Math 21A
Vogler

Inverse Trig Functions

1) $D[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$

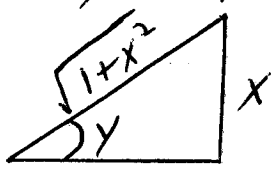
2) $D[\arctan x] = \frac{1}{1+x^2}$

3) $D[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$

Pf of 2 Let $y = \arctan x \Rightarrow \tan y = x$ (*)

$\overset{D}{\Rightarrow} (\sec^2 y)\, y' = 1 \Rightarrow y' = \frac{1}{\sec^2 y}$

From (*), $\tan y = \frac{x}{1}$

$\Rightarrow$ 

$\Rightarrow \cos y = \frac{1}{\sqrt{1+x^2}}$

$\Rightarrow \sec y = \sqrt{1+x^2}$

sub $\Rightarrow y' = \frac{1}{1+x^2}$

$\Rightarrow D[\tan^{-1}(x)] = D[\arctan x] = \frac{1}{1+x^2}$ ∎

Ex 1) $D[x^2 \arctan x^2] = 2x \arctan x^2 + x^2 \frac{1}{1+(x^2)^2} \cdot 2x$

$= 2x \arctan x^2 + \frac{2x^3}{1+x^4}$

2) $D[\operatorname{arcsec}(10^{\sin 4x})] = \frac{1}{|10^{\sin 4x}|\sqrt{(10^{\sin 4x})^2 - 1}} \cdot 10^{\sin 4x} \cdot \ln 10 \cdot \cos 4x \cdot 4$

$= \frac{4 \ln 10 \cdot \cos 4x \cdot 10^{\sin 4x}}{10^{\sin 4x}\sqrt{10^{2\sin 4x} - 1}}$