Beverton - Holt Growth Model

Recall: For discrete exponential growth/decay we have
\[ N_t = N_0 R^t \quad \text{for } t = 0, 1, 2, 3, \ldots \]
for some constant \( R \). Note: 
- \( N_t \) increases if \( R > 1 \)
- \( N_t \) decreases if \( 0 < R < 1 \)
- \( N_t \) is constant if \( R = 1 \)

Also, the corresponding recursion relation is
\[ N_{t+1} = RN_t \quad \text{for } t = 0, 1, 2, 3, \ldots \]
which can be rewritten as
\[ \frac{N_t}{N_{t+1}} = \frac{1}{R} \]

Notes:
1) \( \frac{N_t}{N_{t+1}} \) is called the parent-offspring ratio.

2) Since \( \frac{1}{R} \) is constant, we say the growth is density independent.
   (i.e. the growth rate does NOT depend on the size of \( N_t \))

3) It's useful to plot \( \frac{N_t}{N_{t+1}} \) vs. \( N_t \). For this model

\[ \frac{N_t}{N_{t+1}} \]
\[ \frac{1}{R} \]
\[ N_t \]
\[ N_{t+1} \]

Weakness of this Model

In many cases, this model is unrealistic due to limitations imposed by space, habitat, food, energy, etc. As an attempt to fix this weakness, it is more realistic to assume that the growth rate DECREASES as \( N_t \) increases, which leads to the Beverton-Holt Growth Model.
Beverton-Holt Growth Model

This discrete model assumes the graph of $\frac{N_t}{N_{t+1}}$ vs. $N_t$ is an increasing linear function, which implies:

1) $R > 1$, but decreases over time to $1$
2) The growth rate of $N_t$ decreases over time
3) The growth of $N_t$ is density dependent

Note: If $\frac{N_t}{N_{t+1}} = 1 \Rightarrow N_{t+1} = N_t$. This means there are no new members and $N_t$ has reached its carrying capacity, $K$ (i.e. $\lim_{t \to \infty} N_t = K$).

The following $\frac{N_t}{N_{t+1}}$ vs. $N_t$ graph displays this information:

\[
\begin{align*}
Y &= \frac{N_t}{N_{t+1}} \\
(0, \frac{1}{R}) &= (K, 1)
\end{align*}
\]

The equation of this line is: $Y = mX + b$

\[
\Rightarrow \quad \frac{N_t}{N_{t+1}} = \frac{1 - \frac{1}{R}}{K - 0} \cdot N_t + \frac{1}{R} \Rightarrow \quad \frac{R \cdot N_t}{N_{t+1}} = \frac{R - 1}{K} \cdot N_t + 1
\]

\[
\Rightarrow \quad N_{t+1} = \frac{R \cdot N_t}{1 + \frac{R - 1}{K} \cdot N_t}
\]

for $t = 0, 1, 2, \ldots$

This is the Beverton-Holt Growth Recursion