1.

(a) State the PRECISE definition of the limit for

$$\lim_{x \to a} f(x) = L$$

(b) Give an ε, δ-proof for the following limit. This is a writing exercise.

$$\lim_{x \to -2} (x^2 + 3) = 7$$

2. Determine the following limits

(a) $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 1}$

(b) $\lim\limits_{h \to 0} \dfrac{\sin 5h}{2h}$

(c) $\lim\limits_{x \to 1^-} \dfrac{\sqrt{2x}(x-1)}{|x-1|}$

(d) $\lim\limits_{x \to 4} \dfrac{\sqrt{5+x} - 3}{x - 4}$

(e) $\lim\limits_{x \to \infty} \dfrac{3 + \cos(x)}{x^4}$ (HINT : Use the Squeeze Principle.)

3. Sketch the following graph. Make sure to CLEARLY state intercepts, asymptotes, along with the domain and range.

$$y = \frac{x+2}{x^2(x+1)}$$

4. Use the Intermediate Value Theorem to show that the equation $x^3 = 10 + \sqrt{x}$ is solvable. This is a writing exercise.

4.) Consider the following function $f(x) = \begin{cases} \dfrac{x^2 - 3x}{x^2 - 9}, & \text{if } x \neq 3, -3 \\ \frac{1}{2}, & \text{if } x = 3 \\ 0, & \text{if } x = -3. \end{cases}$

Determine if f is continuous at $x = 3$.

4. Consider the following function

$$f(x) = \begin{cases} -4 & \text{if } x \leq 1 \\ Ax^2 + B & \text{if } 1 < x \leq 2 \\ 5 & \text{if } x > 2 \end{cases}$$

Use limits and a "fake graph" to determine the value of constants A and B so that the following function is continuous for all values of x.

2.) a.) Determine the domain of $f(x) = \sqrt{7 - x}$.

b.) Determine the range of $f(x) = 3 + 5\sin x$.

3.) Consider a three-dimensional cube with side length x.

a.) Write the volume V of the cube as a function of x.

b.) Write the surface area S of the cube as a function of x.

c.) Write the surface area S of the cube as a function of the volume V.

The following EXTRA CREDIT PROBLEM is worth points. This problems is OPTIONAL.

1.) Determine the following limit : $\lim_{x \to \infty} \left(x - \sqrt{x^2 + 9x}\,\right)$