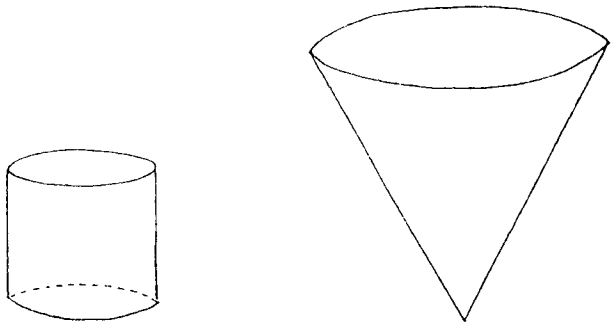
Math 16A
Vogler
Supplemenal Algebra Problems

SA1. Consider the following two empty water tanks (Fig. 1). One is a right circular cylindrical tank of height 5 feet and radius 2 feet. The other is a right circular conical tank of height 10 feet and radius 4 feet. If 2000 pounds of water is poured into each tank, how deep will the water be in each tank ? Assume that water weighs 62.5 pounds per cubic foot.

Fig. 1



SA2. An open rectangular box 4 feet by 6 feet by 3 feet is filled with spherical balls each of volume $\pi/6$ cubic feet (Fig. 2).

a. How many balls will fit in the box in such a way that they are stacked on top of each other and a lid will fit snugly over the top of the box ?

b. If the box has a waterproof lining and the balls are waterproof, how many pounds of water can be poured into the box packed with balls in order to completely fill the box ?

Fig. 2

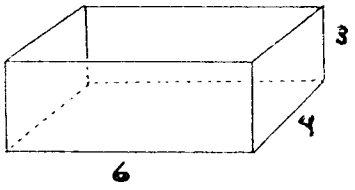
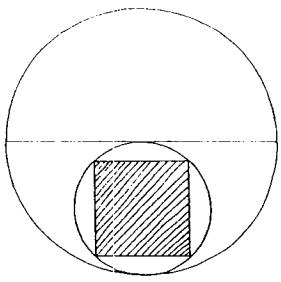


Fig. 3



SA3. In the given diagram (Fig. 3) the smaller circle is the largest one that can be inscribed in the given semi-circle. If the larger circle has circumference 4π in., what is the area of the shaded square ?

SA4. An iceberg in the shape of a right circular cylinder of height 15 meters and circumference 25 meters is sitting on a perfectly flat parking lot of area one acre. What will be the depth of the water in inches on the parking lot after the iceberg melts ? Assume that the density of ice is 0.917 times that of water.

SA5. Find the perimeter of the largest square which can be inscribed in a circle of circumference 2π ft.

SA6. Find the surface area of the largest cube which can be inscribed in a sphere of surface area 4π ft.^{2}.

SA7. A field is in the shape of an equilateral triangle of side length 4 miles (Fig. 4).

a. Find the distance from any corner of the field to the center of the field.

b. Find the radius of the largest circle which can be inscribed in this field.

FACT : In a 30/60 degree right triangle the length of the side opposite the 30 degree angle is one-half the length of the hypotenuse.

Fig. 4

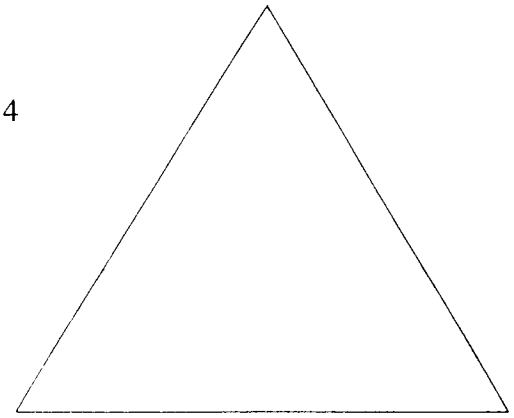
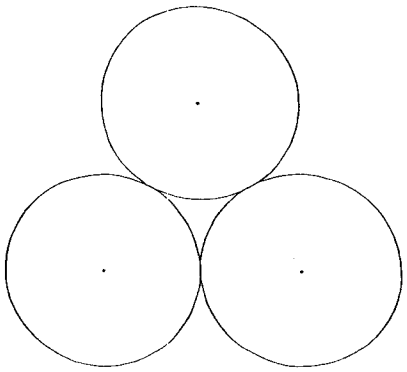


Fig. 5



SA8. You are given three circles, each circle of diameter 4 and each circle tangent to the other two circles (Fig. 5).

a. Find the radius of the smallest circle which is simultaneously tangent to all three circles.

b. Find the radius of the largest circle which is simultaneously tangent to all three circles.

SA9. Find the circumference of the largest circle which can be inscribed in the given right triangle (Fig. 6).

Fig. 6

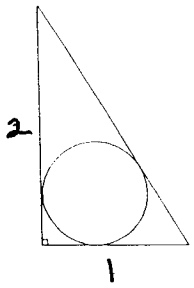
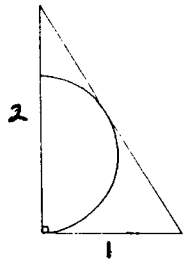


Fig. 7



SA10. Find the radius of the largest semi-circle which can be inscribed in the given right triangle (Fig. 7).

SA11. The kinetic energy, K.E., of an object of mass m and velocity v is directly proportional to the product of m and the square of v. In particular, K.E. = 1/2 m v^2.

a. If the mass of an object doubles and its velocity triples, by what factor will the kinetic energy change ?

b. If the mass of an object increases by 40% aand its velocity decreases by 25%, does the kinetic energy increase or decrease and by what percent ?

c. If the velocity of an object increases by 100% and its mass decreases by 75%, does the kinetic energy increase or decrease and by what percent ?

SA12. Asssume that the number of rabbits in a specific habitat is directly proportional to the number of mice in the habitat divided by the square of the number of birds of prey in the habitat.

a. If the number of mice increases by 40% and the number of birds of prey decreases by 5%, will the number of rabbits increase or decrease and by what percent ?

b. If the number of mice decreases by 25% each year and the number of birds of prey increases by 12 % each year, by what percent will the number of rabbits change in

i. 2 years ?

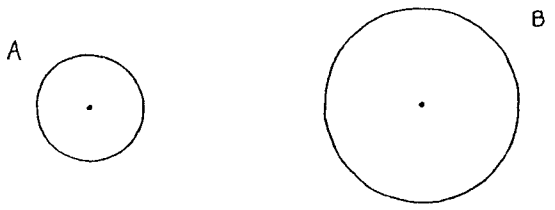
ii. 5 years ?

SA13. Wheel A has a radius of 2 inches. Wheel B has a radius of 6 inches. See Fig. 8.

a. If wheel A and wheel B begin rolling on a flat surface and each completes exactly ten rotations, what is the total distance (inches) traveled by each wheel ?

b. How many rotations are required for each wheel to travel exactly 10 feet ?

Fig. 8



SA14. Sprocket A has a radius of 2 inches and is firmly attached to a wheel of radius 6 inches. Sprocket B has a radius of 6 inches and is firmly attached to a wheel of radius 30 inches. Sprockets A and B are connected with a closed belt about their circumferences. See Fig. 9.

a. If sprocket A rotates exactly 3 times, how many times will sprocket B rotate ?

b. If sprocket B rotates exactly 3 times, how many times will sprocket A rotate ?

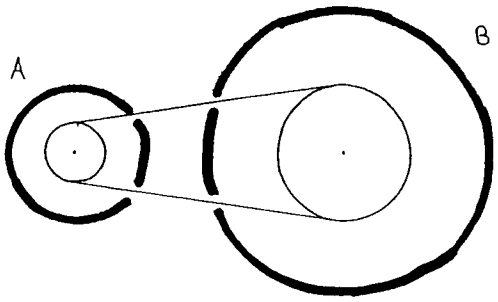
c. If sprocket A rotates exactly 3 times per second, how fast (in./sec. and m.p.h.) is a fixed point on the circumference of

i. sprocket A traveling ? ii. wheel B traveling ?

d. If sprocket B rotates exactly 3 times per second, how fast (in./sec. and m.p.h.) is a fixed point on the circumference of

i. sprocket B traveling ? ii. wheel A traveling ?

Fig. 9



SA15. A large tank initially holds 50 liters of solution (water and salt) containing 5 grams of dissolved salt. A solution (water and salt) containing 1/2 gram of salt per liter begins flowing into the tank at the rate of 2 liters per minute.

a. How many liters of solution are in the tank after 10 minutes ?

b. How many grams of salt are in the tank after 10 minutes ?

c. What is the concentration (grams per liter) of salt in the tank after 10 minutes ?

d. How long will it take the concentration of salt in the tank to reach exactly 1/3 gram per liter, and how many liters of solution will be in the tank at that time ? Let x be the unknown number of minutes and use algebra to solve for x.

SA16. A large tank initially holds 200 liters of pure water. A solution (water and salt) containing 30 grams of salt per liter begins flowing into the tank at the rate of 5 liters per minute. After exactly 20 minutes, 250 liters of the well-stirred mixture is instantaneously removed from the tank. However, the salt water solution continues to flow into the tank and the tank once again begins to fill up.

a. How many liters of solution are in the tank 40 minutes after the salt water begins entering the tank ?

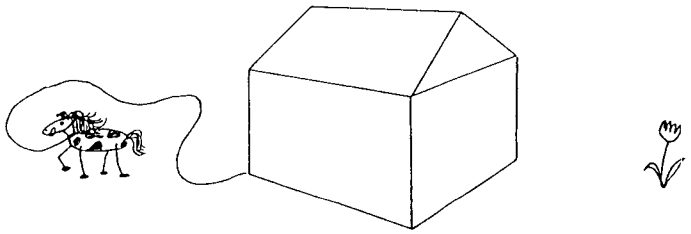
b. How many grams of salt are in the tank 40 minutes after the salt water begins entering the tank ?

c. What is the concentration (grams per liter) of salt in the tank 40 minutes after the salt water begins entering the tank ?

d. How long after the salt water begins entering the tank will it take the concentration of salt in the tank to reach exactly 28 grams per liter, and how many grams of salt and liters of solution will be in the tank at that time ?

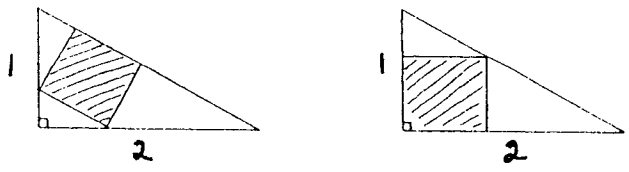
SA17. A horse is tethered by a rope to the corner of a shed with a 10 ft. by 10 ft. floor (Fig. 10). If the rope is 40 ft. long, draw a "bird's eye view" of the shape of the horse's grazing area. How close can you plant flowers to the shed and keep the horse from eating them ?

Fig. 10



SA18. Each of the following right triangles has a shaded square inscribed in its interior (Fig. 11). Which square has the larger area ?

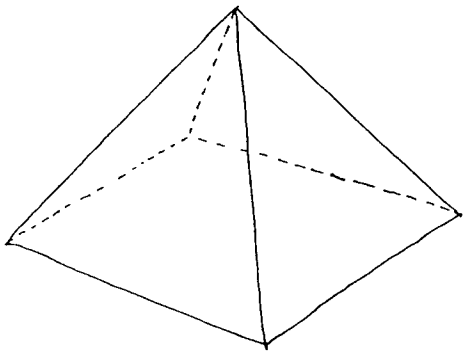
Fig. 11



SA19. Consider the given symmetrical 4-sided pyramid of height 3 ft. and with square base of area 16 ft. ^{2} (Fig. 12).

a. Compute the total surface area of the pyramid.
b. Find the radius of the largest sphere which can be inscribed inside the pyramid.

Fig. 12



SA20. Consider the given regular tetrahedron (each of the four faces of this pyramid is an equilateral triangle of exactly the same size) of side length 1 ft. (Fig. 13).

a. Compute the total surface area of the pyramid.
b. Find the radius of the largest sphere which can be inscribed inside the pyramid.

Fig. 13