21.6) Suppose \( f : S_1 \to S_2 \) and \( g : S_2 \to S_3 \) are both continuous. Consider the function \( g \circ f : S_1 \to S_3 \). Let \( U \subseteq S_3 \) be open. Since \( g \) is continuous, \( g^{-1}(U) \subseteq S_2 \) is open by Theorem 21.3. Since \( f \) is continuous, \( f^{-1}(g^{-1}(U)) \subseteq S_1 \) is open by another application of Theorem 21.3. Then \( (g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U)) \) by the definition of composition and inverse. Thus, \( (g \circ f)^{-1}(U) \subseteq S_1 \) is open. Since \( U \) was arbitrary, this holds for all \( U \subseteq S_3 \). Therefore, \( g \circ f \) is continuous by Theorem 21.3.

21.8) Suppose \( f : S \to S^* \) is uniformly continuous and \( \{s_n\} \subseteq S \) is a Cauchy sequence. Let \( \epsilon > 0 \) be given. Since \( f \) is uniformly continuous,
\[
\exists \delta > 0 \text{ such that } \forall x, y \in S \text{ with } d(x, y) < \delta \Rightarrow d^*(f(x), f(y)) < \epsilon.
\] (1)
Since \( \{s_n\} \) is Cauchy, for this \( \delta \)
\[
\exists N \text{ such that } \forall m, n > N \Rightarrow d(s_m, s_n) < \delta.
\]
Then, for this \( N \), we have
\[
\forall m, n > N \Rightarrow d(s_m, s_n) < \delta \Rightarrow d^*(f(s_m), f(s_n)) < \epsilon,
\]
by (1). Therefore, \( \{f(s_n)\} \) is a Cauchy sequence as well.

21.10) (a) Let \( f : (0, 1) \to [0, 1] \) be defined as
\[
f(x) = \begin{cases} 
0 & \text{if } 0 < x < \frac{1}{2} \\
2x - \frac{1}{2} & \text{if } \frac{1}{2} \leq x \leq \frac{3}{4} \\
1 & \text{if } \frac{3}{4} < x < 1
\end{cases}
\]
(b) Let \( g : (0, 1) \to \mathbb{R} \) be defined as \( g(x) = \tan(\pi x - \frac{\pi}{2}) \).
(c) Let \( h : [01] \cup [2, 3] \to [0, 1] \) be defined as \( h(x) = -\frac{1}{2}x^2 + \frac{3}{2}x \)
\]