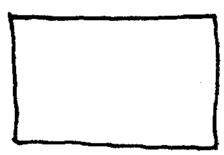
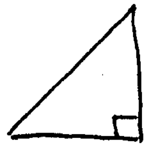
Section 4.5

1) Let x and y be numbers:
$x+y=5 \rightarrow y=5-x$ then maximize
product $P=xy=x(5-x)=5x-x^2=-(x^2-5x)$
$=-(x^2-5x+\frac{25}{4})+\frac{25}{4}=-(x-\frac{5}{2})^2+\frac{25}{4}$;
maximum $P=\frac{25}{4}$ when $x=\frac{5}{2}$ and $y=5/2$.

3) Let x and y be numbers:
$y-x=1 \rightarrow y=x+1$ then minimize
sum $S=x^2+y^2=x^2+(x+1)^2=x^2+x^2+2x+1$
$=2x^2+2x+1=2(x^2+x+1/4)+1-1/2=2(x+\frac{1}{2})^2+\frac{1}{2}$;
minimum $S=1/2$ when $x=-1/2$ and $y=1/2$.

6) x  y

perimeter
$2x+2y=80 \rightarrow$
$x+y=40 \rightarrow y=40-x$
then maximize area $A=xy=x(40-x)$
$=40x-x^2=-(x^2-40x)=-(x^2-40x+400)+400$
$=-(x-20)^2+400$;
maximum $A=400$ cm.^{2} when
$x=20$ cm. and $y=20$ cm.

7)  y x

$x+y=100 \rightarrow y=100-x$;
maximize area
$A=\frac{1}{2}xy=\frac{1}{2}x(100-x)$
$=50x-\frac{1}{2}x^2=-\frac{1}{2}(x^2-100x)=-\frac{1}{2}(x^2-100x+2500)+1250$
$=-\frac{1}{2}(x-50)^2+1250$; maximum $A=1250$ in.^{2}
when $x=50$ in. and $y=50$ in.

10) $h(t) = 512t - 16t^2 = -16(t^2 - 32t)$

$= -16(t^2 - 32t + 256) + 4096 = -16(t-16)^2 + 4096$;

maximum $h(t) = 4096$ ft. when $t = 16$ sec. ;

hit ground : $h(t) = 0$ ft. $\rightarrow 512t - 16t^2 = 0 \rightarrow$

$16t(32 - t) = 0 \rightarrow t = 32$ sec.

11) height $h(t) = -16t^2 + 32t$ ft.

a.) $h(1) = -16 + 32 = 16$ ft.

$h(3/2) = -16(9/4) + 32(3/2) = -36 + 48 = 12$ ft.

b.) $h(t) = -16(t^2 - 2t) = -16(t^2 - 2t + 1) + 16$

$= -16(t-1)^2 + 16$ then maximum

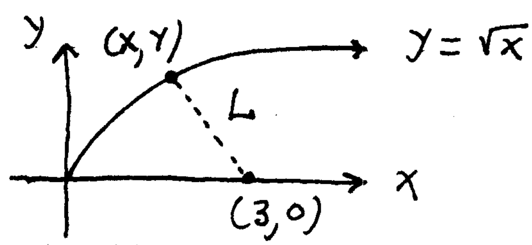
$h(t) = 16$ ft. when $t = 1$ sec.

c.) $h(t) = 7$ ft. $\rightarrow -16t^2 + 32t = 7 \rightarrow$

$0 = 16t^2 - 32t + 7 = (4t-1)(4t-7) = 0 \rightarrow$

$t = 1/4$ sec. or $t = 7/4$ sec.

12)



minimize distance

$L = \sqrt{(x-3)^2 + (y-0)^2}$

$= \sqrt{(x-3)^2 + y^2}$

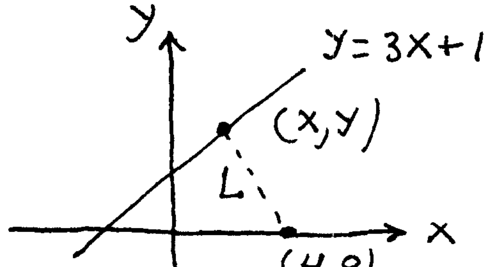
$= \sqrt{(x-3)^2 + (\sqrt{x})^2} = \sqrt{x^2 - 6x + 9 + x} = \sqrt{x^2 - 5x + 9}$

$= \sqrt{(x^2 - 5x + \frac{25}{4}) - \frac{25}{4} + 9} = \sqrt{(x - 5/2)^2 + 11/4}$;

minimum $L = \sqrt{11/4}$ when $x = 5/2$ and

$y = \sqrt{5/2}$.

14)



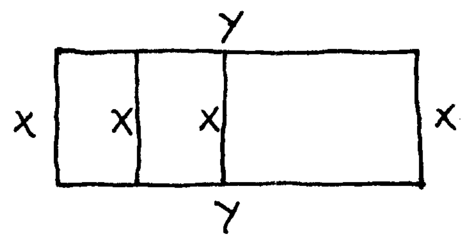
minimize distance

$L = \sqrt{(x-4)^2 + (y-0)^2}$

$= \sqrt{(x-4)^2 + (3x+1)^2}$

$= \sqrt{x^2 - 8x + 16 + 9x^2 + 6x + 1} = \sqrt{10x^2 - 2x + 17}$
$= \sqrt{10(x^2 - \frac{1}{5}x + \frac{1}{100}) - \frac{1}{10} + 17} = \sqrt{10(x - \frac{1}{10})^2 + \frac{169}{10}}$;
minimum $L = \sqrt{169/10}$ when $x = \frac{1}{10}$ and
$y = 13/10$.

16)

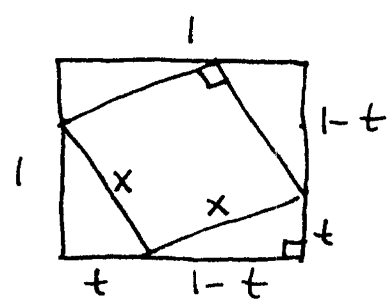


fencing
$4x + 2y = 1800$ m. $\rightarrow$
$2x + y = 900 \rightarrow$

$y = 900 - 2x$; maximize area $A = xy$
$= x(900 - 2x) = 900x - 2x^2 = -2(x^2 - 450x)$
$= -2(x^2 - 450x + 50{,}625) + 101{,}250$
$= -2(x - 225)^2 + 101{,}250$; maximum area is
$A = 101{,}250$ m.^{2} when $x = 225$ m. and
$y = 450$ m.

20) maximize revenue $R = \frac{-1}{5}x^2 + 200x$
$= \frac{-1}{5}(x^2 - 1000x) = \frac{-1}{5}(x^2 - 1000x + 250{,}000) + 50{,}000$
$= -\frac{1}{5}(x - 500)^2 + 50{,}000$; maximum
$R = \$50{,}000$ when $x = 500$ units .

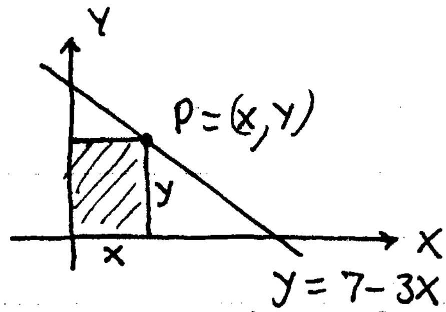
29)



by Pythagorean Theorem
$x^2 = (1 - t)^2 + t^2$;
maximize area
$A = x^2 = (1 - t)^2 + t^2 = 1 - 2t + t^2 + t^2$
$= 2t^2 - 2t + 1 = 2(t^2 - t + \frac{1}{4}) + \frac{-1}{2} + 1$
$= 2(t - \frac{1}{2})^2 + \frac{1}{2}$; maximum $A = \frac{1}{2}$ when

$t = \frac{1}{2}$ and $x = \sqrt{(\frac{-1}{2})^2 + (\frac{1}{2})^2} = \sqrt{1/2}$.

31)



maximize area

$A = xy = x(7-3x)$

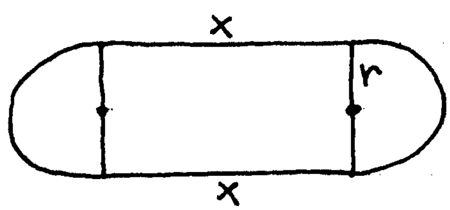
$= 7x - 3x^2$

$= -3(x^2 - \frac{7}{3}x)$

$= -3(x^2 - \frac{7}{3}x + 49/36) + \frac{49}{12} = -3(x - \frac{7}{6})^2 + 49/12$;

maximum $A = 49/12$ when $x = 7/6$ and $y = 7/2$.

34)



perimeter

$2x + 2\pi r = 1/4$ mi. $\rightarrow$

$x + \pi r = 1/8 \rightarrow$

$x = 1/8 - \pi r$;

maximize area of rectangle $A = x(2r)$

$= (\frac{1}{8} - \pi r)(2r) = \frac{1}{4}r - 2\pi r^2 = -2\pi(r^2 - \frac{1}{8\pi}r)$

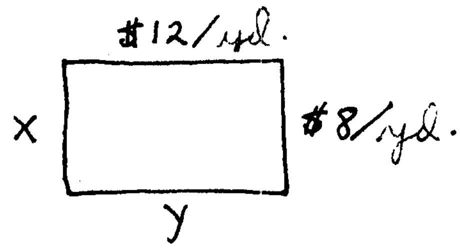
$= -2\pi(r^2 - \frac{1}{8\pi}r + \frac{1}{256\pi^2}) + \frac{1}{128\pi}$

$= -2\pi(r - \frac{1}{16\pi})^2 + 1/128\pi$; maximum

$A = \frac{1}{128\pi}$ mi.^{2} when $r = \frac{1}{16\pi}$ mi. and

$x = \frac{1}{8} - \pi(\frac{1}{16\pi}) = \frac{1}{16}$ mi.

35)



cost

$C = (2x)(\$8) + (2y)(\$12)$

$= 16x + 24y$

$= \$4800 \rightarrow$

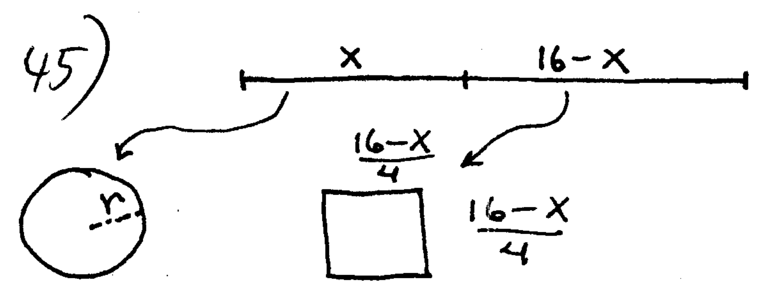
$2x + 3y = 600 \rightarrow 3y = 600 - 2x \rightarrow y = 200 - \frac{2}{3}x$;

maximize area $A = xy = x(200 - \frac{2}{3}x)$

$= 200x - \frac{2}{3}x^2 = -\frac{2}{3}(x^2 - 300x)$
$= -\frac{2}{3}(x^2 - 300x + 22{,}500) + 15{,}000$
$= -\frac{2}{3}(x - 150)^2 + 15{,}000$; maximum
$A = 15{,}000$ yd.^{2} when $x = 150$ yd. and
$y = 100$ yds.

42a) $f(x) = x - 3$, $g(x) = x^2 - 4x + 1$ then
$(g \circ f)(x) = g(f(x)) = g(x-3) = (x-3)^2 - 4(x-3) + 1$
$= x^2 - 6x + 9 - 4x + 12 + 1 = x^2 - 10x + 22$
$= (x^2 - 10x + 25) - 25 + 22 = (x-5)^2 - 3$;
minimum $(g \circ f)(x) = -3$ when $x = 5$.

45)



$2\pi r = x \rightarrow r = \frac{x}{2\pi}$

a.) total area $A = A_{\circ} + A_{\square}$
$= \pi r^2 + \left(\frac{16-x}{4}\right)^2 = \pi\left(\frac{x}{2\pi}\right)^2 + \left(\frac{16-x}{4}\right)^2$