

Understanding Implicit Differentiation

Ex Consider the equation $x^2 + y^2 = 9$, and the following cases:

1) Assume $y = f(x)$ and $D = \frac{d}{dx}$,
 $\Rightarrow \frac{d}{dx} [x^2 + y^2 = 9] \Rightarrow 2x + 2y \frac{dy}{dx} = 0$

2) Assume $x = f(y)$ and $D = \frac{d}{dy}$,
 $\Rightarrow \frac{d}{dy} [x^2 + y^2 = 9] \Rightarrow 2x \frac{dx}{dy} + 2y = 0$

3) Assume $x = f(t)$ but y is NOT a function of t and $D = \frac{d}{dt} \Rightarrow \frac{d}{dt} [x^2 + y^2 = 9] \Rightarrow 2x \frac{dx}{dt} + 0 = 0$

4) Assume $x = f(t)$, $y = h(t)$, and $D = \frac{d}{dt}$
 $\Rightarrow \frac{d}{dt} [x^2 + y^2 = 9] \Rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

Moral: The outcome of implicit differentiation depends on both the 'invisible' (assumed) function(s) and the variable you are taking the derivative with respect to.