Section 3.6

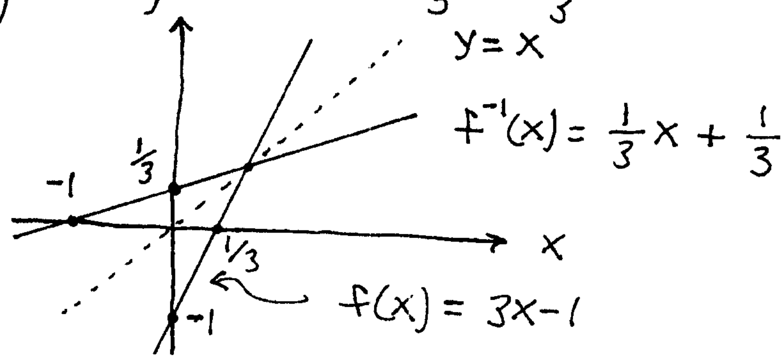
8) $f(x) = 3x - 1$

a.) $Y = 3x - 1 \to$ (switch variables and solve for y) $\to$ $x = 3Y - 1 \to 3Y = x + 1 \to$
$Y = \frac{1}{3}x + \frac{1}{3} \to f^{-1}(x) = \frac{1}{3}x + \frac{1}{3}$

c) $f(f^{-1}(x)) = f\left(\frac{1}{3}x + \frac{1}{3}\right) = 3\left(\frac{1}{3}x + \frac{1}{3}\right) - 1$
$= x + 1 - 1 = x$

$f^{-1}(f(x)) = f^{-1}(3x - 1) = \frac{1}{3}(3x - 1) + \frac{1}{3}$
$= x - \frac{1}{3} + \frac{1}{3} = x$

d)



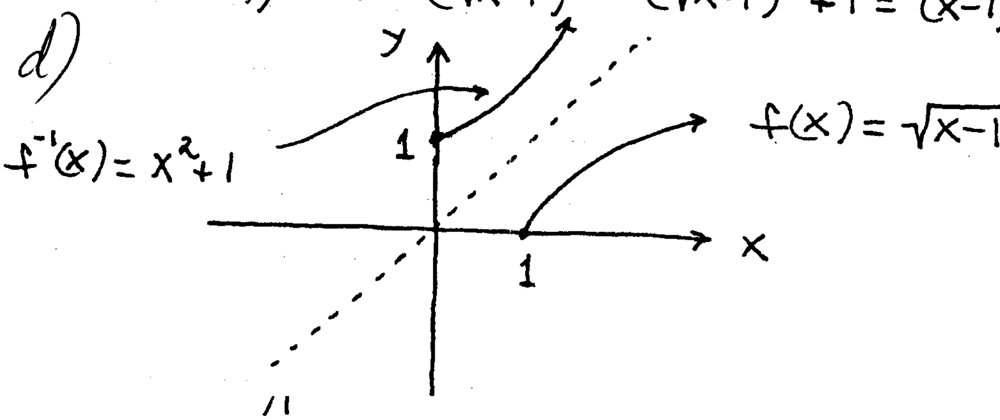
10) $f(x) = \sqrt{x-1}$ (for $x \geq 1$ and $Y \geq 0$)

a.) $Y = \sqrt{x-1} \to$ (switch variables and solve for y) $\to$ $x = \sqrt{Y-1} \to x^2 = Y - 1 \to$
$Y = x^2 + 1 \to f^{-1}(x) = x^2 + 1$ (for $Y \geq 1$ and $x \geq 0$)

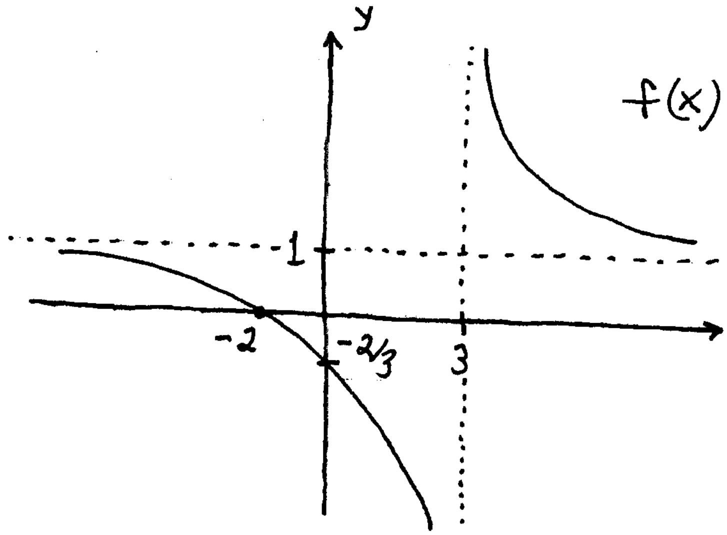
c) $f(f^{-1}(x)) = f(x^2+1) = \sqrt{(x^2+1)-1} = \sqrt{x^2}$
$= |x| = x$ (since $x \geq 0$)

$f^{-1}(f(x)) = f^{-1}(\sqrt{x-1}) = (\sqrt{x-1})^2 + 1 = (x-1) + 1 = x$

d)



15)



$f(x) = \frac{x+2}{x-3}$

a.) Domain: all $x \neq 3$
Range: all $y \neq 1$

b.) f^{-1} exists by horizontal line test
Domain for f^{-1}: all $x \neq 1$ (Range of f)
Range for f^{-1}: all $y \neq 3$ (Domain of f)

c) $y = \frac{x+2}{x-3}$ $\rightarrow$ (switch variables and solve for y) $\rightarrow$

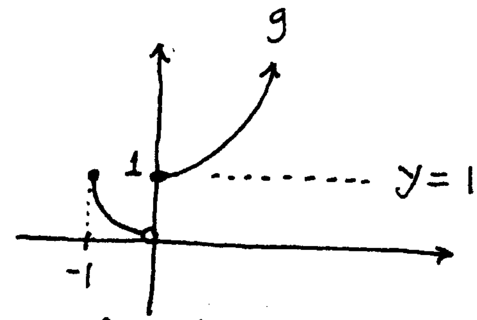
$x = \frac{y+2}{y-3}$ $\rightarrow$ $x(y-3) = y+2$ $\rightarrow$

$xy - 3x = y+2$ $\rightarrow$ $xy - y = 3x+2$ $\rightarrow$
$y(x-1) = 3x+2$ $\rightarrow$ $y = \frac{3x+2}{x-1}$ $\rightarrow$

$f^{-1}(x) = \frac{3x+2}{x-1}$

27)

$$g(x) = \begin{cases} x^2 & \text{if } -1 \le x < 0 \\ x^2+1 & \text{if } x \ge 0 \end{cases}$$

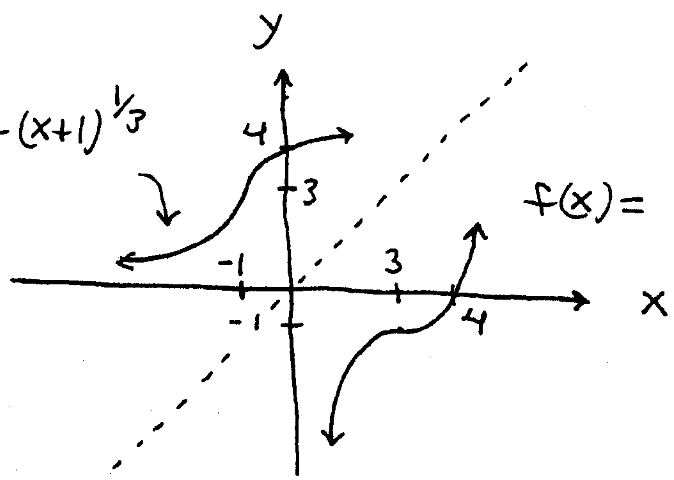


g is not 1-1 since it fails the horizontal line test at y = 1

36) $f(x) = (x-3)^3 - 1$

d.) $y = (x-3)^3 - 1 \rightarrow$ (switch variables and solve for y) $\rightarrow x = (y-3)^3 - 1 \rightarrow (y-3)^3 = x+1 \rightarrow (y-3)^{3 \cdot \frac{1}{3}} = (x+1)^{1/3} \rightarrow y - 3 = (x+1)^{1/3} \rightarrow y = 3 + (x+1)^{1/3} \rightarrow f^{-1}(x) = 3 + (x+1)^{1/3}$

c) $f^{-1}(x) = 3 + (x+1)^{1/3}$



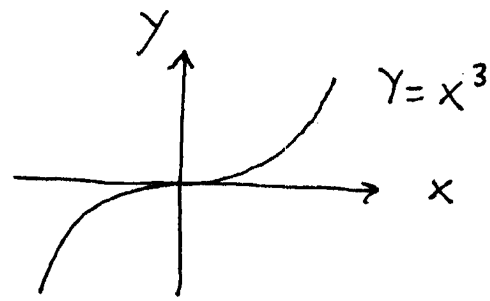
$f(x) = (x-3)^3 - 1$

40) $A = (a, f(a)) \longrightarrow B = (f(a), f(a))$;
at point C the y-value is $f(a)$ so
$f^{-1}(x) = f(a) \rightarrow f(f^{-1}(x)) = f(f(a)) \rightarrow x = f(f(a))$
and $C = (f(f(a)), f(a)) \longrightarrow$
$D = (f(a), f(f(a)))$

41) $A = (b, f^{-1}(b)) \longrightarrow B = (f^{-1}(b), f^{-1}(b)) \longrightarrow$
$C = (f^{-1}(b), f(f^{-1}(b))) = (f^{-1}(b), b)$; at point D the
y-value is b so $b = f^{-1}(x) \rightarrow f(b) = f(f^{-1}(x)) = x$
and $D = (f(b), b) \longrightarrow E = (b, f(b))$; at point
F the y-value is $f(b)$ so $f(b) = f^{-1}(x) \rightarrow$
$f(f(b)) = f(f^{-1}(x)) = x$ and $F = (f(f(b)), f(b))$

Worksheet 5

1)



y is 1-1.

2) $g(x) = \frac{1}{x} + 3 \rightarrow g(b) = g(a) \rightarrow$
$\frac{1}{b} + 3 = \frac{1}{a} + 3 \rightarrow \frac{1}{b} = \frac{1}{a} \rightarrow b = a$ so g is 1-1.

3) $g(x) = \sqrt{2x+1} \rightarrow g(b) = g(a) \rightarrow$
$\sqrt{2b+1} = \sqrt{2a+1} \rightarrow (\sqrt{2b+1})^2 = (\sqrt{2a+1})^2 \rightarrow$
$2b+1 = 2a+1 \rightarrow 2b = 2a \rightarrow b = a$ so g is 1-1.

4) $f(t) = \frac{t+4}{2t-7} \rightarrow f(b) = f(a) \rightarrow$
$\frac{b+4}{2b-7} = \frac{a+4}{2a-7} \rightarrow (b+4)(2a-7) = (a+4)(2b-7) \rightarrow$
$2ab - 7b + 8a - 28 = 2ab - 7a + 8b - 28 \rightarrow 15a = 15b$
$\rightarrow a = b$ so f is 1-1.

5) $f(x) = \frac{3x-2}{5x-3}$ then

$$f(f(x)) = f\left(\frac{3x-2}{5x-3}\right) = \frac{3\left(\frac{3x-2}{5x-3}\right) - 2}{5\left(\frac{3x-2}{5x-3}\right) - 3} \cdot \frac{5x-3}{5x-3}$$

$$= \frac{3(3x-2) - 2(5x-3)}{5(3x-2) - 3(5x-3)} = \frac{9x - 6 - 10x + 6}{15x - 10 - 15x + 9} = \frac{-x}{-1} = x$$

6) Show $f(g(x)) = x$ and $g(f(x)) = x$:

$f(g(x)) = f(\frac{1}{4}x + \frac{1}{4}) = 4(\frac{1}{4}x + \frac{1}{4}) - 1$

$= x + 1 - 1 = x$

$g(f(x)) = g(4x-1) = \frac{1}{4}(4x-1) + \frac{1}{4}$

$= x - \frac{1}{4} + \frac{1}{4} = x$.

7) $f(x) = 2x+1$

a.) $Y = 2X+1 \rightarrow$ (switch variables and solve for y) $\rightarrow X = 2Y+1 \rightarrow 2Y = X-1 \rightarrow$

$Y = \frac{1}{2}X - \frac{1}{2} \rightarrow f^{-1}(x) = \frac{1}{2}X - \frac{1}{2}$

b.) $f^{-1}(5) = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$,

$\frac{1}{f(5)} = \frac{1}{11}$.