Defn. Let \( \{a_n\} \) be a sequence and \( x \) a variable. Then \( \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots \) is called a **power series**.

Note: One can replace \( x \) with \( (x-a) \) to obtain the power series centered at \( x=\alpha \).

**Thm. (23.1)**
For the power series \( \sum_{n=0}^{\infty} a_n x^n \), let
\[
\beta = \lim_{n \to \infty} \sup_n \left| a_n \right|^{1/n} \quad \text{and} \quad R = \frac{1}{\beta}
\]
Then
a) the power series converges for \( |x| < R \).
b) the power series diverges for \( |x| > R \).

Notes:
1) If \( \beta = 0 \), we set \( R = \infty \). If \( \beta = \infty \), we set \( R = 0 \).
2) \( R \) is called the **radius of convergence**.
3) Most times it's easier to compute \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| \) (i.e. ratio test). Corollary 12.3 tells us \( \beta = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| \).
4) There are exactly 3 scenarios for a power series:
   a) it converges \( \forall x \in \mathbb{R} \) (i.e. \( R = \infty \))
   b) it converges for only \( x=0 \) (i.e. \( R = 0 \))
   c) it converges for \( x \) in a bounded interval centered at \( 0 \) (i.e. \( R \) is finite). This interval may be open, half-open, or closed, and it is called the **interval of convergence**.