Section 3.1

11) a.) $f(x) = x^2 - 9$; Domain : all x-values

b.) $g(x) = \dfrac{1}{x^2-9} = \dfrac{1}{(x-3)(x+3)}$; Domain : all x-values except $x=3, x=-3$

c.) $h(x) = \sqrt{x^2-9} = \sqrt{(x-3)(x+3)}$; do sign chart for

$(x-3)(x+3)$ $\;\; + \; 0 \; - \; 0 \; +$ at $x=-3$, $x=3$; Domain : $x \geq 3$ or $x \leq -3$

d.) $k(x) = \sqrt[3]{x^2-9}$; Domain : all x-values

14) a.) $F(x) = 2x^2 + x - 6$; Domain : all x-values

b.) $G(x) = \dfrac{1}{2x^2+x-6} = \dfrac{1}{(2x-3)(x+2)}$; Domain: all x-values except $x = \frac{3}{2}, x = -2$

c.) $H(x) = \sqrt{2x^2+x-6} = \sqrt{(2x-3)(x+2)}$; do sign chart for

$(2x-3)(x+2)$ $\;\; + \; 0 \; - \; 0 \; +$ at $x=-2$, $x=3/2$; Domain : $x \geq 3/2$ or $x \leq -2$

d.) $K(x) = \sqrt[3]{2x^2+x-6}$; Domain : all x-values

5&6) a) function : f, g, F, H

not function : h, G

b) range of f : $\{1,2,3\}$, range of g : $\{2,3\}$,

range of F : $\{1\}$, range of H : $\{1,2\}$

29) $f(x) = x^2 - 3x + 1$

a.) $f(1) = 1 - 3 + 1 = -1$ b.) $f(0) = 0 - 0 + 1 = 1$

c.) $f(-1) = 1 - 3(-1) + 1 = 5$ d.) $f\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} + 1 = \frac{-9}{4} + \frac{4}{4} = \frac{-5}{4}$

e.) $f(z) = z^2 - 3z + 1$

f.) $f(x+1) = (x+1)^2 - 3(x+1) + 1 = x^2 + 2x + 1 - 3x - 3 + 1 = x^2 - x - 1$

g.) $f(a+1) = (a+1)^2 - 3(a+1) + 1 = a^2 + 2a + 1 - 3a - 3 + 1 = a^2 - a - 1$

h.) $f(-x) = (-x)^2 - 3(-x) + 1 = x^2 + 3x + 1$

i.) $|f(1)| = |-1| = 1$ j.) $f(\sqrt{3}) = (\sqrt{3})^2 - 3\sqrt{3} + 1 = 4 - 3\sqrt{3}$

k.) $f(1+\sqrt{2}) = (1+\sqrt{2})^2 - 3(1+\sqrt{2}) + 1$

$= 1 + 2\sqrt{2} + 2 - 3 - 3\sqrt{2} + 1 = 1 - \sqrt{2}$

l.) $|1 - f(2)| = |1 - (4 - 6 + 1)| = |1 - (-1)| = |2| = 2$

31) $f(x) = 3x^2$

a.) $f(2x) = 3(2x)^2 = 3(4x^2) = 12x^2$

b.) $2f(x) = 2(3x^2) = 6x^2$

c.) $f(x^2) = 3(x^2)^2 = 3x^4$

d.) $[f(x)]^2 = [3x^2]^2 = 9x^4$

e.) $f\left(\frac{x}{2}\right) = 3\left(\frac{x}{2}\right)^2 = 3\left(\frac{x^2}{4}\right) = \frac{3}{4}x^2$

f.) $\frac{f(x)}{2} = \frac{3x^2}{2} = \frac{3}{2}x^2$

Section 3.2

8) Domain : $[-4, 4]$
Range : $\{4\} \cup [-2, 3)$

9) Domain : $[-3, 4]$
Range : $[-2, 2]$

10) Domain : $[-4, 4]$
Range : $[-2, 2)$

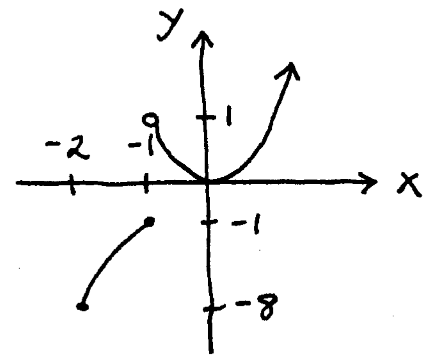
14) Domain : $(-3, 6)$
Range : $\{-1\}$

16) a.) $F(4) = 3$ b.) $F(-1) = 0$
c.) $F(-4) = 2 > 0$ d.) $F(x) = 5$ when $x = -6$
e.) $F(5) - F(-3) = 3 - 3 = 0$

19) a.) $f(-2) = 0$, $g(-2) = 1$ so $f(-2) < g(-2)$
b.) $f(0) - g(0) = 2 - (-3) = 5$
c.) $f(1) - g(1) = 1 - (-1) = 2$, $f(2) - g(2) = 1 - 0 = 1$,
$f(3) - g(3) = 4 - 1 = 3$ so $f(2) - g(2)$ is smallest

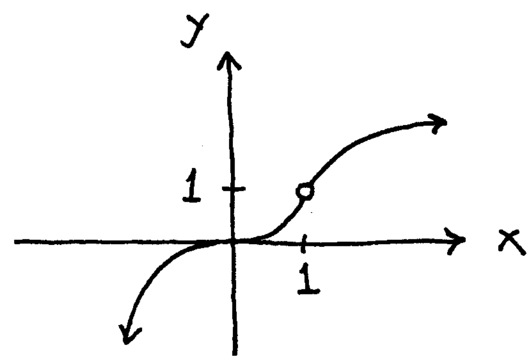
23)

$$A(x) = \begin{cases} x^3 \text{ if } -2 \le x \le -1 \\ x^2 \text{ if } x > -1 \end{cases}$$



25)

$$C(x) = \begin{cases} x^3 \text{ if } x < 1 \\ \sqrt{x} \text{ if } x > 1 \end{cases}$$



35) $x = \frac{2}{5}$ ~~so~~ $g\left(\frac{2}{5}\right) = -\sqrt{1-\left(\frac{2}{5}\right)^2} = -\sqrt{\frac{25}{25} - \frac{4}{25}} = -\sqrt{\frac{21}{25}}$.

$\rightarrow P = \left(\frac{2}{5}, -\sqrt{\frac{21}{25}}\right)$; then $Q = \left(-\sqrt{\frac{21}{25}}, -\sqrt{\frac{21}{25}}\right)$;

if $x = -\sqrt{\frac{21}{25}}$ then $g\left(-\sqrt{\frac{21}{25}}\right) = -\sqrt{1-\left(-\sqrt{\frac{21}{25}}\right)^2}$

$= -\sqrt{1 - \frac{21}{25}} = -\sqrt{\frac{25}{25} - \frac{21}{25}} = -\sqrt{\frac{4}{25}} = -\frac{2}{5} \rightarrow R = \left(-\sqrt{\frac{21}{25}}, -\frac{2}{5}\right)$

Section 3.3

9) $f(x) = x^2 + 2x$ on $[3, 5] \longrightarrow$

$$ARC = \frac{f(5) - f(3)}{5 - 3} = \frac{35 - 15}{2} = \frac{20}{2} = 10$$

10) $f(x) = \sqrt{x}$ on $[4, 9] \longrightarrow$

$$ARC = \frac{f(9) - f(4)}{9 - 4} = \frac{\sqrt{9} - \sqrt{4}}{9 - 4} = \frac{3 - 2}{5} = \frac{1}{5}$$

15) a.) $ARC = \frac{G(3) - G(0)}{3 - 0} = \frac{23 - 22}{3 - 0} = \frac{1}{3}$ °C/min.

b.) $ARC = \frac{G(6) - G(3)}{6 - 3} = \frac{27 - 23}{3} = \frac{4}{3}$ °C/min.

c.) $ARC = \frac{G(8) - G(6)}{8 - 6} = \frac{28 - 27}{2} = \frac{1}{2}$ °C/min.

Worksheet 3

1) a) $f(x) = -2x+9$,

$$\frac{f(x+h)-f(x)}{h} = \frac{-2(x+h)+9-(-2x+9)}{h}$$

$$= \frac{\cancel{-2x}-2h+\cancel{9}+\cancel{2x}-\cancel{9}}{h} = \frac{-2\cancel{h}}{\cancel{h}} = -2$$

b) $f(x) = 2x^2-3x+1$,

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2-3(x+h)+1-(2x^2-3x+1)}{h}$$

$$= \frac{2(x^2+2hx+h^2)-\cancel{3x}-3h+\cancel{1}-2x^2+\cancel{3x}-\cancel{1}}{h}$$

$$= \frac{\cancel{2x^2}+4hx+2h^2-3h-\cancel{2x^2}}{h} = \frac{\cancel{h}(4x+2h-3)}{\cancel{h}} = 4x+2h-3$$

c) $f(x) = \frac{3}{x}$,

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{3}{x+h}-\frac{3}{x}}{\frac{h}{1}} = \left[\frac{3x}{x(x+h)}-\frac{3(x+h)}{x(x+h)}\right]\cdot\frac{1}{h}$$

$$= \frac{\cancel{3x}-\cancel{3x}-3h}{x(x+h)h} = \frac{-3\cancel{h}}{x(x+h)\cancel{h}} = \frac{-3}{x(x+h)}$$

d) $H(x) = 1-2x^2 \rightarrow$

$$\frac{H(x+h)-H(x)}{h} = \frac{1-2(x+h)^2-(1-2x^2)}{h}$$

$$= \frac{\cancel{1}-2(x^2+2hx+h^2)-\cancel{1}+2x^2}{h} = \frac{\cancel{-2x^2}-4hx-2h^2+\cancel{2x^2}}{h}$$

$$= \frac{\cancel{h}(-4x-2h)}{\cancel{h}} = -4x-2h$$

2) $g(x) = 2x^2 - x + 1$,

$$\frac{g(x)-g(a)}{x-a} = \frac{2x^2 - x + 1 - (2a^2 - a + 1)}{x-a}$$

$$= \frac{2x^2 - 2a^2 - x + a + 1 - 1}{x-a} = \frac{2(x^2-a^2) - (x-a)}{x-a}$$

$$= \frac{2(x-a)(x+a) - (x-a)}{x-a} = \frac{\cancel{(x-a)}\left[2(x+a)-1\right]}{\cancel{x-a}} = 2(x+a) - 1$$

3) a.) $f(x) = 4x - 3$, $g(x) = 8 - x \rightarrow$

$4x - 3 = 8 - x \rightarrow 5x = 11 \rightarrow x = 11/5$

$f(x) = 2x^2 - x$, $g(x) = 3 \rightarrow$

$2x^2 - x = 3 \rightarrow 2x^2 - x - 3 = 0 \rightarrow (2x-3)(x+1) = 0 \rightarrow$

$x = 3/2$ or $x = -1$

4) $f(n) = n^2 - n + 2$ and

if $f(n^2+k) = f(n) \cdot f(n+1)$ for all values of n then it must be true for $n = 0 \rightarrow$

$f(k) = f(0) \cdot f(1) \rightarrow k^2 - k + 2 = (2)(2) = 4 \rightarrow$

$k^2 - k - 2 = 0 \rightarrow (k-2)(k+1) = 0 \rightarrow k = 2$ or $k = -1$

<u>TEST $k=-1$</u> : Does $f(n^2-1) = f(n)f(n+1)$?

$f(n)f(n+1) = (n^2 - n + 2)((n+1)^2 - (n+1) + 2)$

$= \cdots = n^4 + 3n^2 + 4$ and

$f(n^2-1) = (n^2-1)^2 - (n^2-1) + 2 = \cdots = n^4 - 3n^2 + 4$, NO.

<u>TEST $k=2$:</u> Does $f(n^2+2) = f(n)f(n+1)$?

$f(n^2+2) = (n^2+2)^2 - (n^2+2) + 2 = \cdots = n^4 + 3n^2 + 4$, YES.