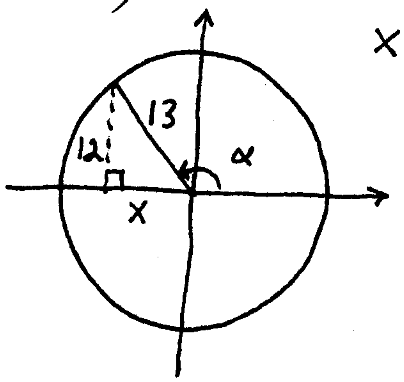
$$= \cos\theta\cdot\frac{\sqrt{2}}{2} + \cos\theta\cdot\frac{\sqrt{2}}{2} = \cos\theta\cdot\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)$$

$$= \cos\theta\cdot\frac{2\sqrt{2}}{2} = \sqrt{2}\cdot\cos\theta$$

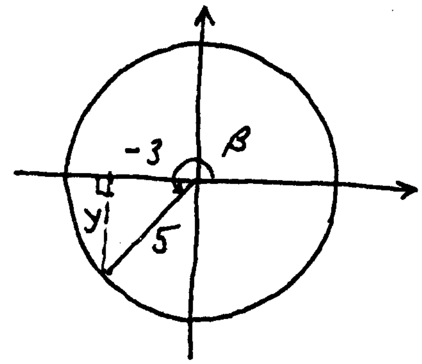
25)

$x^2 + 12^2 = 13^2 \rightarrow x^2 = 169 - 144 = 25 \rightarrow$
$x = -5$

and



$3^2 + y^2 = 5^2 \rightarrow$
$y^2 = 25 - 9 \rightarrow$
$y^2 = 16 \rightarrow$
$y = -4$



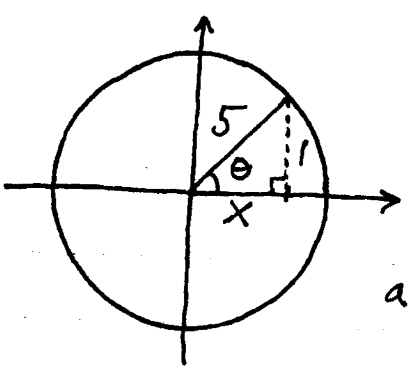
a.) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{12}{13}\cdot\frac{-3}{5} + \frac{-5}{13}\cdot\frac{-4}{5} = \frac{-36-20}{65} = -\frac{56}{65}$

b.) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{-5}{13}\cdot\frac{-3}{5} - \frac{12}{13}\cdot\frac{-4}{5} = \frac{15+48}{65} = \frac{63}{65}$

29)



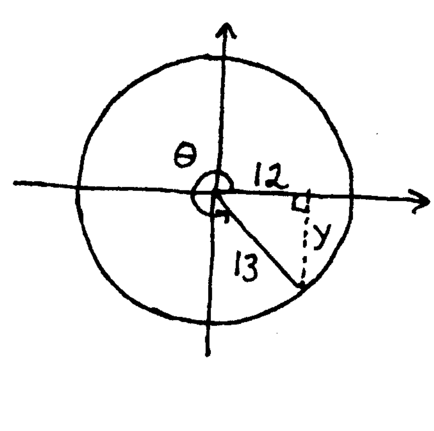
$1^2 + x^2 = 5^2 \rightarrow$
$x^2 = 25 - 1 = 24 \rightarrow$
$x = \sqrt{24}$

a.) $\cos\theta = \frac{x}{5} = \frac{\sqrt{24}}{5}$

b.) $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{1}{5}\cdot\frac{\sqrt{24}}{5} = \frac{2\sqrt{24}}{25}$

30) $12^2 + y^2 = 13^2 \rightarrow$

$y^2 = 169 - 144 = 25 \rightarrow y = -5$



a.) $\sin\theta = \frac{y}{12} = \frac{-5}{12}$

b.) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \left(\frac{12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2 = \frac{144-25}{169} = \frac{119}{169}$$

36) Show $\sec(\alpha+\beta) = \dfrac{\sec\alpha \sec\beta}{1-\tan\alpha\tan\beta}$:

$$\frac{\sec\alpha \sec\beta}{1-\tan\alpha\tan\beta} = \frac{\frac{1}{\cos\alpha}\cdot\frac{1}{\cos\beta}}{1-\frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\cos\beta}} \cdot \frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}$$

$$= \frac{1}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} = \frac{1}{\cos(\alpha+\beta)} = \sec(\alpha+\beta)$$

37) $\tan(s+t) = \dfrac{\tan s + \tan t}{1-\tan s \tan t}$

$$= \frac{2+3}{1-(2)(3)} = \frac{5}{-5} = -1 \quad ;$$

$$\tan(s-t) = \frac{\tan s - \tan t}{1+\tan s\tan t} = \frac{2-3}{1+(2)(3)} = \frac{-1}{7}$$

43) $\dfrac{\tan 70° - \tan 10°}{1+\tan 70° \tan 10°} = \tan(70° - 10°)$

$$= \tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

44) $\dfrac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}} = \tan\left(\frac{\pi}{12}+\frac{\pi}{12}\right) = \tan\left(\frac{2\pi}{12}\right)$

$$= \tan\left(\frac{\pi}{6}\right) = \frac{\sin(\pi/6)}{\cos(\pi/6)} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

48) $\tan 15° = \tan(45° - 30°)$

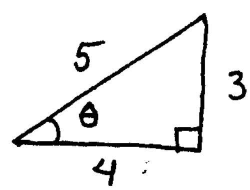
$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ} = \frac{1 - 1/\sqrt{3}}{1 + (1)\cdot\left(\frac{1}{\sqrt{3}}\right)} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

50) Show $\dfrac{\cos(s-t)}{\cos s \sin t} = \cot t + \tan s$:

$$\frac{\cos(s-t)}{\cos s \sin t} = \frac{\cos s \cos t + \sin s \sin t}{\cos s \sin t}$$

$$= \frac{\cancel{\cos s} \cos t}{\cancel{\cos s} \sin t} + \frac{\sin s \cancel{\sin t}}{\cos s \cancel{\sin t}} = \cot t + \tan s$$

62) 

Show
$5 \sin(x+\theta) = 4 \sin x + 3 \cos x$:

$$5 \sin(x+\theta) = 5\left[\sin x \cos\theta + \cos x \sin\theta\right]$$

$$= 5\left[\sin x \cdot \frac{4}{5} + \cos x \cdot \frac{3}{5}\right]$$

$$= 4 \sin x + 3 \cos x$$