Math 17A
Voyler
Beverton - Holt Growth Model

Recall: For discrete exponential growth/decay we have

$$N_t = N_0 R^t \quad \text{for } t = 0, 1, 2, 3, \ldots$$

for some constant R. Note: - N_t increases if $R > 1$
Growth Rate
- N_t decreases if $0 < R < 1$
- N_t is constant if $R = 1$

Also, the corresponding recursion relation is

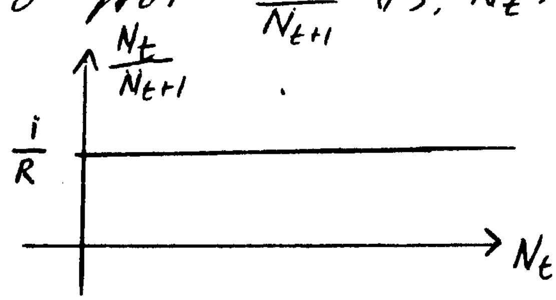
$$N_{t+1} = R N_t \quad \text{for } t = 0, 1, 2, 3, \ldots$$

which can be rewritten as $\boxed{\frac{N_t}{N_{t+1}} = \frac{1}{R}}$

Notes: 1) $\frac{N_t}{N_{t+1}}$ is called the parent-offspring ratio.

2) Since $\frac{1}{R}$ is constant, we say the growth is density independent. (i.e. the growth rate does NOT depend on the size of N_t)

3) It's useful to plot $\frac{N_t}{N_{t+1}}$ vs. N_t. For this model



Weakness of this Model

In many cases, this model is unrealistic due to limitations imposed by space, habitat, food, energy, etc... As an attempt to fix this weakness, it is more realistic to assume that the growth rate DECREASES as N_t increases, which leads to the Beverton - Holt Growth Model.

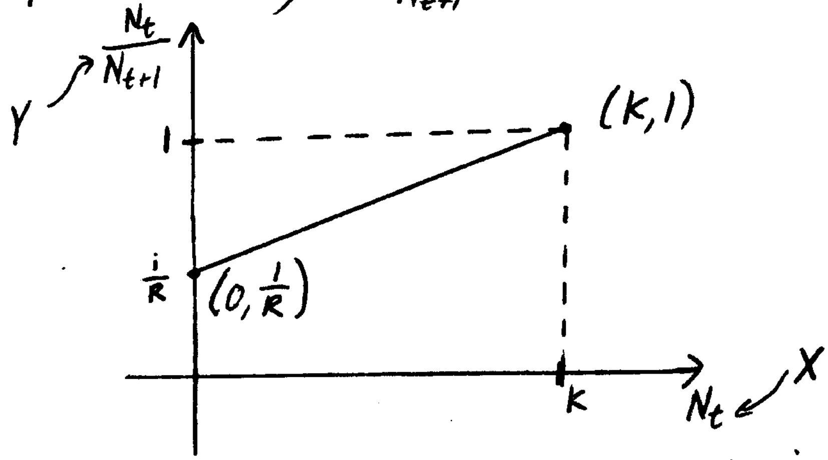
Beverton-Holt Growth Model

This discrete model assumes the graph of $\frac{N_t}{N_{t+1}}$ vs. N_t is an increasing linear function, which implies:

1) $R>1$, but decreases over time to 1
2) The growth rate of N_t decreases over time
3) The growth of N_t is density dependent

Note: If $\frac{N_t}{N_{t+1}} = 1 \Rightarrow N_{t+1} = N_t$. This means there are no new members and N_t has reached its carrying capacity, K (i.e. $\boxed{\lim_{t\to\infty} N_t = K}$).

The following $\frac{N_t}{N_{t+1}}$ vs. N_t graph displays this information:



The equation of this line is: $Y = mX + b$

$$\Rightarrow \frac{N_t}{N_{t+1}} = \frac{1-1/R}{K-0}\cdot N_t + \frac{1}{R} \Rightarrow \frac{R N_t}{N_{t+1}} = \frac{R-1}{K}\cdot N_t + 1$$

$$\Rightarrow \boxed{N_{t+1} = \frac{R\cdot N_t}{1+\frac{R-1}{K}\cdot N_t}} \quad \text{for } t = 0,1,2,\ldots$$

This is the Beverton-Holt Growth Recursion.