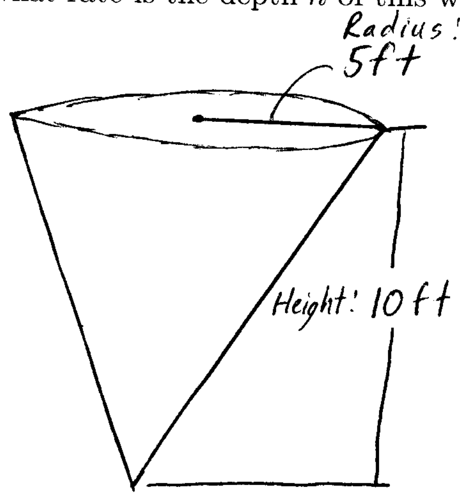
Math 17A
Vogler
Related Rates Examples

1.) Claire Redfield is at the bottom of a cylindrical tank with a base radius of 10 ft. The tank is filling with acid at a rate of 3,000 ft^3/sec. There is a ladder Claire can climb at a rate of 10 ft/sec. Can she climb faster than the rate of the acid rising to escape the tank?

2.) If the bottom of a 100 ft ladder is pushed toward the wall at a rate of 2 ft/sec, how fast is the top of the ladder moving up the wall when the bottom of the ladder is 60 ft from the wall?

3.) Water is filling in the given conical (i.e cone-shaped) tank at a rate of 5 ft^3/sec. At what rate is the depth h of this water changing when $h = 1$ ft?



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Procedure for Solving Related Rates Problems

1) Draw a picture (if one is not provided) and define the variables. Assign ALL numbers to variables. Remember, rates ALWAYS correspond to a derivative.

2) Determine and CLEARLY STATE goal of the problem, which is ALWAYS finding a rate/derivative.

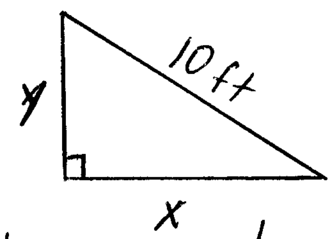
3) Build your related rates equation. Usually, this involves the implicit differentiation of a equation from geometry. Note: If there's only ONE rate/derivative in your related rates equation, you did something wrong!

4) Isolate the goal rate in 2) and make sure you have the numbers for all the other variables and rates/derivatives. If not, you have to find the missing numbers, usually by solving another equation.

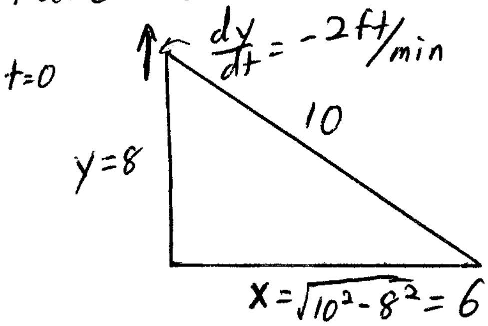
5) Plug in numbers and solve, making sure to INCLUDE UNITS. Think about the solution and its plausibility!

Related Rate Example

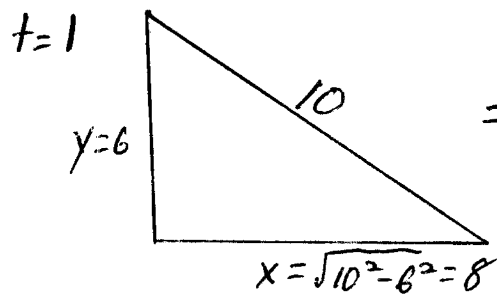
Consider following triangle:



Assume edge y is decreasing @ rate of 2 ft/min. At what rate is the area of triangle changing when $y = 8$ ft?



$\Rightarrow$



$\Rightarrow$ y & x are functions of time

Let $A :=$ Area of Triangle

Goal: Find $\frac{dA}{dt}$ when $y = 8$ ft & $\frac{dy}{dt} = -2$ ft/min

Assume $x(t), y(t),$ & $A(t)$

$$A = \tfrac{1}{2}xy \Rightarrow \frac{d}{dt}\left[A = \tfrac{1}{2}xy\right] \Rightarrow \frac{dA}{dt} = \frac{1}{2}\overset{?}{x}\overset{\checkmark}{\frac{dy}{dt}} + \frac{1}{2}\overset{?}{\frac{dx}{dt}}\overset{\checkmark}{y} \quad (*)$$

Problem ?!? Need to find x & $\frac{dx}{dt}$

$$\boxed{\begin{array}{l} \text{Recall: } x^2 + y^2 = 10^2 \Rightarrow x^2 = 100 - 64 = 36 \Rightarrow x = 6 \text{ ft} \\ \text{Also, } \frac{d}{dt}\left[x^2 + y^2 = 10^2\right] \Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \\ \Rightarrow \frac{dx}{dt} = \frac{-y\frac{dy}{dt}}{x} = \frac{-8(-2)}{6} = 8/3 \text{ ft/min} \end{array}}$$

Hence, $(*)$ becomes $\frac{dA}{dt} = \frac{1}{2}(6)(-2) + \frac{1}{2}\left(\frac{8}{3}\right)8 = \boxed{\frac{14}{3} \text{ ft/min}}$