Math 16A
Vogler

Vertical/Horizontal Asypmptote Example

Ex Use intercepts & vertical/horizontal asymptotes to sketch the graph of $y = \frac{2-x}{x^2-1}$

Intercepts

y-int: $x=0 \Rightarrow y = \frac{2}{-1} = -2$, $\boxed{y=-2}$

x-int: $y=0 \Rightarrow 0 = \frac{2-x}{x^2-1} \Rightarrow \boxed{x=2}$

H.A.'s (Look at behavoir as $x \to \pm\infty$)

$$\lim_{x\to\pm\infty} \frac{2-x}{x^2-1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\pm\infty} \frac{\frac{2}{x^2} - \frac{1}{x}}{1 - \frac{1}{x^2}} = 0$$

So $\boxed{\text{H.A. at } y=0}$

V.A.'s (Look for division by $\emptyset$. Candidates are $x = \pm 1$)

$$\lim_{x\to 1^-} \frac{2-x}{x^2-1} = \frac{1}{0^-} = -\infty$$

$$\lim_{x\to 1^+} \frac{2-x}{x^2-1} = \frac{1}{0^+} = +\infty$$

$\boxed{\text{V.A. at } x=1}$

$$\lim_{x\to -1^-} \frac{2-x}{x^2-1} = \frac{1}{0^+} = +\infty$$

$$\lim_{x\to -1^+} \frac{2-x}{x^2-1} = \frac{1}{0^-} = -\infty$$

$\boxed{\text{V.A. at } x=-1}$

With all this info., we can now sketch