

Section 2.3

$$1) \quad 2x - 7 < 11 \rightarrow 2x < 18 \rightarrow x < 9 : (-\infty, 9)$$

$$3) \quad 4x + 6 < 3(x-1) - x \rightarrow 4x + 6 < 3x - 3 - x \rightarrow \\ 4x + 6 < 2x - 3 \rightarrow 2x < -9 \rightarrow x < -\frac{9}{2} : (-\infty, -\frac{9}{2})$$

$$7) \quad \frac{2x+1}{2} + \frac{x-1}{3} < x + \frac{1}{2} \quad (\text{mult. by } 6) \rightarrow \\ 3(2x+1) + 2(x-1) < 6x + 3 \rightarrow 6x + 3 + 2x - 2 < 6x + 3 \rightarrow \\ 8x + 1 < 6x + 3 \rightarrow 2x < 2 \rightarrow x < 1 : (-\infty, 1)$$

$$10) \quad \frac{2}{3} \leq \frac{5-3t}{-2} \leq \frac{3}{4} \quad (\text{mult. by } 12) \rightarrow \\ 4(2) \leq -6(5-3t) \leq 3(3) \rightarrow 8 \leq -30 + 18t \leq 9 \rightarrow \\ 38 \leq 18t \leq 39 \rightarrow \frac{38}{18} \leq t \leq \frac{39}{18} \rightarrow \frac{19}{9} \leq t \leq \frac{13}{6} \\ \left[\frac{19}{9}, \frac{13}{6} \right]$$

$$15) \quad \text{a.) } |x| \leq \frac{1}{2} \rightarrow -\frac{1}{2} \leq x \leq \frac{1}{2} : \left[-\frac{1}{2}, \frac{1}{2} \right] \\ \text{b.) } |x| > \frac{1}{2} \rightarrow x > \frac{1}{2} \text{ or } x < -\frac{1}{2} : (-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty)$$

19) a.) $x-2 < 1 \rightarrow x < 3 : (-\infty, 3)$
 b.) $|x-2| < 1 \rightarrow -1 < x-2 < 1 \rightarrow 1 < x < 3 : (1, 3)$
 c.) $|x-2| > 1 \rightarrow x-2 > 1$ or $x-2 < -1 \rightarrow$
 $x > 3$ or $x < 1 : (-\infty, 1) \cup (3, \infty)$

22) a.) $3x+5 < 17 \rightarrow 3x < 12 \rightarrow x < 4 : (-\infty, 4)$
 b.) $|3x+5| < 17 \rightarrow -17 < 3x+5 < 17 \rightarrow -22 < 3x < 12 \rightarrow$
 $-\frac{22}{3} < x < 4 : (-\frac{22}{3}, 4)$
 c.) $|3x+5| > 17 \rightarrow 3x+5 > 17$ or $3x+5 < -17 \rightarrow$
 $3x > 12$ or $3x < -22 \rightarrow x > 4$ or $x < -\frac{22}{3} :$
 $(-\infty, -\frac{22}{3}) \cup (4, \infty)$

26) $|\frac{4-5x}{2}| > 1 \rightarrow \frac{4-5x}{2} > 1$ or $\frac{4-5x}{2} < -1 \rightarrow$
 $4-5x > 2$ or $4-5x < -2 \rightarrow -5x > -2$ or $-5x < -6 \rightarrow$
 $x < \frac{2}{5}$ or $x > \frac{6}{5} : (-\infty, \frac{2}{5}) \cup (\frac{6}{5}, \infty)$

27) $|\frac{x+1}{2} - \frac{x-1}{3}| < 1 \rightarrow |\frac{3x+3}{6} - \frac{2x-2}{6}| < 1 \rightarrow$
 $|\frac{(3x+3)-(2x-2)}{6}| < 1 \rightarrow \frac{|3x+3-2x+2|}{6} < 1 \rightarrow$
 $|x+5| < 6 \rightarrow -6 < x+5 < 6 \rightarrow -11 < x < 1 : (-11, 1)$

40) b.) $\sqrt{ab} \leq \frac{a+b}{2}$ equiv. $2\sqrt{ab} \leq (a+b)$ equiv.
 $(2\sqrt{ab})^2 \leq (a+b)^2$ equiv. $4(ab) \leq a^2 + 2ab + b^2$ equiv.
 $0 \leq a^2 - 2ab + b^2$ equiv. $0 \leq (a-b)^2$
 which is true, so $\sqrt{ab} \leq \frac{a+b}{2}$.

$$38) \quad |x-1| + |x-2| < 3$$

case 1: $\boxed{x \geq 2}$ then $x-1 \geq 0$ and $x-2 \geq 0$ so
 $|x-1| + |x-2| < 3 \rightarrow x-1 + x-2 < 3 \rightarrow 2x-3 < 3 \rightarrow$
 $2x < 6 \rightarrow \underline{x < 3}$ so $[2, 3)$ is part of solution;

case 2: $\boxed{x \leq 1}$ then $x-1 \leq 0$ and $x-2 \leq 0$ so
 $|x-1| + |x-2| < 3 \rightarrow -(x-1) + -(x-2) < 3 \rightarrow$
 $-x+1-x+2 < 3 \rightarrow -2x+3 < 3 \rightarrow -2x < 0 \rightarrow$
 $\underline{x > 0}$ so $(0, 1]$ is part of solution;

case 3: $\boxed{1 < x < 2}$ then $x-1 > 0$ and $x-2 < 0$ so
 $|x-1| + |x-2| < 3 \rightarrow (x-1) + -(x-2) < 3 \rightarrow$
 $x-1-x+2 < 3 \rightarrow 1 < 3$ so $(1, 2)$ is part of
solution;

solution: $[2, 3) \cup (0, 1] \cup (1, 2) = (0, 3)$

Section 2.4

1) a.) $x \leq -1$ or $x \geq 4$: $(-\infty, -1) \cup (4, \infty)$
 b.) $-1 \leq x \leq 4$: $[-1, 4]$

2) a.) $-5 < x < 2$: $(-5, 2)$
 b.) $x < -5$ or $x > 2$: $(-\infty, -5) \cup (2, \infty)$

5) a.) $-1 \leq x \leq 1$ or $x \geq 3$: $[-1, 1] \cup [3, \infty)$
 b.) $x < -1$ or $1 < x < 3$: $(-\infty, -1) \cup (1, 3)$

9) $x^2 - 11x + 18 > 0 \rightarrow (x-9)(x-2) > 0$

$$\begin{array}{ccccccc} + & 0 & - & 0 & + \\ | & | & | & | & | \\ \hline & x=2 & & x=9 & \end{array}$$
 $x < 2$ or $x > 9$: $(-\infty, 2) \cup (9, \infty)$

18) $x^4 - 16 < 0 \rightarrow (x^2 - 4)(x^2 + 4) < 0 \rightarrow$
 $(x-2)(x+2)(x^2+4) < 0$

$$\begin{array}{ccccccc} + & 0 & - & 0 & + \\ | & | & | & | & | \\ \hline & x=-2 & & x=2 & \end{array}$$
 $-2 < x < 2$: $(-2, 2)$

20) $2x^3 - 9x^2 + 4x \geq 0 \rightarrow x(2x^2 - 9x + 4) \geq 0 \rightarrow$
 $x(2x-1)(x-4) \geq 0$

$$\begin{array}{ccccccc} - & 0 & + & 0 & - & 0 & + \\ | & | & | & | & | & | & | \\ \hline & x=0 & x=\frac{1}{2} & & x=4 & \end{array}$$
 $0 \leq x \leq \frac{1}{2}$ or $x \geq 4$:
 $[0, \frac{1}{2}] \cup [4, \infty)$

23) $12x^3 + 17x^2 + 6x < 0 \rightarrow x(12x^2 + 17x + 6) < 0 \rightarrow$
 $x(4x+3)(3x+2) < 0$

$$\begin{array}{ccccccc} - & 0 & + & 0 & - & 0 & + \\ | & | & | & | & | & | & | \\ \hline & x=-\frac{3}{4} & x=-\frac{2}{3} & & x=0 & \end{array}$$

$$x < -\frac{3}{4} \text{ or } -\frac{2}{3} < x < 0 : (-\infty, -\frac{3}{4}) \cup (-\frac{2}{3}, 0)$$

$$31) (x+4)(x+5)(x+6) < 0$$

$$x < -6 \text{ or } -5 < x < -4 :$$

$$(-\infty, -6) \cup (-5, -4)$$

$$\begin{array}{ccccccc} - & 0 & + & 0 & - & 0 & + \\ | & | & | & | & | & | & | \\ x & = & -6 & x & = & -5 & x & = & -4 \end{array}$$

$$40) (x+1)^2 - 5(x+1) > 14$$

$$\rightarrow x^2 + 2x + 1 - 5x - 5 > 14 \rightarrow x^2 - 3x - 18 > 0 \rightarrow$$

$$(x-6)(x+3) > 0 \rightarrow$$

$$x < -3 \text{ or } x > 6 :$$

$$(-\infty, -3) \cup (6, \infty)$$

$$\begin{array}{ccccccc} + & 0 & - & 0 & + \\ | & | & | & | & | \\ x & = & -3 & x & = & 6 \end{array}$$

$$43) \frac{x-1}{x+1} \leq 0$$

$$-1 < x \leq 0 : (-1, 0]$$

$$\begin{array}{ccccccc} & \text{No} & & & & & \\ + & | & - & 0 & + \\ | & | & | & | & | \\ x & = & -1 & x & = & 1 \end{array}$$

$$46) \frac{x^2-1}{x^2+8x+15} \geq 0 \rightarrow \frac{(x-1)(x+1)}{(x+3)(x+5)} \geq 0$$

$$\begin{array}{ccccccc} \text{No} & \text{No} & & & & & \\ + & | & - & | & + & 0 & - & 0 & + \\ | & | & | & | & | & | & | & | & | \\ x & = & -5 & x & = & -3 & x & = & -1 & x & = & 1 \end{array}$$

$$x < -5 \text{ or } -3 < x \leq -1 \text{ or } x \geq 1 :$$

$$(-\infty, -5) \cup (-3, -1] \cup [1, \infty)$$

$$53) \frac{1}{x} \leq \frac{1}{x+1} \rightarrow \frac{1}{x} - \frac{1}{x+1} \leq 0 \rightarrow \frac{x+1-x}{x(x+1)} \leq 0 \rightarrow$$

$$\frac{1}{x(x+1)} \leq 0$$

$$\begin{array}{ccccccc} & \text{No} & & \text{No} & & & \\ + & | & - & | & + \\ | & | & | & | & | \\ x & = & -1 & x & = & 0 \end{array}$$

$$-1 < x < 0 : (-1, 0)$$

$$58) \frac{x+1}{x+2} > \frac{x-3}{x+4} \rightarrow \frac{x+1}{x+2} - \frac{x-3}{x+4} > 0$$

$$\frac{(x+1)(x+4) - (x+2)(x-3)}{(x+2)(x+4)} > 0 \rightarrow$$

$$\frac{\cancel{x^2} + 5x + 4 - (\cancel{x^2} - x - 6)}{(x+2)(x+4)} > 0 \rightarrow \frac{6x+10}{(x+2)(x+4)} > 0 \rightarrow$$

$$\frac{6(x + \frac{5}{3})}{(x+2)(x+4)} > 0$$

$$\begin{array}{ccccccc} & \text{No} & & \text{No} & & & \\ & \downarrow & & \downarrow & & & \\ - & | & + & | & - & 0 & + \\ & x = -4 & & x = -2 & & x = -\frac{5}{3} & \end{array}$$

$$-4 < x < -2 \text{ or } x > -\frac{5}{3} : (-4, -2) \cup (-\frac{5}{3}, \infty)$$

$$80) \frac{n(n+1)}{2} < 1225 \rightarrow n^2 + n < 2450 \rightarrow$$

$$n^2 + n - 2450 < 0 \rightarrow (n+50)(n-49) < 0$$

$$\begin{array}{ccccccc} + & 0 & - & 0 & + \\ | & & & & | \\ n = -50 & & & & n = 49 \end{array}$$

$$-50 < n < 49 : (-50, 49) \text{ or } -49, -48, -47, \dots, 48.$$

$$84) \quad \frac{x}{\frac{x}{4} \quad \frac{x}{4}} \quad \frac{12-x}{\frac{12-x}{4} \quad \frac{12-x}{4}}$$

Combined area of squares is

$$\left(\frac{x}{4}\right)^2 + \left(\frac{12-x}{4}\right)^2 > 5 \rightarrow \frac{x^2}{16} + \frac{(x-12)^2}{16} > 5 \rightarrow$$

$$x^2 + (x^2 - 24x + 144) > 80 \rightarrow 2x^2 - 24x + 64 > 0 \rightarrow$$

$$2(x^2 - 12x + 32) > 0 \rightarrow 2(x-8)(x-4) > 0$$

$$\begin{array}{ccccccc} + & 0 & - & 0 & + \\ | & & & & | \\ x = 4 & & & & x = 8 \end{array}$$

$$x > 8 \text{ or } x < 4 : (-\infty, 4) \cup (8, \infty)$$