Math 16A
Vogler
Instructions for DETAILED GRAPHING

1.) State the DOMAIN of the function.

2.) Take the FIRST derivative and set up a SIGN CHART for $f'(x)$. Clearly mark the solutions to $f'(x) = 0$ and their y-values, and identify all RELATIVE and ABSOLUTE maximum and minimum values.

3.) State the OPEN INTERVALS on which f is INCREASING and DECREASING.

4.) Take the SECOND derivative and set up a SIGN CHART for $f''(x)$. Clearly mark the solutions to $f''(x) = 0$ and their y-values, and identify all INFLECTION POINTS.

5.) State the OPEN INTERVALS on which f is CONCAVE UP and CONCAVE DOWN.

6.) Determine all x-INTERCEPTS and y-INTERCEPTS.

7.) If appropriate, determine all HORIZONTAL ASYMPTOTES (H.A.).

8.) If appropriate, determine all VERTICAL ASYMPTOTES (V.A.).

9.) DRAW a rough SKETCH of the graph of $y = f(x)$ and CLEARLY identify the coordinates of all important points on the graph.

Graphing Using First and Second Derivatives

1. If f' is $+$, then f is increasing ($\uparrow$).
2. If f' is $-$, then f is decreasing ($\downarrow$).
3. If f'' is $+$ (means f' is $\uparrow$), then f is concave up ($\cup$).
4. If f'' is $-$ (means f' is $\downarrow$), then f is concave down ($\cap$).

$+ \quad 0 \quad -$ $\quad f'$

$x=a$

relative (or absolute) maximum

$- \quad 0 \quad +$ $\quad f'$

$x=a$

relative (or absolute) minimum

$+ \quad 0 \quad -$ $\quad f''$

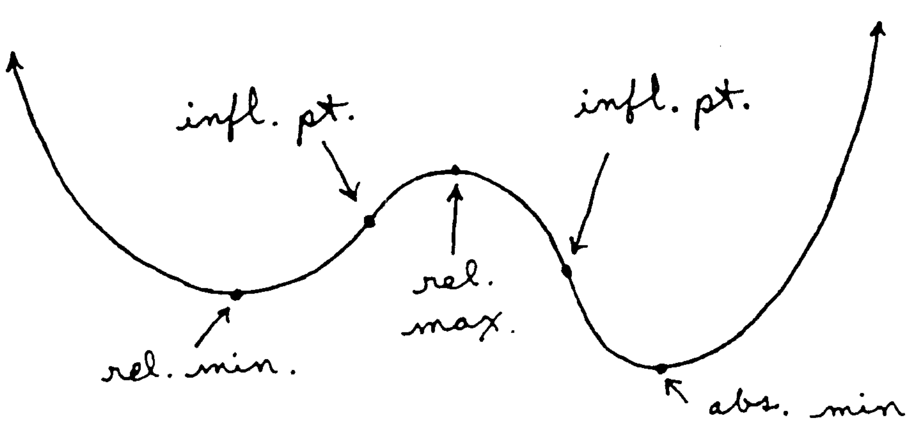
$x=a$

inflection point

$- \quad 0 \quad +$ $\quad f''$

$x=a$

inflection point



For each of the following functions begin by finding the domain of the function. Determine all relative and absolute maximum and minimum values and inflection points. State clearly the intervals on which the function is increasing (↑), decreasing (↓), concave up (∪), and concave down (∩). Determine all vertical and horizontal asymptotes (when appropriate) and x- and y-intercepts. Neatly sketch the graph.

<u>Example 1</u>: $f(x) = (x-1)^3(x-5)$ domain : all x-values

$$f'(x) = (x-1)^3\cdot(1) + 3(x-1)^2\cdot(x-5)$$
$$= (x-1)^2\cdot[(x-1) + 3(x-5)]$$
$$= (x-1)^2[4x-16] = 0$$

y': $-$ 0 $-$ 0 $+$

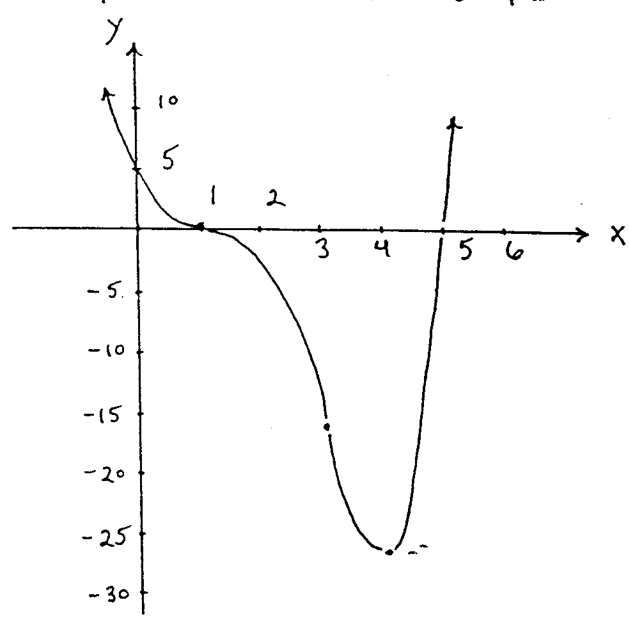
? $\left\{\begin{matrix} x=1 \\ y=0 \end{matrix}\right.$ $\left.\begin{matrix} x=4 \\ y=-27 \end{matrix}\right\}$ abs. min.

$$f''(x) = (x-1)^2(4) + 2(x-1)[4x-16]$$
$$= 4(x-1)[(x-1) + 2(x-4)]$$
$$= 4(x-1)[3x-9] = 0$$

y'': $+$ 0 $-$ 0 $+$

inf. pt. $\left\{\begin{matrix} x=1 \\ y=0 \end{matrix}\right.$ $\left.\begin{matrix} x=3 \\ y=-16 \end{matrix}\right\}$ inf. pt.

f is ↑ for $x > 4$,
f is ↓ for $x < 4$,
f is ∪ for $x < 1$, $x > 3$,
f is ∩ for $1 < x 3$,
$y = 0$: $x = 1$, $x = 5$
$x = 0$: $y = 5$



<u>Example 2</u>: $y = 3x^{2/3} - 2x$ domain : all x-values

$$y' = 3\cdot\frac{2}{3}x^{-1/3} - 2 = 2x^{-1/3} - 2$$
$$= 2\left(\frac{1}{x^{1/3}} - 1\right) = 2\left(\frac{1 - x^{1/3}}{x^{1/3}}\right) = 0$$

y': $-$ ¦ $+$ 0 $-$

rel. min. $\left\{\begin{matrix} x=0 \\ y=0 \end{matrix}\right.$ $\left.\begin{matrix} x=1 \\ y=1 \end{matrix}\right\}$ rel. max.

$$y'' = 2\cdot\frac{-1}{3}x^{-4/3} = \frac{-2}{3x^{4/3}}$$

y'': $- -$ ¦ $- -$

$x = 0$
$y = 0$

Y is ↑ for $0<x<1$,
Y is ↓ for $x<0$, $x>1$,
Y is ∩ for $x<0$, $x>0$

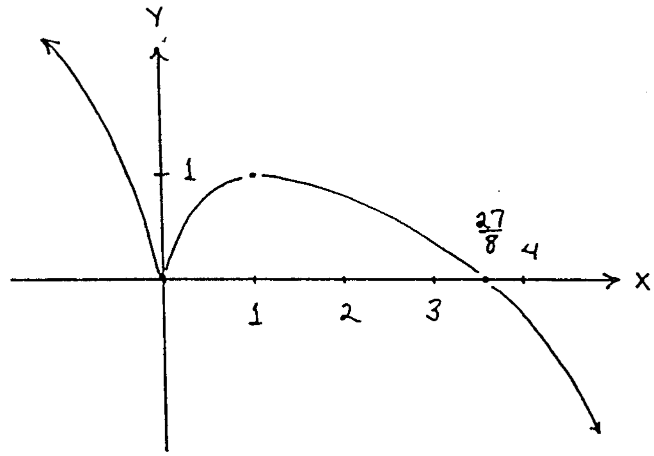
$x=0: \; Y=0$

$Y=0: \; 3x^{2/3}-2x=0$

$\to x^{2/3}\left(3-2x^{1/3}\right)=0$

$\downarrow \qquad \downarrow$

$x=0 \qquad x=\frac{27}{8}$



<u>*Example*</u> 3: $\quad Y=\dfrac{x^2+1}{x^2-2}$ $\qquad$ domain : all $x \neq \pm\sqrt{2}$

$$Y'=\frac{(x^2-2)(2x)-(x^2+1)(2x)}{(x^2-2)^2}=\frac{-6x}{(x^2-2)^2}=0$$

Y': $+$ | $+$ $\;0\;$ $-$ | $-$ at $x=-\sqrt{2}$, $x=0$, $x=\sqrt{2}$

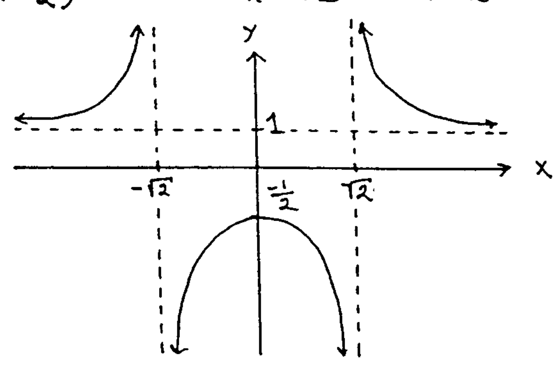
$Y=\frac{-1}{2}$ rel. max.

$$Y''=\frac{(x^2-2)^2(-6)-(-6x)\cdot 2(x^2-2)\cdot 2x}{(x^2-2)^4}$$

$$=\frac{-6(x^2-2)\cdot\left[(x^2-2)-4x^2\right]}{(x^2-2)^4}=\frac{6(2+3x^2)}{(x^2-2)^3}=0$$

Y'': $+$ | $-$ | $+$ at $x=-\sqrt{2}$, $x=\sqrt{2}$

Y is ↑ for $x<-\sqrt{2}$, $-\sqrt{2}<x<0$,
Y is ↓ for $0<x<\sqrt{2}$, $x>\sqrt{2}$,
Y is ∪ for $x<-\sqrt{2}$, $x>\sqrt{2}$,
Y is ∩ for $-\sqrt{2}<x<\sqrt{2}$



$x=0: \; Y=-1/2$

$Y=0$: none

$$\lim_{x\to+\sqrt{2}^+}\frac{x^2+1}{x^2-2}=+\infty, \quad \lim_{x\to+\sqrt{2}^-}\frac{x^2+1}{x^2-2}=-\infty$$

$$\lim_{x\to-\sqrt{2}^+}\frac{x^2+1}{x^2-2}=-\infty, \quad \lim_{x\to-\sqrt{2}^-}\frac{x^2+1}{x^2-2}=+\infty$$

vertical asymptotes $x=\pm\sqrt{2}$

$$\lim_{x\to\pm\infty}\frac{x^2+1}{x^2-2}=\lim_{x\to\pm\infty}\frac{1+\frac{1}{x^2}}{1-\frac{2}{x^2}}=1$$

: horizontal asymptote $Y=1$