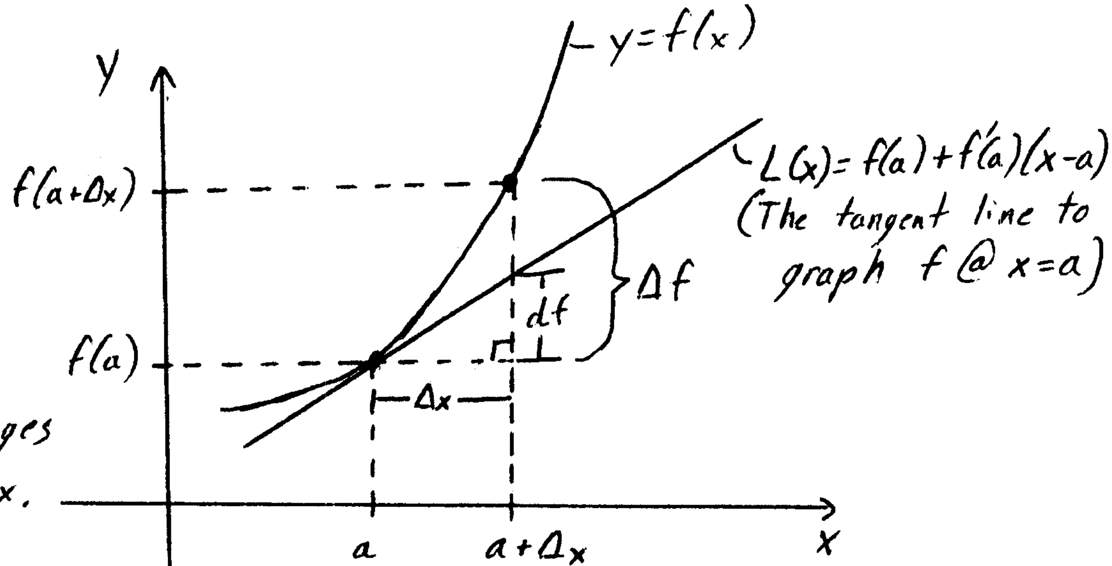
Math 16A
Vogler
The Differential

- Let Δx be the <u>change (error) in x</u>, & assume x changes from a to $a+\Delta x$. ($x: a \to a+\Delta x$)



- Define the <u>exact change</u> in f as

$$\boxed{\Delta f = \Delta y = f(a+\Delta x) - f(a)}$$

- Note: slope of line $L = \frac{\text{rise}}{\text{run}} \Rightarrow f'(a) = \frac{df}{\Delta x} \Rightarrow df = f'(a)\Delta x$
- Define the <u>differential</u> (approximate change) of f as

$$\boxed{df = f'(a)\cdot\Delta x}$$

- Fact: If Δx is 'small', then $\boxed{df \doteq \Delta f}$

With this fact & the differential, we can approximate or simply functions by the following eqn for a line

$$\boxed{f(a+\Delta x) \doteq f(a) + df = f(a) + f'(a)\Delta x}$$

Note: To use this equation effectively, you must choose 'a' such that f(a) & f'(a) can be easily determined.

Examples using differentials

Example 1: Use differentials to estimate $\sqrt[3]{26.5}$.

Let $f(x) = \sqrt[3]{x}$ and $a = 27$ ← (b/c $27 \approx 26.5$ and $\sqrt[3]{27}$ is computable)

$x: \overset{a}{27} \to 26.5 \Rightarrow \Delta x = -\frac{1}{2}$; $\quad f'(x) = \frac{1}{3(\sqrt[3]{x})^2}$

$$\Delta y = f(26.5) - f(27) = \sqrt[3]{26.5} - \sqrt[3]{27} = \sqrt[3]{26.5} - 3$$

$$dy = f'(27) \cdot \Delta x = \frac{1}{3(\sqrt[3]{27})^2} \cdot \left(-\frac{1}{2}\right) = -\frac{1}{54}$$

$$\Delta y \approx dy \Rightarrow \sqrt[3]{26.5} - 3 \approx -\frac{1}{54}$$

$$\Rightarrow \sqrt[3]{26.5} \approx 3 - \frac{1}{54} = 2\frac{53}{54}$$

Example 2: If the radius of a circle is measured w/ an absolute percentage error of @ most 3%, use differentials to estimate the maximum absolute percentage error in computing the circle's

a) circumference b) area

Solution Assume that $\frac{|\Delta r|}{r} \le 3\%$

a) $C = 2\pi r \Rightarrow C' = 2\pi$, find $\frac{|\Delta C|}{C}$:

$$\frac{|\Delta C|}{C} \approx \frac{|dC|}{C} = \frac{|C'\Delta r|}{C} = \frac{|2\pi \cdot \Delta r|}{2\pi r} = \frac{|\Delta r|}{r} \le 3\%$$

b) $A = \pi r^2 \Rightarrow A' = 2\pi r$, find $\frac{|\Delta A|}{A}$

$$\frac{|\Delta A|}{A} \approx \frac{|dA|}{A} = \frac{|A'\Delta r|}{A} = \frac{|2\pi r \Delta r|}{\pi r^2} = 2\frac{|\Delta r|}{r} \le 2(3\%) = 6\%$$