Math 16A

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**Definition** A function \( y = f(x) \) is **continuous at** \( x = a \) if

1. \( f(a) \) exists (finite #)
2. \( \lim_{{x \to a}} f(x) \) exists (finite #) \( \Leftrightarrow \) \( \lim_{{x \to a^-}} f(x) = \lim_{{x \to a^+}} f(x) \)
3. \( \lim_{{x \to a}} f(x) = f(a) \)

Note: This definition is equivalent to not lifting your pen/pencil when drawing the function's graph.

**Fact:** Sums, differences, products, quotients (denominator \( \neq 0 \)), and compositions of continuous functions are continuous.

**Fact:** The following list of functions are continuous:

1. polynomials for all \( x \)-values
2. Roots \( \sqrt[n]{\cdots} \) when defined
3. \( \sin x \) and \( \cos x \) for all \( x \)-values

**Example** Let \( f(x) = \frac{x^3 - 5x + 6}{2x^2 + x - 3} \); since \( y = x^3 - 5x + 6 \) (polynomial) and \( y = 2x^2 + x - 3 \) (polynomial) are continuous for all \( x \), it follows \( f(x) = \frac{x^3 - 5x + 6}{2x^2 + x - 3} \) (quotient) is continuous for all \( x \) except when \( y = 2x^2 + x - 3 = (2x+3)(x-1) = 0 \), that is, except at \( x = 1 \) and \( x = \frac{-3}{2} \).

**Example:** Let \( f(x) = (3 + \sin x)^{50} \); since \( g(x) = 3 + \sin x \) (well-known) and \( h(x) = x^{50} \) (polynomial) are continuous for all \( x \), it follows their composition \( f(x) = h(g(x)) = (3 + \sin x)^{50} \) is continuous for all \( x \).