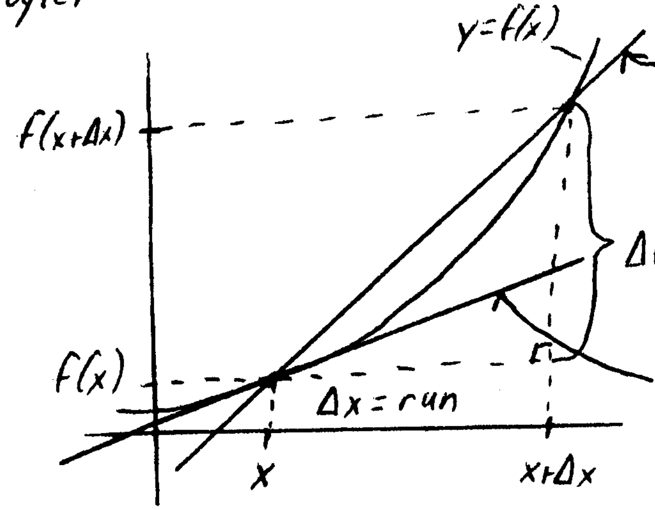
Math 21A
Vogler

The Derivative of a Function



Secant line with slope $= \dfrac{f(x+h)-f(x)}{\Delta x}$ $=$ ARC

$\Delta f = f(x+\Delta x) - f(x)$ $=$ rise

Tangent line with slope $f'(x)$ $=$ IRC

Note that the slope of the secant line is

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{f(x+h)-f(x)}{\Delta x}$$

We can obtain slope of tangent line by letting Δx get 'small' (i.e. $\Delta x \to 0$) which leads to the following:

Defn The <u>derivative of f at x</u> is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

Notes: 1) $f'(x)$ is the slope of tangent line through point $(x, f(x))$

2) $\displaystyle\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} \approx \frac{\Delta f}{\Delta x}$ can be considered 'infintesmal division' (i.e. division by two 'small' numbers

3) The derivative is sometimes refered to as the <u>Instantaneous Rate of Change</u> (IRC)

Defn The <u>Average Rate of Change</u> (ARC) of a function $y = f(x)$ on interval $[a,b]$ is

$$A.R.C = \frac{f(b)-f(a)}{b-a}$$

Note: ARC is slope of secant line between $(a, f(a))$ & $(b, f(b))$