The Derivative of a Function

\[
\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}
\]

Note that the slope of the secant line is

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{f(x+h) - f(x)}{h}
\]

We can obtain slope of tangent line by letting \( h \to 0 \) which leads to the following:

**Defn** The derivative of \( f \) at \( x \) is

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

Notes: 1) \( f'(x) \) is the slope of tangent line through point \( (x, f(x)) \)

2) \( \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{\Delta f}{\Delta x} \) can be considered "infinitesimal division" (i.e., division by two "small" numbers)

3) The derivative is sometimes referred to as the **Instantaneous Rate of Change (IRC)**

**Defn** The **Average Rate of Change (ARC)** of a function \( y = f(x) \) on interval \([a, b]\) is

\[
A.R.C = \frac{f(b) - f(a)}{b - a}
\]

Note: ARC is slope of secant line between \((a, f(a)) \) & \((b, f(b))\)