Math 16A

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Continuity

Defn A function $y=f(x)$ is continuous at $x=a$ if

i) $f(a)$ exists (finite #)

ii) $\lim_{x\to a} f(x)$ exists (finite #) $\Leftrightarrow \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

and iii) $\lim_{x\to a} f(x) = f(a)$

Note: This definition is equivalent to not lifting your pen/pencil when drawing the function's graph.

Fact: Sums, differences, products, quotients (denominator $\neq 0$), and compositions of continuous functions are continuous.

Fact: The following list of functions are continuous:

1) polynomials for all x-values
2) Roots ($\sqrt{\ }$, $\sqrt[3]{\ }$, ...) when defined
3) $\sin x$ and $\cos x$ for all x-values

Ex Let $f(x) = \frac{x^3-5x+6}{2x^2+x-3}$; since $y=x^3-5x+6$ (polynomial) and $y=2x^2+x-3$ (polynomial) are continuous for all x, it follows $f(x)=\frac{x^3-5x+6}{2x^2+x-3}$ (quotient) is continuous for all x except when $y=2x^2+x-3=(2x+3)(x-1)=0$, that is, except at $x=1$ and $x=\frac{-3}{2}$.

Ex: Let $f(x)=(3+\sin x)^{50}$; since $g(x)=3+\sin x$ (well-known) and $h(x)=x^{50}$ (polynomial) are continuous for all x, it follows their composition $f(x)=h(g(x))=(3+\sin x)^{50}$ is continuous for all x.