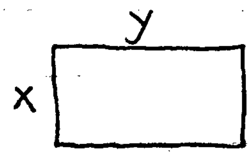
Section 4.4

1) 

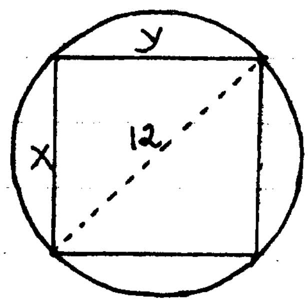
a.) perimeter:

$2x+2y=16 \rightarrow x+y=8 \rightarrow$

$y=8-x$ so area $A=xy=x(8-x) \rightarrow A=8x-x^2$

b.) area: $xy=85 \rightarrow y=85/x$ so perimeter

$P=2x+2y=2x+2\left(\frac{85}{x}\right) \rightarrow P=2x+\frac{170}{x}$

2) 

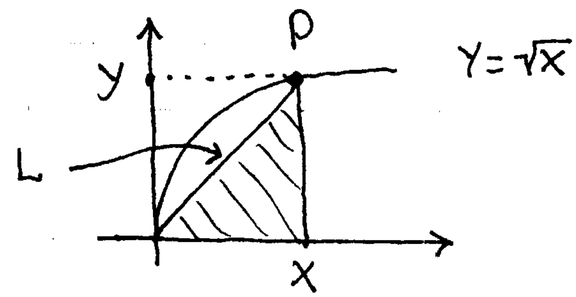
$x^2+y^2=12^2 \rightarrow y=\sqrt{144-x^2}$

a.) perimeter:

$P=2x+2y=2x+2\sqrt{144-x^2} \rightarrow$

$P=2x+2\sqrt{144-x^2}$

b.) area: $A=xy=x\cdot\sqrt{144-x^2} \rightarrow A=x\sqrt{144-x^2}$

4) 

a.) area:

$A=\frac{1}{2}(\text{base})(\text{height})$

$=\frac{1}{2}xy=\frac{1}{2}x(\sqrt{x})$

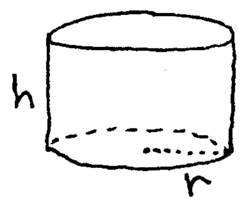
$=\frac{1}{2}x^1\cdot x^{1/2}=\frac{1}{2}x^{3/2} \rightarrow$

$A=\frac{1}{2}x^{3/2}$

b.) $L^2=x^2+y^2=x^2+(\sqrt{x})^2=x^2+x \rightarrow$

$L=\sqrt{x^2+x}$ so perimeter $P=x+y+L$

$=x+\sqrt{x}+\sqrt{x^2+x} \rightarrow P=x+\sqrt{x}+\sqrt{x^2+x}$

6) 

volume: $\pi r^2 h=12 \rightarrow$

a) $h=\frac{12}{\pi r^2}$ so surface area is

$S=S_{top}+S_{bottom}+S_{side}$

$= \pi r^2 + \pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r\left(\frac{12}{\pi r^2}\right) \rightarrow$

$S = 2\pi r^2 + \frac{24}{r}$

7) Let x and y represent numbers. Then

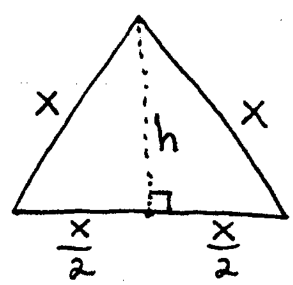
$x + y = 16 \rightarrow y = 16 - x$

a.) product : $P = xy = x(16-x) \rightarrow P = 16x - x^2$

b.) sum of squares : $S = x^2 + y^2 = x^2 + (16-x)^2$

$= x^2 + (x-16)^2$ (why?) $\rightarrow S = x^2 + (x-16)^2$

13)



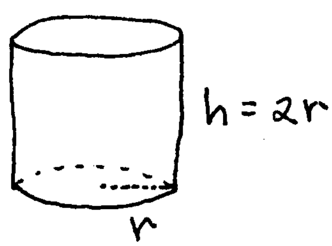
$x^2 = h^2 + \left(\frac{x}{2}\right)^2 = h^2 + \frac{1}{4}x^2 \rightarrow$

$h^2 = \frac{3}{4}x^2 \rightarrow h = \frac{\sqrt{3}}{2}x$ so

area $A = \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}xh = \frac{1}{2}x \cdot \frac{\sqrt{3}}{2}x \rightarrow A = \frac{\sqrt{3}}{4}x^2$

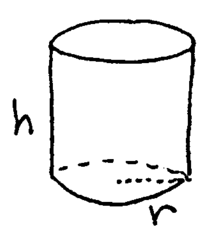
14)



Volume $V = \pi r^2 h = \pi r^2(2r)$

$\rightarrow V = 2\pi r^3$

16)



Surface area :

$2\pi r^2 + 2\pi r h = 14 \rightarrow$

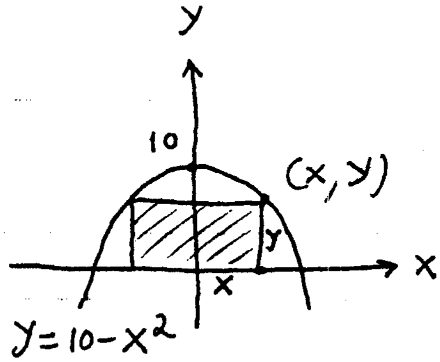
$\pi r^2 + \pi r h = 7 \rightarrow$

$\pi r h = 7 - \pi r^2 \rightarrow h = \frac{7 - \pi r^2}{\pi r}$

so volume $V = \pi r^2 h = \cancel{\pi} r^{\cancel{2}} \cdot \frac{(7 - \pi r^2)}{\cancel{\pi}\cancel{r}} \rightarrow$

$V = 7r - \pi r^3$

18)

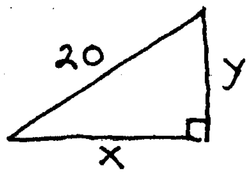


Area $A = (2x)y$

$= 2x(10 - x^2) \rightarrow$

$A = 20x - 2x^3$

19)



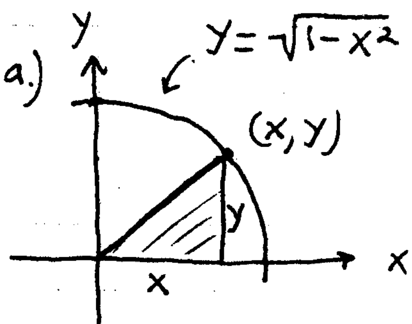
$x^2 + y^2 = 20^2 \rightarrow y^2 = 400 - x^2 \rightarrow$

$y = \sqrt{400 - x^2}$ so area

$A = \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}(x)(y) = \frac{1}{2}x \cdot \sqrt{400 - x^2} \rightarrow A = \frac{1}{2}x\sqrt{400 - x^2}$

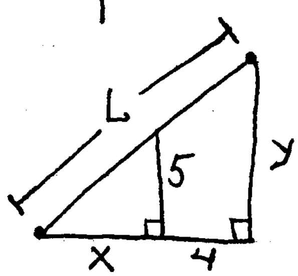
20) a.)



Area $A = \frac{1}{2}xy \rightarrow$

$A = \frac{1}{2}x\sqrt{1 - x^2}$.

21)



By similar triangles:

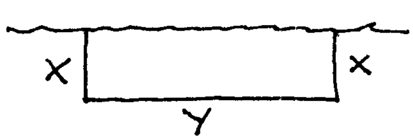
$\frac{5}{x} = \frac{y}{x+4} \rightarrow y = \frac{5}{x}(x+4)$

$\rightarrow y = 5 + \frac{20}{x}$ then

$L^2 = (x+4)^2 + y^2 = (x+4)^2 + \left(5 + \frac{20}{x}\right)^2 \rightarrow$

$L = \sqrt{(x+4)^2 + \left(5 + \frac{20}{x}\right)^2}$.

22)



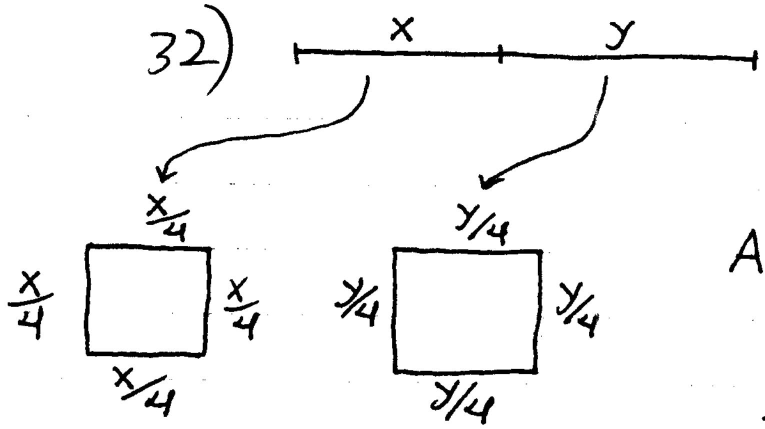
fencing:

$2x + y = 500 \rightarrow$

$y = 500 - 2x$ then

area $A = xy = x(500 - 2x) \rightarrow A = 500x - 2x^2$

32)



$x + y = 4 \rightarrow$
$y = 4 - x$;

total area is

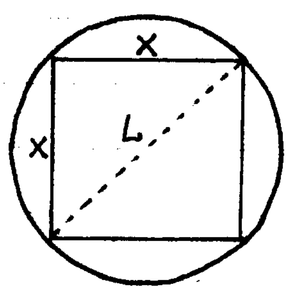
$$A = \left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2$$
$$= \frac{1}{4}x^2 + \frac{1}{4}y^2$$
$$= \frac{1}{4}x^2 + \frac{1}{4}(4-x)^2$$
$$= \frac{1}{4}x^2 + \frac{1}{4}(16 - 8x + x^2) = \frac{1}{4}x^2 + 4 - 2x + \frac{1}{4}x^2 \rightarrow$$
$$A = \frac{1}{2}x^2 - 2x + 4$$

34) volume : $\frac{1}{3}\pi r^2 h = 12\pi \rightarrow r^2 h = 36$

a.) $h = 36/r^2$

b.) $r^2 = 36/h \rightarrow r = \sqrt{36/h} \rightarrow r = \frac{6}{\sqrt{h}}$

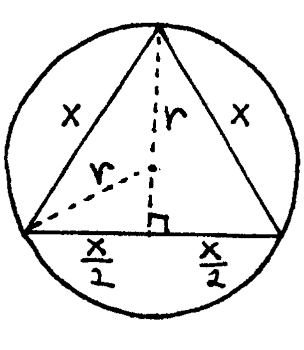
42)

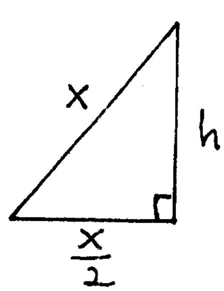


$L^2 = x^2 + x^2 = 2x^2 \rightarrow$
$L = \sqrt{2x^2} = \sqrt{2}\,x$ is diameter of circle so radius $r = \frac{1}{2}\sqrt{2}\,x$ then

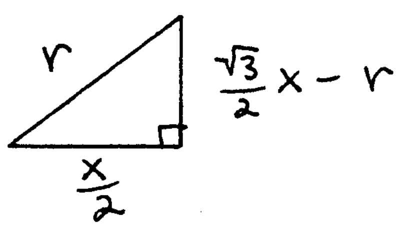
$$\text{Area} = \pi r^2 = \pi\left(\frac{1}{2}\sqrt{2}\,x\right)^2 = \pi\left(\frac{1}{4}\cdot 2 \cdot x^2\right) \rightarrow$$
$$A = \frac{1}{2}\pi x^2$$

43)





$$h^2 + \left(\frac{x}{2}\right)^2 = x^2 \rightarrow$$
$$h^2 + \frac{1}{4}x^2 = x^2 \rightarrow$$
$$h^2 = \frac{3}{4}x^2 \rightarrow$$
$$h = \frac{\sqrt{3}}{2}x$$;



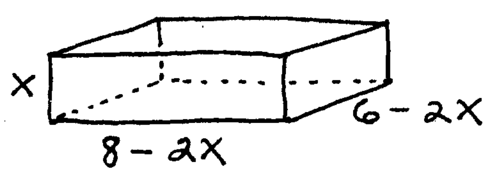
$$\left(\frac{x}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}x - r\right)^2 = r^2 \rightarrow$$

$$\frac{1}{4}x^2 + \frac{3}{4}x^2 - \sqrt{3}\,rx + \cancel{r^2} = \cancel{r^2} \rightarrow$$

$$\sqrt{3}\,rx = x^2 \rightarrow \quad r = \frac{1}{\sqrt{3}}x \quad \text{then}$$

area $A = \pi r^2 = \pi\left(\frac{1}{\sqrt{3}}x\right)^2 \rightarrow \quad A = \frac{\pi}{3}x^2$.

45a)

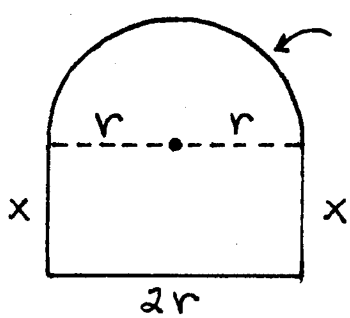


Volume

$V = (l)(w)(h) \rightarrow$

$V = (8-2x)(6-2x)(x)$.

47a)



$\frac{1}{2}(2\pi r) = \pi r$;

perimeter :

$2x + 2r + \pi r = 32 \rightarrow$

$2x = 32 - 2r - \pi r \rightarrow$

$x = 16 - r - \frac{1}{2}\pi r$ then total area

$$A = A_{\square} + A_{\triangle} = (x)(2r) + \frac{1}{2}\pi r^2$$

$$= \left(16 - r - \frac{1}{2}\pi r\right)(2r) + \frac{1}{2}\pi r^2$$

$$= 32r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2 \rightarrow$$

$$A = 32r - 2r^2 - \frac{1}{2}\pi r^2 .$$