

1.

- (a) State the PRECISE definition of the limit for

$$\lim_{x \rightarrow a} f(x) = L$$

- (b) Give an ε, δ -proof for the following limit. This is a writing exercise.

$$\lim_{x \rightarrow -1} \frac{3}{x+7} = \frac{1}{2}$$

2. Determine the following limits

(a) $\lim_{x \rightarrow 0} \frac{(e^x - 1)^2}{\sin(x^2)}$

(b) $\lim_{x \rightarrow 1^-} \frac{\sqrt{2x}(x-1)}{|x-1|}$

(c) $\lim_{x \rightarrow \infty} (3 + 2x)^{\frac{1}{x}}$

3. (8 pts) Consider the following function

$$f(x) = \begin{cases} Ax + B & \text{if } x < 0 \\ 12 & \text{if } 0 \leq x \leq 2 \\ Bx^2 - A & \text{if } x > 2 \end{cases}$$

Using limits, determine all constants A and B such that the function is continuous for all values of x .

4. Differentiate each of the following functions. DO NOT SIMPLIFY ANSWERS.

(a) $y = \arcsin(2^x + \log x)$

(b) $f(t) = \cot\left(\frac{\sin t}{t}\right)$

(c) $y = (2x)^{x+3}$

5. Use differentials to estimate the value of $\sqrt[3]{26.5}$.

6.

(a) State the definition of the derivative.

(b) Use the definition of the derivative to differentiate the function

$$f(x) = \sqrt{2x + 1}$$

7. Use Newton's Method to approximate the solution to $x^3 + x - 3 = 0$. Let $x_0 = 0$ and perform ONLY TWO iterations of Newton's method (i.e. find x_2).

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8. Use the Intermediate Value Theorem to show that the equation $x^3 + x = \sqrt{x + 4}$ is solvable. This is a writing exercise.

9. Find the slope and concavity of the graph $xy + y^2 = 3x + 1$ at the point $(0, -1)$

9. Use a differential to linearize the following function at the center point of $x = 5$

$$f(x) = \sqrt{x + 4}$$

- (b) Determine if the following function satisfies the assumptions of the MVT on the given closed interval. If so, find all values of c guaranteed by the conclusion of the MVT.

$$f(x) = \ln(x - 1) \text{ on } [2, 4]$$

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13. Find the point (x, y) on the graph of $y = \sqrt{x}$ which is nearest to the point $(4, 0)$
14. Gifted White Rabbit has done a market analysis on pricing verses number of apps sold. This analysis shows that 400 apps are sold when the price is \$2. For each \$1 increase in price, 25 fewer apps are sold. What price per app will maximize total revenue?

15. Consider the following function with its corresponding first and second derivatives

$$f(x) = (x - 1)^3(x - 5), \quad f'(x) = (x - 1)^2(4x - 16), \quad f''(x) = 4(x - 1)(3x - 9)$$

Determine where f is increasing, decreasing, concave up, and concave down. Identify all relative and absolute extrema, inflection points, vertical and horizontal asymptotes, and x- and y- intercepts. Neatly sketch the graph.

9.) Car B is 34 miles directly east of car A and begins moving west at 90 mph. At the same moment car A begins moving north at 60 mph.

a.) At what rate is the distance between the cars changing after $t = \frac{1}{5}$ hr. ?

b.) What is the minimum distance between the cars and at what time t does the minimum distance occur ?

The following EXTRA CREDIT PROBLEM is worth
TIONAL.

This problem is OP-

1.) You are standing at point A on the edge of a river 1 mile wide. You are to get to point B, which is 4 miles from the point directly across the river from you. You can paddle a canoe in the water at a speed of 10 miles per hour and you can ride a bicycle on land at a speed of 15 miles per hour. Determine x so that the time it takes to go from point A to point B is a minimum and determine the minimum time.