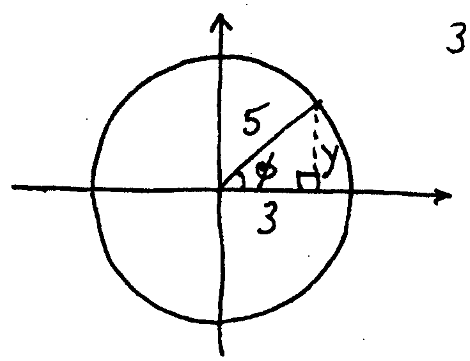
Section 9.2

2)



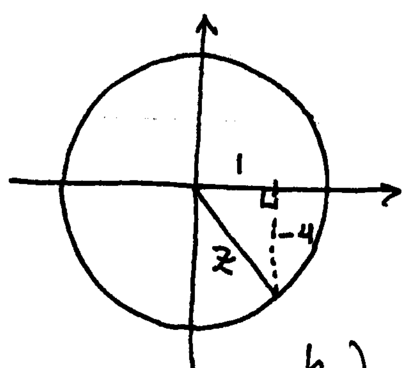
$3^2 + y^2 = 5^2 \rightarrow y^2 = 25 - 9 \rightarrow$
$y^2 = 16 \rightarrow y = 4$; then

a.) $\sin 2\phi = 2 \sin\phi \cos\phi$
$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{24}{25}$

b.) $\cos 2\phi = \cos^2\phi - \sin^2\phi = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{-7}{25}$

c.) $\tan 2\phi = \frac{2\tan\phi}{1-\tan^2\phi} = \frac{2(4/3)}{1-(4/3)^2} = \frac{8/3}{1-16/9} = \frac{8/3}{-7/9}$
$= \frac{8}{3} \cdot \frac{-9}{7} = \frac{-24}{7}$

3)



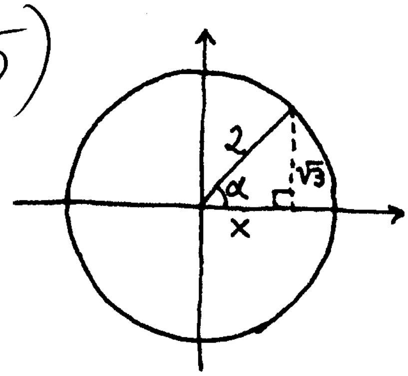
$1^2 + 4^2 = z^2 \rightarrow z^2 = 17 \rightarrow$
$z = \sqrt{17}$; then

a.) $\sin 2u = 2 \sin u \cos u$
$= 2\left(\frac{-4}{\sqrt{17}}\right)\left(\frac{1}{\sqrt{17}}\right) = -8/17$

b.) $\cos 2u = 2\cos^2 u - 1 = 2\left(1/\sqrt{17}\right)^2 - 1$
$= \frac{2}{17} - \frac{17}{17} = \frac{-15}{17}$

c.) $\tan 2u = \frac{2\tan u}{1-\tan^2 u} = \frac{2(-4/1)}{1-(-4/1)^2} = \frac{-8}{1-16} = \frac{8}{15}$

5)



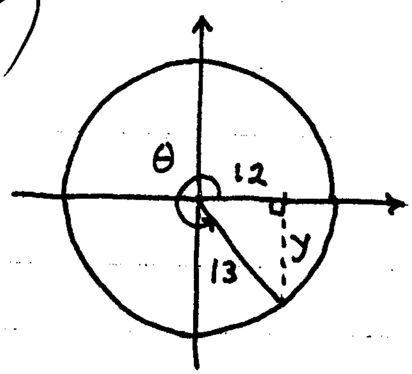
$x^2 + (\sqrt{3})^2 = 2^2 \rightarrow x^2 + 3 = 4 \rightarrow$
$x^2 = 1 \rightarrow x = 1$; then

a.) $\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1-\cos\alpha}{2}}$
$= \sqrt{\frac{1-1/2}{2}} = \sqrt{1/4} = 1/2$

b.) $\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+1/2}{2}} = \sqrt{3/4} = \sqrt{3}/2$

c.) $\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\alpha}{1+\cos\alpha} = \frac{\sqrt{3}/2}{1+\ 1/2}\cdot\frac{2}{2} = \frac{\sqrt{3}}{3}$

8)



$12^2 + y^2 = 13^2 \rightarrow y^2 = 169 - 144 \rightarrow$
$y^2 = 25 \rightarrow y = -5$; then

a.) $\sin\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1-\cos\theta}{2}}$
$= -\sqrt{\frac{1-12/13}{2}} = -\sqrt{\frac{1}{26}} = \frac{-1}{\sqrt{26}}$

b.) $\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+12/13}{2}} = \sqrt{\frac{25}{26}} = \frac{5}{\sqrt{26}}$

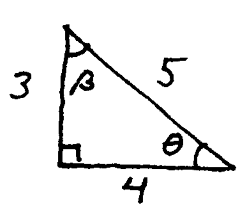
c.) $\tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1+\cos\theta} = \frac{-5/13}{1+12/13} = \frac{-5/13}{25/13} = \frac{-5}{25} = -1/5$

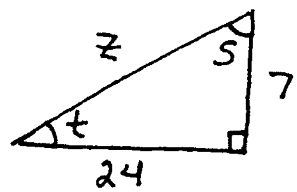
13) a.) $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi/6}{2}\right) = \sqrt{\frac{1-\cos(\pi/6)}{2}}$
$= \sqrt{\frac{1-\sqrt{3}/2}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

b.) $\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi/6}{2}\right) = \sqrt{\frac{1+\cos(\pi/6)}{2}} = \sqrt{\frac{1+\sqrt{3}/2}{2}}$
$= \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$

14) c.) $\tan\left(\frac{\pi}{8}\right) = \tan\left(\frac{\pi/4}{2}\right) = \frac{\sin(\pi/4)}{1+\cos(\pi/4)}$
$= \frac{\sqrt{2}/2}{1+\sqrt{2}/2}\cdot\frac{2}{2} = \frac{\sqrt{2}}{2+\sqrt{2}}$

17) a.)





$24^2 + 7^2 = z^2 \rightarrow$
$z = 25$

$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$

18) b.) $\cos 2t = 2\cos^2 t - 1 = 2\left(\frac{24}{25}\right)^2 - 1$

$= 2\left(\frac{576}{625}\right) - 1 = \frac{1152}{625} - \frac{625}{625} = \frac{527}{625}$

19) c.) $\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2}$

$= \frac{8/3}{1-\frac{16}{9}} = \frac{8/3}{-7/9} = \frac{8}{3} \cdot \frac{-9}{7} = -\frac{24}{7}$

20) a.) $\sin 2s = 2\sin s\cos s$

$= 2\left(\frac{24}{25}\right)\left(\frac{7}{25}\right) = \frac{336}{625}$

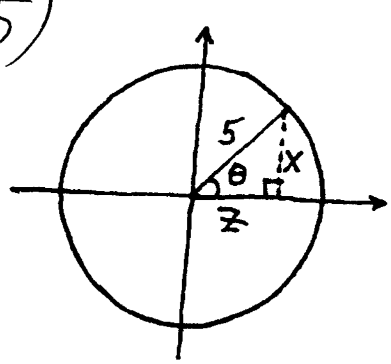
23) a.) $\sin\left(\frac{\beta}{2}\right) = \sqrt{\frac{1-\cos\beta}{2}}$

$= \sqrt{\frac{1-3/5}{2}} = \sqrt{\frac{2/5}{2}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$

b.) $\cos\left(\frac{\beta}{2}\right) = \sqrt{\frac{1+\cos\beta}{2}} = \sqrt{\frac{1+3/5}{2}} = \sqrt{\frac{8/5}{2}} = \frac{2}{\sqrt{5}}$

c.) $\tan\left(\frac{\beta}{2}\right) = \frac{\sin\beta}{1+\cos\beta} = \frac{4/5}{1+3/5} = \frac{4/5}{8/5} = \frac{1}{2}$

25)



$x = 5\sin\theta \rightarrow \sin\theta = \frac{x}{5}$;

$z^2 + x^2 = 5^2 \rightarrow$

$z^2 = 25 - x^2 \rightarrow$

$z = \sqrt{25 - x^2}$; then

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{x}{5}\cdot\frac{\sqrt{25-x^2}}{5}$$
$$= \frac{2}{25}x\sqrt{25-x^2}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 2\cdot\left(\frac{\sqrt{25-x^2}}{5}\right)^2 - 1$$
$$= \frac{2}{25}(25-x^2) - 1 = 2 - \frac{2}{25}x^2 - 1$$
$$= 1 - \frac{2}{25}x^2$$

29) $\sin^4\theta = \sin^2\theta \cdot \sin^2\theta$

$$= \frac{1}{2}(1-\cos 2\theta)\cdot\frac{1}{2}(1-\cos 2\theta)$$
$$= \frac{1}{4}(1 - 2\cos 2\theta + \cos^2 2\theta)$$
$$= \frac{1}{4}\left(1 - 2\cos 2\theta + \frac{1}{2}(1+\cos 2(2\theta))\right)$$
$$= \frac{1}{4}\left(\frac{3}{2} - 2\cos 2\theta + \frac{1}{2}\cos 4\theta\right)$$
$$= \frac{3}{8} - \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta$$

31) $\sin^4\left(\frac{\theta}{2}\right) = \sin^2\left(\frac{\theta}{2}\right)\cdot\sin^2\left(\frac{\theta}{2}\right)$

$$= \frac{1}{2}\left(1-\cos 2\left(\frac{\theta}{2}\right)\right)\cdot\frac{1}{2}\left(1-\cos 2\left(\frac{\theta}{2}\right)\right)$$
$$= \frac{1}{4}(1-\cos\theta)(1-\cos\theta)$$
$$= \frac{1}{4}(1 - 2\cos\theta + \cos^2\theta)$$
$$= \frac{1}{4}\left(1 - 2\cos\theta + \frac{1}{2}(1+\cos 2\theta)\right)$$
$$= \frac{3}{8} - \frac{1}{2}\cos\theta + \frac{1}{8}\cos 2\theta$$

35) Show $\cos 2s = \dfrac{1-\tan^2 s}{1+\tan^2 s}$:

$$\frac{1-\tan^2 s}{1+\tan^2 s} = \frac{1-\dfrac{\sin^2 s}{\cos^2 s}}{1+\dfrac{\sin^2 s}{\cos^2 s}} \cdot \frac{\cos^2 s}{\cos^2 s}$$

$$= \frac{\cos^2 s - \sin^2 s}{\cos^2 s + \sin^2 s} = \frac{\cos 2s}{1} = \cos 2s$$

36) Show $1+\cos 2t = \cot t \cdot \sin 2t$:

$$\cot t \cdot \sin 2t = \frac{\cos t}{\cancel{\sin t}} \cdot 2\cancel{\sin t}\cos t = 2\cos^2 t$$

$$= 2 \cdot \tfrac{1}{2}(1+\cos 2t) = 1+\cos 2t$$

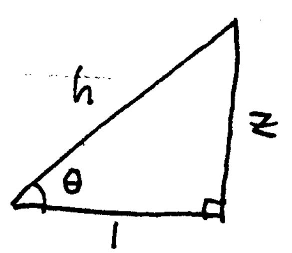
38) Show $\dfrac{\sin 2\theta}{\sin\theta} - \dfrac{\cos 2\theta}{\cos\theta} = \sec\theta$:

$$\frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta} = \frac{2\cancel{\sin\theta}\cos\theta}{\cancel{\sin\theta}} - \frac{2\cos^2\theta - 1}{\cos\theta}$$

$$= 2\cos\theta - \left(\frac{2\cos^2\theta}{\cos\theta} - \frac{1}{\cos\theta}\right) = 2\cos\theta\ (2\cos\theta - \sec\theta)$$

$$= 2\cancel{\cos}\theta - 2\cancel{\cos}\theta + \sec\theta = \sec\theta$$

52)



$z = \tan\theta \rightarrow$

$\tan\theta = \dfrac{z}{1}$ then

$1^2 + z^2 = h^2 \rightarrow$

$h = \sqrt{1+z^2}$;

a.) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \left(\frac{1}{\sqrt{1+z^2}}\right)^2 - \left(\frac{z}{\sqrt{1+z^2}}\right)^2 = \frac{1}{1+z^2} - \frac{z^2}{1+z^2} = \frac{1-z^2}{1+z^2}$$

b.) $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{z}{\sqrt{1+z^2}}\cdot\frac{1}{\sqrt{1+z^2}} = \frac{2z}{1+z^2}$

55) a.) $\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

$$= \frac{CD/AC}{AD/AC} = \frac{CD}{\cancel{AC}}\cdot\frac{\cancel{AC}}{AD}$$

$$= \frac{CD}{AD} = \frac{CD}{AO+OD}$$

$$= \frac{CD}{1+OD}$$ (note that $OC = 1$.)

$$= \frac{CD/1}{1+OD/1}$$

$$= \frac{CD/OC}{1+OD/OC}$$

$$= \frac{\sin\theta}{1+\cos\theta}$$