

PREREQUISITE
REVIEW 1.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, simplify each expression.

1. $\sqrt{(3-6)^2 + [1-(-5)]^2}$

3. $\frac{5+(-4)}{2}$

5. $\sqrt{27} + \sqrt{12}$

In Exercises 7–10, solve for x or y .

7. $\sqrt{(3-x)^2 + (7-4)^2} = \sqrt{45}$

9. $\frac{x+(-5)}{2} = 7$

2. $\sqrt{(-2-0)^2 + [-7-(-3)]^2}$

4. $\frac{-3+(-1)}{2}$

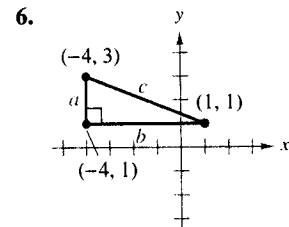
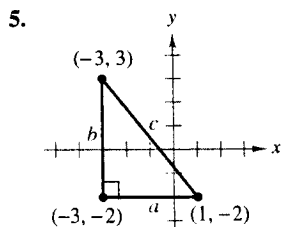
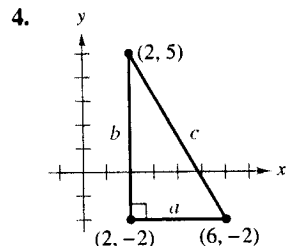
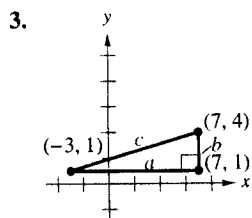
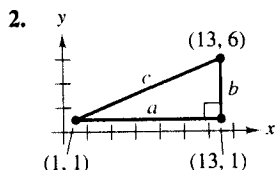
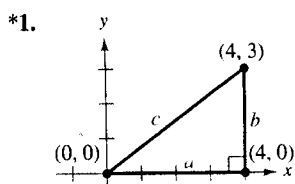
6. $\sqrt{8} - \sqrt{18}$

8. $\sqrt{(6-2)^2 + (-2-y)^2} = \sqrt{52}$

10. $\frac{-7+y}{2} = -3$

EXERCISES 1.1

In Exercises 1–6, (a) find the length of each side of the right triangle and (b) show that these lengths satisfy the Pythagorean Theorem.



In Exercises 7–14, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.

7. (3, 1), (5, 5)

8. (-3, 2), (3, -2)

9. $(\frac{1}{2}, 1)$, $(-\frac{3}{2}, -5)$

10. $(\frac{2}{3}, -\frac{1}{3})$, $(\frac{5}{6}, 1)$

11. (2, 2), (4, 14)

12. (-3, 7), (1, -1)

13. $(1, \sqrt{3})$, $(-1, 1)$

14. (-2, 0), $(0, \sqrt{2})$

In Exercises 15–18, show that the points form the vertices of the given figure. (A rhombus is a quadrilateral whose sides have the same length.)

Vertices	Figure
15. (0, 1), (3, 7), (4, -1)	Right triangle
16. (1, -3), (3, 2), (-2, 4)	Isosceles triangle
17. (0, 0), (1, 2), (2, 1), (3, 3)	Rhombus
18. (0, 1), (3, 7), (4, 4), (1, -2)	Parallelogram

In Exercises 19–22, use the Distance Formula to determine whether the points are collinear (lie on the same line).

19. (0, -4), (2, 0), (3, 2)

20. (0, 4), (7, -6), (-5, 11)

21. (-2, -6), (1, -3), (5, 2)

22. (-1, 1), (3, 3), (5, 5)

In Exercises 23 and 24, find x such that the distance between the points is 5.

23. (1, 0), $(x, -4)$

24. (2, -1), $(x, 2)$

In Exercises 25 and 26, find y such that the distance between the points is 8.

25. (0, 0), (3, y)

26. (5, 1), (5, y)

* The answers to the odd-numbered and selected even exercises are given in the back of the text. Worked-out solutions to the odd-numbered exercises are given in the *Student Solutions Guide*.

27. Use the Midpoint Formula to find the midpoint of the line segment joining the points that divide the segment into four equal parts.

28. Show that $(\frac{1}{3}[2x_1 + x_2], \frac{1}{3}[2y_1 + y_2])$ is the second trisection of the line segment joining the points (x_1, y_1) and (x_2, y_2) . Then, find the second midpoint of the segment joining the points $(1, -2)$ and $(4, -1)$.

$$(\frac{1}{3}[2x_1 + x_2], \frac{1}{3}[2y_1 + y_2])$$

29. Use Exercise 27 to find the midpoint joining the given points.

(a) (1, -2), (4, -1)

(b) (-2, -3), (0, 0)

30. Use Exercise 28 to find the second midpoint of the segment joining the given points.

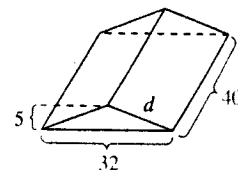
(a) (1, -2), (4, 1)

(b) (-2, -3), (0, 0)

31. **Building Dimension** Trusses for the roof of a building are shown in the figure. Find the distance d between the points $(1, -2)$ and $(4, -1)$.

(a) Find the distance d between the points $(1, -2)$ and $(4, -1)$.

(b) The length of the part (a) to find the distance d .



32. **Wire Length** A guy wire is attached to a tower at a point 200 feet from the base of the tower. The distance from the base of the tower to the point where the wire is attached is 125 feet. Find the length of the wire.



The symbol $\oplus$ indicates an exercise that involves graphing technology or a symbol that indicates that other exercises may also involve graphing technology.

17. Use the Midpoint Formula repeatedly to find the three points that divide the segment joining (x_1, y_1) and (x_2, y_2) into four equal parts.

18. Show that $(\frac{1}{3}[2x_1 + x_2], \frac{1}{3}[2y_1 + y_2])$ is one of the points of trisection of the line segment joining (x_1, y_1) and (x_2, y_2) . Then, find the second point of trisection by finding the midpoint of the segment joining

$$(\frac{1}{3}[2x_1 + x_2], \frac{1}{3}[2y_1 + y_2]) \text{ and } (x_2, y_2).$$

19. Use Exercise 27 to find the points that divide the line segment joining the given points into four equal parts.

(a) $(1, -2), (4, -1)$

(b) $(-2, -3), (0, 0)$

20. Use Exercise 28 to find the points of trisection of the line segment joining the given points.

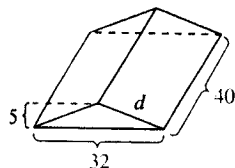
(a) $(1, -2), (4, 1)$

(b) $(-2, -3), (0, 0)$

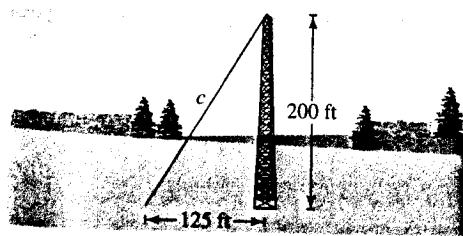
31. **Building Dimensions** The base and height of the trusses for the roof of a house are 32 feet and 5 feet, respectively (see figure).

- (a) Find the distance d from the eaves to the peak of the roof.

- (b) The length of the house is 40 feet. Use the result of part (a) to find the number of square feet of roofing.



32. **Wire Length** A guy wire is stretched from a broadcasting tower at a point 200 feet above the ground to an anchor 125 feet from the base (see figure). How long is the wire?



33. **Consumer Trends** The numbers (in millions) of cable television subscribers in the United States for 1992–2001 are shown in the table. (Source: Nielsen Media Research)

Year	1992	1993	1994	1995	1996
Subscribers	57.2	58.8	60.5	63.0	64.6

Year	1997	1998	1999	2000	2001
Subscribers	65.9	67.0	68.5	69.3	73.0

34. **Consumer Trends** The numbers (in millions) of cellular telephone subscribers in the United States for 1993–2002 are shown in the table. (Source: Cellular Telecommunications & Internet Association)

Year	1993	1994	1995	1996	1997
Subscribers	16.0	24.1	33.8	44.0	55.3

Year	1998	1999	2000	2001	2002
Subscribers	69.2	86.0	109.5	128.4	140.8

- Dow Jones Industrial Average** In Exercises 35 and 36, use the figure below showing the Dow Jones Industrial Average for common stocks. (Source: Dow Jones, Inc.)

35. Estimate the Dow Jones Industrial Average for each date.

(a) March 2002

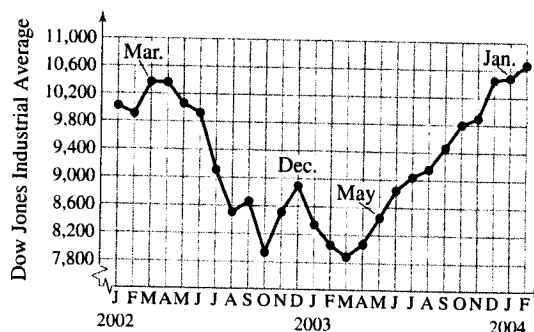
(b) December 2002

(c) May 2003

(d) January 2004

36. Estimate the percent increase or decrease in the Dow Jones Industrial Average (a) from April 2002 to November 2002 and (b) from June 2003 to February 2004.

Figure for 35 and 36

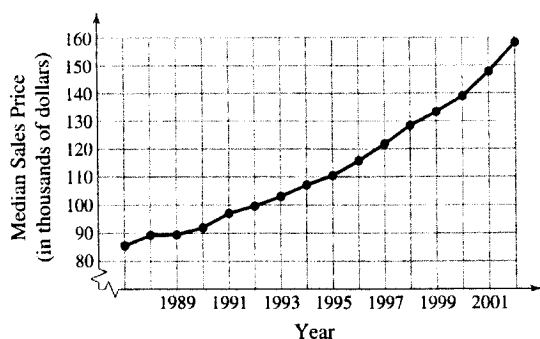


The symbol $\oplus$ indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by use of appropriate technology.

Construction In Exercises 37 and 38, use the figure, which shows the median sales prices of existing one-family homes sold (in thousands of dollars) in the United States from 1987 to 2002. (Source: National Association of Realtors)

37. Estimate the median sales price of existing one-family homes for each year.
- (a) 1987 (b) 1992
- (c) 1997 (d) 2002
38. Estimate the percent increases in the value of existing one-family homes (a) from 1993 to 1994 and (b) from 2001 to 2002.

Figure for 37 and 38



Research Project In Exercises 39 and 40, (a) use the Midpoint Formula to estimate the revenue and profit of the company in 2001. (b) Then use your school's library, the Internet, or some other reference source to find the actual revenue and profit for 2001. (c) Did the revenue and profit increase in a linear pattern from 1999 to 2003? Explain your reasoning. (d) What were the company's expenses during each of the given years? (e) How would you rate the company's growth from 1999 to 2003? (Source: Walgreen Company and The Yankee Candle Company)

39. Walgreen Company

Year	1999	2001	2003
Revenue (millions of \$)	17,839		32,505
Profit (millions of \$)	624.1		1157.3

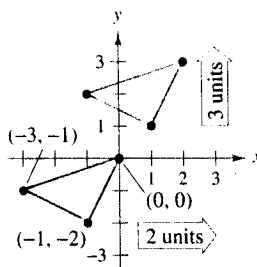
40. The Yankee Candle Company

Year	1999	2001	2003
Revenue (millions of \$)	256.6		508.6
Profit (millions of \$)	34.3		74.8

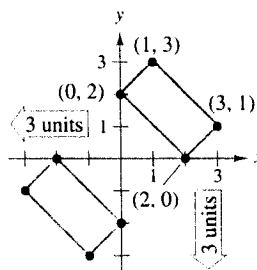
Computer Graphics In Exercises 41 and 42, the red figure is translated to a new position in the plane to form the blue figure.

(a) Find the vertices of the transformed figure. (b) Then use a graphing utility to draw both figures.

41.



42.



43. Economics The table shows the numbers of ear infections treated by doctors at HMO clinics of three different sizes: small, medium, and large.

Cases per small clinic	Cases per medium clinic	Cases per large clinic	Number of doctors
0	0	0	0
20	30	35	1
28	42	49	2
35	53	62	3
40	60	70	4

- (a) Show the relationship between doctors and treated ear infections using *three* curves, where the number of doctors is on the horizontal axis and the number of ear infections treated is on the vertical axis.

- (b) Compare the three relationships.

(Source: Adapted from Taylor, *Economics, Fourth Edition*)

The symbol indicates an exercise that contains material from textbooks in other disciplines.

The Graph of an

In Section 1.1, you learned about the relationship between two variables. A collection of points in a plane is called a graph.

Frequently, a relationship between two variables can be described by an equation. For instance, degrees Celsius can be converted to degrees Fahrenheit by the equation

$$F = \frac{9}{5}C + 32.$$

In this section, you will learn how to graph such equations. The solutions of the equation

EXAMPLE 1

Sketch the graph of $y = 7 - 3x$.

SOLUTION The simplest way to graph a linear equation is the *plotting method*. With this method, you find several solution points of the equation and plot them. When $x = 0$,

$$y = 7 - 3(0) = 7$$

which implies that $(0, 7)$ is a solution point.

x	0	1
$y = 7 - 3x$	7	4

From the table, it follows that $(0, 7)$, $(1, 4)$, $(2, 1)$, and $(3, -2)$ are solution points of the equation.

These points appear to lie on a line. The line that passes through these points is the graph of the equation $y = 7 - 3x$.

STUDY TIP

Even though we refer to the graph of $y = 7 - 3x$ as a line, it actually is a line that would extend infinitely in both directions.

**PREREQUISITE
REVIEW 1.2**

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, solve for y .

1. $5y - 12 = x$

2. $-y = 15 - x$

3. $x^3y + 2y = 1$

4. $x^2 + x - y^2 - 6 = 0$

5. $(x - 2)^2 + (y + 1)^2 = 9$

6. $(x + 6)^2 + (y - 5)^2 = 81$

In Exercises 7–10, complete the square to write the expression as a perfect square trinomial.

7. $x^2 - 4x +$

8. $x^2 + 6x +$

9. $x^2 - 5x +$

10. $x^2 + 3x +$

In Exercises 11–14, factor the expression.

11. $x^2 - 3x + 2$

12. $x^2 + 5x + 6$

13. $y^2 - 3y + \frac{9}{4}$

14. $y^2 - 7y + \frac{49}{4}$

EXERCISES 1.2

In Exercises 1–6, determine whether the points are solution points of the given equation.

1. $2x - y - 3 = 0$

(a) $(1, 2)$

(b) $(1, -1)$

(c) $(4, 5)$

2. $7x + 4y - 6 = 0$

(a) $(6, -9)$

(b) $(-5, 10)$

(c) $(\frac{1}{2}, \frac{5}{8})$

3. $x^2 + y^2 = 4$

(a) $(1, -\sqrt{3})$

(b) $(\frac{1}{2}, -1)$

(c) $(\frac{3}{2}, \frac{7}{2})$

4. $x^2y + x^2 - 5y = 0$

(a) $(0, \frac{1}{5})$

(b) $(2, 4)$

(c) $(-2, -4)$

5. $x^2 - xy + 4y = 3$

(a) $(0, 2)$

(b) $(-2, -\frac{1}{6})$

(c) $(3, -6)$

6. $3y + 2xy - x^2 = 5$

(a) $(-7, -5)$

(b) $(-1, 6)$

(c) $(1, \frac{6}{5})$

In Exercises 7–12, match the equation with its graph. Use a graphing utility, set for a square setting, to confirm your result. [The graphs are labeled (a)–(f).]

7. $y = x - 2$

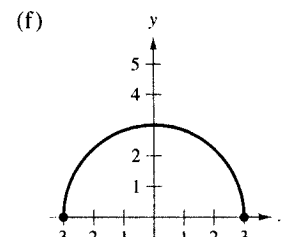
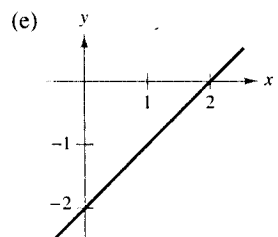
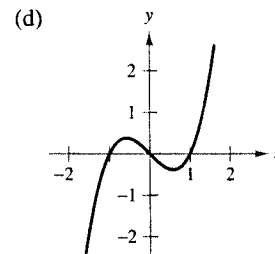
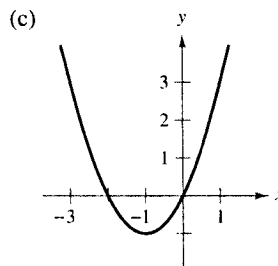
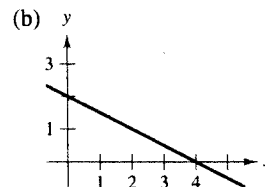
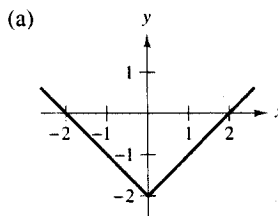
8. $y = -\frac{1}{2}x + 2$

9. $y = x^2 + 2x$

10. $y = \sqrt{9 - x^2}$

11. $y = |x| - 2$

12. $y = x^3 - x$



In Exercises 13–22, find the x - and y -intercepts of the graph of the equation.

13. $2x - y - 3 = 0$

14. $4x - 2y - 5 = 0$

15. $y = x^2 + x - 2$

16. $y = x^2 - 4x + 3$

17. $y = x^2\sqrt{9 - x^2}$

18. $y^2 = x^3 - 4x$

19. $y = \frac{x^2 - 4}{x - 2}$

20. $y = \frac{x^2 + 3x}{(3x + 1)^2}$

21. $x^2y - x^2 + 4y = 0$

22. $2x^2y + 8y - x^2 = 1$

In Exercises 23–38, sketch the graph of the equation and label the intercepts. Use a graphing utility to verify your results.

23. $y = 2x + 3$

24. $y = -3x + 2$

25. $y = x^2 - 3$

26. $y = x^2 + 6$

27. $y = (x - 1)^2$

28. $y = (5 - x)^2$

29. $y = x^3 + 2$

30. $y = 1 - x^3$

31. $y = -\sqrt[3]{x + 1}$

32. $y = \sqrt{x + 1}$

33. $y = |x + 1|$

34. $y = -|x - 2|$

35. $y = 1/(x - 3)$

36. $y = 1/(x^2 + 1)$

37. $x = y^2 - 4$

38. $x = 4 - y^2$

In Exercises 39–46, write the general form of the equation of the circle.

39. Center: (0, 0); radius: 3

40. Center: (0, 0); radius: 5

41. Center: (2, -1); radius: 4

42. Center: (-4, 3); radius: 3

43. Center: (-1, 2); solution point: (0, 0)

44. Center: (3, -2); solution point: (-1, 1)

45. Endpoints of a diameter: (3, 3), (-3, 3)

46. Endpoints of a diameter: (-4, -1), (4, 1)

 In Exercises 47–54, complete the square to write the equation of the circle in standard form. Then use a graphing utility to graph the circle.

47. $x^2 + y^2 - 2x + 6y + 6 = 0$

48. $x^2 + y^2 - 2x + 6y - 15 = 0$

49. $x^2 + y^2 + 4x + 6y - 3 = 0$

50. $x^2 + y^2 - 4x + 2y + 3 = 0$

51. $2x^2 + 2y^2 - 2x - 2y - 3 = 0$

52. $4x^2 + 4y^2 - 4x + 2y - 1 = 0$

53. $16x^2 + 16y^2 + 16x + 40y - 7 = 0$

54. $3x^2 + 3y^2 - 6y - 1 = 0$

In Exercises 55–62, find the points of intersection (if any) of the graphs of the equations. Use a graphing utility to check your results.

55. $x + y = 2$, $2x - y = 1$

56. $x + y = 7$, $3x - 2y = 11$

57. $x^2 + y^2 = 25$, $2x + y = 10$

58. $x^2 + y = 4$, $2x - y = 1$

59. $y = x^3$, $y = 2x$

60. $y = \sqrt{x}$, $y = x$

61. $y = x^4 - 2x^2 + 1$, $y = 1 - x^2$

62. $y = x^3 - 2x^2 + x - 1$, $y = -x^2 + 3x - 1$

63. Break-Even Analysis You are setting up a part-time business with an initial investment of \$15,000. The unit cost of the product is \$11.80, and the selling price is \$19.30.

(a) Find equations for the total cost C and total revenue R for x units.

(b) Find the break-even point by finding the point of intersection of the cost and revenue equations.

(c) How many units would yield a profit of \$1000?

64. Break-Even Analysis A 2004 Chevrolet Malibu costs \$20,930 with a gasoline engine. A 2004 Toyota Prius costs \$22,052 with a hybrid engine. The Malibu gets 16 miles per gallon of gasoline and the Prius gets 35 miles per gallon of gasoline. Assume that the price of gasoline is \$1.759. (Source: Adapted from Consumer Reports, May 2004)

(a) Show that the cost C_g of driving the Chevrolet Malibu x miles is

$$C_g = 20,930 + 1.759x/16$$

and the cost C_h of driving the Toyota Prius x miles is

$$C_h = 22,052 + 1.759x/35.$$

(b) Find the break-even point. That is, find the mileage at which the hybrid-powered Toyota Prius becomes more economical than the gasoline-powered Chevrolet Malibu.

 **Break-Even Analysis** In Exercises 65–68, find the sales necessary to break even for the given cost and revenue equations. (Round your answer up to the nearest whole unit.) Use a graphing utility to graph the equations and then find the break-even point.

65. $C = 0.85x + 35,000$, $R = 1.55x$

66. $C = 6x + 500,000$, $R = 35x$

67. $C = 8650x + 250,000$, $R = 9950x$

68. $C = 5.5\sqrt{x} + 10,000$, $R = 3.29x$

69. Supply and Demand The demand and supply equations for an electronic organizer are given by

$$p = 180 - 4x \quad \text{Demand equation}$$

$$p = 75 + 3x \quad \text{Supply equation}$$

where p is the price in dollars and x represents the number of units, in thousands. Find the equilibrium point for this market.

70. Supply and Demand The demand and supply equations for a portable CD player are given by

$$p = 190 - 15x \quad \text{Demand equation}$$

$$p = 75 + 8x \quad \text{Supply equation}$$

where p is the price in dollars and x represents the number of units, in hundreds of thousands. Find the equilibrium point for this market.

71. Consumer Trends The number of billions of dollars spent on books in the United States in the years 1955 to 2000 is given by the function

Year	1995
Expense	2708

Year	1999
Expense	3773

A mathematical model for the data is

$$y = 2.177t^3 - 41t^2 + 27,080t - 15,000$$

where t represents the year.

(a) Compare the actual data with the model. How good is the model?

(b) Use the model to predict the expense in 2005.

72. Farm Work Force The percentage of the U.S. farm work force in the United States from 1955 to 2000 is given by the function

Year	1955
Percent	9.9

Year	1980
Percent	3.1

A mathematical model for the data is

$$y = \frac{-4.97 + 0.02t}{1 - 0.025t}$$

where y represents the percentage of the farm work force with $t = 55$ corresponding to 1955.

(a) Compare the actual data with the model. How good is the model?

(b) Use the model to predict the percentage of the farm work force in 2005.

(c) Discuss the validity of the model.

73. Weekly Salary A person's weekly salary y of a person in the year t is given by the function

$$y = \frac{292.48 + 37.7t}{1 + 0.02t}$$

where t represents the year. (Source: U.S. Bureau of Economic Analysis)

- 71. Consumer Trends** The amounts of money y (in millions of dollars) spent on college textbooks in the United States in the years 1995 to 2002 are shown in the table. (Source: Book Industry Study Group, Inc.)

Year	1995	1996	1997	1998
Expense	2708	2920	3110	3365

Year	1999	2000	2001	2002
Expense	3773	3905	4187	4706

A mathematical model for the data is given by

$$y = 2.177t^3 - 41.99t^2 + 497.1t + 985$$

where t represents the year, with $t = 5$ corresponding to 1995.

- (a) Compare the actual expenses with those given by the model. How good is the model? Explain your reasoning.
- (b) Use the model to predict the expenses in 2010.

- 72. Farm Work Force** The numbers of workers in farm work force in the United States for selected years from 1955 to 2000, as percents of the total work force, are shown in the table. (Source: Department of Commerce)

Year	1955	1960	1965	1970	1975
Percent	9.9	7.8	5.9	4.2	3.6

Year	1980	1985	1990	1995	2000
Percent	3.1	2.8	2.6	2.6	1.7

A mathematical model for the data is given by

$$y = \frac{-4.97 + 0.021t}{1 - 0.025t}$$

where y represents the percent and t represents the year, with $t = 55$ corresponding to 1955.

- (a) Compare the actual percents with those given by the model. How good is the model?
- (b) Use the model to predict the farm work force as a percent of the total work force in 2010.
- (c) Discuss the validity of your prediction in part (b).

- 73. Weekly Salary** A mathematical model for the average weekly salary y of a person in finance, insurance, or real estate is given by

$$y = \frac{292.48 + 37.72t}{1 + 0.02t}$$

where t represents the year, with $t = 7$ corresponding to 1997. (Source: U.S. Bureau of Labor Statistics)

- (a) Use the model to complete the table.

Year	1997	1998	1999	2000	2001	2004
Salary						

- (b) This model was created using actual data from 1997 through 2002. How accurate do you think the model is in predicting the 2004 average weekly salary? Explain your reasoning.
- (c) What does this model predict the average weekly salary to be in 2006? Do you think this prediction is valid?

- 74. Medicine** A mathematical model for the numbers of kidney transplants performed in the United States in the years 1998 to 2002 is given by

$$y = 60.64t^2 - 544.0t + 12,624$$

where y is the number of transplants and t is the time in years, with $t = 8$ corresponding to 1998. (Source: United Network for Organ Sharing)

- (a) Enter the model into a graphing utility and use it to complete the table.

Year	1998	1999	2000	2001	2002
Transplants					

- (b) Use your school's library, the Internet, or some other reference source to find the actual numbers of kidney transplants for the years 1998 to 2002. Compare the actual numbers with those given by the model. How good is the model? Explain your reasoning.
- (c) Using this model, what is the prediction for the number of transplants in the year 2008? How valid do you think the prediction is? What factors could affect this model's accuracy?

- 75.** Use a graphing utility to graph the equation $y = cx + 1$ for $c = 1, 2, 3, 4$, and 5 . Then make a conjecture about the x -coefficient and the graph of the equation.

- 76.** Define the break-even point for a business marketing a new product. Give examples of a linear cost equation and a linear revenue equation for which the break-even point is 10,000 units.

- In Exercises 77–82, use a graphing utility to graph the equation. Use the graphing utility to approximate the x - and y -intercepts of the graph.**

77. $y = 0.24x^2 + 1.32x + 5.36$

78. $y = -0.56x^2 - 5.34x + 6.25$

79. $y = \sqrt{0.3x^2 - 4.3x + 5.7}$

80. $y = \sqrt{-1.21x^2 + 2.34x + 5.6}$

81. $y = \frac{0.2x^2 + 1}{0.1x + 2.4}$

82. $y = \frac{0.4x - 5.3}{0.4x^2 + 5.3}$

PREREQUISITE
REVIEW 1.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, simplify the expression.

1. $\frac{5 - (-2)}{-3 - 4}$

2. $\frac{-7 - (-0)}{4 - 1}$

3. Evaluate $-\frac{1}{m}$ when $m = -3$.

4. Evaluate $-\frac{1}{m}$ when $m = \frac{6}{7}$.

In Exercises 5–10, solve for y in terms of x .

5. $-4x + y = 7$

6. $3x - y = 7$

7. $y - 2 = 3(x - 4)$

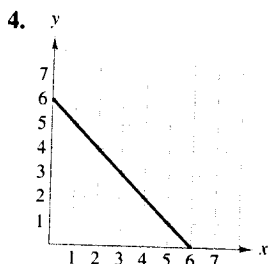
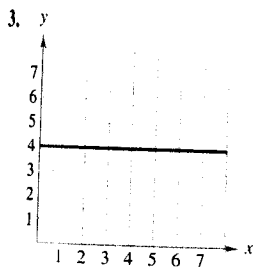
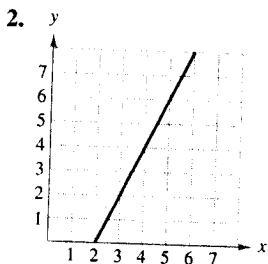
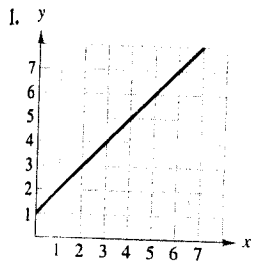
8. $y - (-5) = -1[x - (-2)]$

9. $y - (-3) = \frac{4 - (-3)}{2 - 1}(x - 2)$

10. $y - 1 = \frac{-3 - 1}{-7 - (-1)}[x - (-1)]$

EXERCISES 1.3

In Exercises 1–4, estimate the slope of the line.



In Exercises 17–24, use the point on the line and the slope of the line to find three additional points through which the line passes. (There are many correct answers.)

Point	Slope	Point	Slope
17. (2, 1)	$m = 0$	18. (-3, -1)	$m = 0$
19. (6, -4)	$m = \frac{2}{3}$	20. (-2, -2)	$m = \frac{5}{2}$
21. (1, 7)	$m = -3$	22. (10, -6)	$m = -1$
23. (-8, 1)	m is undefined.		
24. (-3, 4)	m is undefined.		

In Exercises 25–34, find the slope and y-intercept (if possible) of the equation of the line.

- | | |
|--------------------|--------------------|
| 25. $x + 5y = 20$ | 26. $2x + y = 40$ |
| 27. $7x - 5y = 15$ | 28. $6x - 5y = 15$ |
| 29. $3x - y = 15$ | 30. $2x - 3y = 24$ |
| 31. $x = 4$ | 32. $x + 5 = 0$ |
| 33. $y - 4 = 0$ | 34. $y + 1 = 0$ |

In Exercises 35–46, write an equation of the line that passes through the points. Then use the equation to sketch the line.

- | | |
|---|--|
| 35. (4, 3), (0, -5) | 36. (-3, -4), (1, 4) |
| 37. (0, 0), (-1, 3) | 38. (-3, 6), (1, 2) |
| 39. (2, 3), (2, -2) | 40. (6, 1), (10, 1) |
| 41. (3, -1), (-2, -1) | 42. (2, 5), (2, -10) |
| 43. $(-\frac{1}{3}, 1)$, $(-\frac{2}{3}, \frac{5}{6})$ | 44. $(\frac{7}{8}, \frac{3}{4})$, $(\frac{5}{4}, -\frac{1}{4})$ |
| 45. $(-\frac{1}{2}, 4)$, $(\frac{1}{2}, 8)$ | 46. (4, -1), $(\frac{1}{4}, -5)$ |

In Exercises 5–16, plot the points and find the slope of the line passing through the pair of points.

- | | |
|--|--|
| 5. (3, -4), (5, 2) | 6. (1, 2), (-2, 2) |
| 7. $(\frac{1}{2}, 2)$, (6, 2) | 8. $(\frac{11}{3}, -2)$, $(\frac{11}{3}, -10)$ |
| 9. (-8, -3), (-8, -5) | 10. (2, -1), (-2, -5) |
| 11. (-2, 1), (4, -3) | 12. (3, -5), (-2, -5) |
| 13. $(\frac{1}{4}, -2)$, $(-\frac{3}{8}, 1)$ | 14. $(-\frac{3}{2}, -5)$, $(\frac{5}{6}, 4)$ |
| 15. $(\frac{2}{3}, \frac{5}{3})$, $(\frac{1}{4}, -\frac{5}{6})$ | 16. $(\frac{7}{8}, \frac{3}{4})$, $(\frac{5}{4}, -\frac{1}{4})$ |

 In Exercises 47–56, write an equation of the line that passes through the given point and has the given slope. Then use a graphing utility to graph the line.

Point	Slope	Point	Slope
47. (0, 3)	$m = \frac{3}{4}$	48. (0, 0)	$m = \frac{2}{3}$
49. (-1, 2)	m is undefined.		
50. (0, 4)	m is undefined.		
51. (-2, 7)	$m = 0$	52. (-2, 4)	$m = 0$
53. (0, -2)	$m = -4$	54. (-1, -4)	$m = -2$
55. $(0, \frac{2}{3})$	$m = \frac{3}{4}$	56. $(0, -\frac{2}{3})$	$m = \frac{1}{6}$

In Exercises 57 and 58, explain how to use the concept of slope to determine whether the three points are collinear. Then explain how to use the Distance Formula to determine whether the points are collinear.

57. $(-2, 1)$, $(-1, 0)$, $(2, -2)$

58. $(0, 4)$, $(7, -6)$, $(-5, 11)$

59. Write an equation of the vertical line with x -intercept at 3.

60. Write an equation of the horizontal line through $(0, -5)$.

61. Write an equation of the line with y -intercept at -10 and parallel to all horizontal lines.

62. Write an equation of the line with x -intercept at -5 and parallel to all vertical lines.

 In Exercises 63–70, write the equations of the lines through the given point (a) parallel to the given line and (b) perpendicular to the given line. Then use a graphing utility to graph all three equations in the same viewing window.

Point	Line
63. $(-3, 2)$	$x + y = 7$
64. $(2, 1)$	$4x - 2y = 3$
65. $(-\frac{2}{3}, \frac{7}{8})$	$3x + 4y = 7$
66. $(\frac{7}{8}, \frac{3}{4})$	$5x + 3y = 0$
67. $(-1, 0)$	$y + 3 = 0$
68. $(2, 5)$	$y + 4 = 0$
69. $(1, 1)$	$x - 2 = 0$
70. $(12, -3)$	$x + 4 = 0$

In Exercises 71–78, sketch the graph of the equation. Use a graphing utility to verify your result.

- | | |
|-------------------------|------------------------|
| 71. $y = -2$ | 72. $y = -4$ |
| 73. $2x - y - 3 = 0$ | 74. $x + 2y + 6 = 0$ |
| 75. $y = -2x + 1$ | 76. $4x + 5y = 20$ |
| 77. $y + 2 = -4(x + 1)$ | 78. $y - 1 = 3(x + 4)$ |

79. Population The resident population of South Carolina (in thousands) was 3860 in 1997 and 4107 in 2002. Assume that the relationship between the population y and the year t is linear. Let $t = 7$ represent 1997. (Source: U.S. Census Bureau)

- Write a linear model for the data. What is the slope and what does it tell you about the population?
- Estimate the population in 1999.
- Use your model to estimate the population in 2001.
- Use your school's library, the Internet, or some other reference source to find the actual populations in 1999 and 2001. How close were your estimates?
- Do you think your model could be used to predict the population in 2006? Explain.

80. Annual Salary Your annual salary was \$26,300 in 2002 and \$29,700 in 2004. Assume your salary can be modeled by a linear equation.

- Write a linear equation giving your salary S in terms of the year. Let $t = 2$ represent 2002.
- Use the linear model to predict your salary in 2008.

81. Temperature Conversion Write a linear equation that expresses the relationship between the temperature in degrees Celsius C and degrees Fahrenheit F . Use the fact that water freezes at 0°C (32°F) and boils at 100°C (212°F).

 **82. Chemistry** Use the result of Exercise 81 to answer the following:

- A person has a temperature of 102.5°F . What is this temperature on the Celsius scale?
- If the temperature in a room is 74°F , what is this temperature on the Celsius scale?

(Source: Adapted from Zumdahl, Chemistry, Sixth Edition)

83. Reimbursed Expenses A company reimburses its sales representatives \$150 per day for lodging and meals, plus \$0.34 per mile driven. Write a linear equation giving the daily cost C in terms of x , the number of miles driven.

84. Union Negotiation You are on a negotiating panel in a union hearing for a large corporation. The union is asking for a base pay of \$9.25 per hour plus an additional piecework rate of \$0.80 per unit produced. The corporation is offering a base pay of \$6.85 per hour plus a piecework rate of \$1.15.

- Write a linear equation for the hourly wages W in terms of x , the number of units produced per hour, for each pay schedule.

 (b) Use a graphing utility to graph each linear equation and find the point of intersection.

- Interpret the meaning of the point of intersection of the graphs. How would you use this information to advise the corporation and the union?

85. Chemistry Ethylene glycol is used in automobile antifreeze. Assume the relationship between the amount of antifreeze y (in percent) and the temperature x (in $^\circ\text{C}$) is linear. You can base the approximation on the approximate data points below. (Source: Automotive Engineering Handbook, 4th Edition, SAE International)

- Derive an expression for the relationship between y and x .
- Derive an expression for the relationship between y and x .
- At what temperature does the antifreeze freeze in $^\circ\text{C}$ and in $^\circ\text{F}$?
- Your thermometer is in $^\circ\text{C}$ and in $^\circ\text{F}$?
- What is a temperature in $^\circ\text{C}$ and in $^\circ\text{F}$?

(Source: Zumdahl, Chemistry, Sixth Edition)

86. Linear Depreciation A house for \$825,000. It has a useful life of 25 years, and its value decreases linearly to \$75,000. Write a linear equation giving the value V of the house during its life in terms of the time t in years.

87. Linear Depreciation A piece of equipment for \$10,000 will be outdated, having a value of \$2,000 after 5 years.

- Write a linear equation giving the value V of the equipment in terms of the time t in years.
- Use a graphing utility to graph the equation.
- Move the cursor to the point on the graph when $t = 3$. What is the value of the equipment?
- Move the cursor to the point on the graph when $V = 4000$. What is the time t in years?

88. College Enrollment The number of students enrolled in 2002 and 2702 follows a linear growth pattern. Write a linear equation giving the number of students N in terms of the year t .

89. Consumer Awareness A company is planning to build an apartment complex with x units. The average monthly rent for the units is \$1000. However, the average monthly rent for the units is \$1000. Assume that the relationship between the number of units x and the average monthly rent R is linear.

- Write a linear equation for the average monthly rent R in terms of the number of units x .
- Linear Extrapolation: Use the equation to find the average monthly rent when the number of units is 100.
- Linear Interpolation: Use the equation to find the number of units when the average monthly rent is \$1200.

85. **Chemistry** Ethylene glycol is the main component in automobile antifreeze. To monitor the temperature of an auto cooling system, you intend to use a meter that reads from 0 to 100. You devise a new temperature scale ($^{\circ}\text{A}$) based on the approximate melting and boiling points of a typical antifreeze solution (-45°C and 115°C). You wish these points to correspond to 0°A and 100°A , respectively.

- Derive an expression for converting between $^{\circ}\text{A}$ and $^{\circ}\text{C}$.
- Derive an expression for converting between $^{\circ}\text{F}$ and $^{\circ}\text{A}$.
- At what temperature would your thermometer and a Celsius thermometer give the same numerical reading?
- Your thermometer reads 86°A . What is the temperature in $^{\circ}\text{C}$ and in $^{\circ}\text{F}$?
- What is a temperature of 45°C in $^{\circ}\text{A}$?

86. **Linear Depreciation** A company constructs a warehouse for \$825,000. The warehouse has an estimated useful life of 25 years, after which its value is expected to be \$75,000. Write a linear equation giving the value y of the warehouse during its 25 years of useful life. (Let t represent the time in years.)

87. **Linear Depreciation** A small business purchases a piece of equipment for \$1025. After 5 years the equipment will be outdated, having no value.

- Write a linear equation giving the value y of the equipment in terms of the time t in years, $0 \leq t \leq 5$.
- Use a graphing utility to graph the equation.
- Move the cursor along the graph and estimate (to two-decimal-place accuracy) the value of the equipment when $t = 3$.
- Move the cursor along the graph and estimate (to two-decimal-place accuracy) the time when the value of the equipment will be \$600.

88. **College Enrollment** A small college had 2546 students in 2002 and 2702 students in 2004. If the enrollment follows a linear growth pattern, how many students will the college have in 2008?

89. **Consumer Awareness** A real estate office handles an apartment complex with 50 units. When the rent is \$380 per month, all 50 units are occupied. When the rent is \$425, however, the average number of occupied units drops to 47. Assume that the relationship between the monthly rent p and the demand x is linear. (The term *demand* refers to the number of occupied units.)

- Write a linear equation expressing x in terms of p .
- Linear Extrapolation** Predict the number of occupied units when the rent is set at \$455.
- Linear Interpolation** Predict the number of occupied units when the rent is set at \$395.

90. **Profit** You are a contractor and have purchased a piece of equipment for \$26,500. The equipment costs an average of \$5.25 per hour for fuel and maintenance, and the operator is paid \$9.50 per hour.

- Write a linear equation giving the total cost C of operating the equipment for t hours.
- You charge your customers \$25 per hour of machine use. Write an equation for the revenue R derived from t hours of use.
- Use the formula for profit, $P = R - C$, to write an equation for the profit derived from t hours of use.
- Find the number of hours you must operate the equipment before you break even.

91. **Personal Income** Personal income (in billions of dollars) in the United States was 6937 in 1997 and 8685 in 2001. Assume that the relationship between the personal income Y and the time t (in years) is linear. Let $t = 0$ correspond to 1990.

- Write a linear model for the data.
- Linear Interpolation** Estimate the personal income in 1999.
- Linear Extrapolation** Estimate the personal income in 2002.
- Use your school's library, the Internet, or some other reference source to find the actual personal income in 1999 and 2002. How close were your estimates?

92. **Sales Commission** As a salesperson, you receive a monthly salary of \$2000, plus a commission of 7% of sales. You are offered a new job at \$2300 per month, plus a commission of 5% of sales.

- Write a linear equation for your current monthly wage W in terms of your monthly sales S .
- Write a linear equation for the monthly wage W of your job offer in terms of the monthly sales S .

⊕ (c) Use a graphing utility to graph both equations in the same viewing window. Find the point of intersection. What does it signify?

- You think you can sell \$20,000 worth of a product per month. Should you change jobs? Explain.

⊕ In Exercises 93–102, use a graphing utility to graph the cost function. Determine the maximum production level x , given that the cost C cannot exceed \$100,000.

93. $C = 23,500 + 3100x$ 94. $C = 30,000 + 575x$

95. $C = 18,375 + 1150x$ 96. $C = 24,900 + 1785x$

97. $C = 75,500 + 89x$ 98. $C = 83,620 + 67x$

99. $C = 32,000 + 650x$ 100. $C = 53,500 + 495x$

101. $C = 50,000 + 0.25x$ 102. $C = 75,500 + 1.50x$

PREREQUISITE REVIEW 1.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, simplify the expression.

1. $5(-1)^2 - 6(-1) + 9$

2. $(-2)^3 + 7(-2)^2 - 10$

3. $(x - 2)^2 + 5x - 10$

4. $(3 - x) + (x + 3)^3$

5. $\frac{1}{1 - (1 - x)}$

6. $1 + \frac{x - 1}{x}$

In Exercises 7–12, solve for y in terms of x .

7. $2x + y - 6 = 11$

8. $5y - 6x^2 - 1 = 0$

9. $(y - 3)^2 = 5 + (x + 1)^2$

10. $y^2 - 4x^2 = 2$

11. $x = \frac{2y - 1}{4}$

12. $x = \sqrt[3]{2y - 1}$

In Exercises 1–8, decide whether the equation defines y as a function of x .

1. $x^2 + y^2 = 4$

2. $x + y^2 = 4$

3. $\frac{1}{2}x - 6y = -3$

4. $3x - 2y + 5 = 0$

5. $x^2 + y = 4$

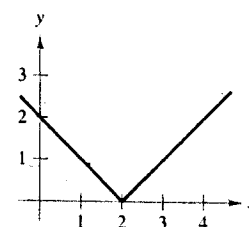
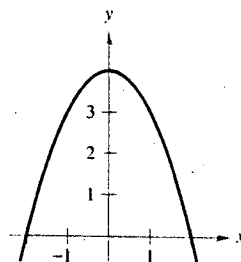
6. $x^2 + y^2 - 2x - 4y + 1 = 0$

7. $y^2 = x^2 - 1$

8. $x^2y - x^2 + 4y = 0$

19. $f(x) = 4 - x^2$

20. $f(x) = |x - 2|$



In Exercises 9–16, use a graphing utility to graph the function. Then determine the domain and range of the function.

9. $f(x) = 2x^2 - 5x + 1$

10. $f(x) = 5x^3 + 6x^2 - 1$

11. $f(x) = \frac{|x|}{x}$

12. $f(x) = \sqrt{9 - x^2}$

13. $f(x) = \frac{x}{\sqrt{x - 4}}$

14. $f(x) = \frac{2x}{\sqrt{x + 1}}$

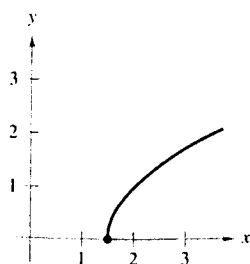
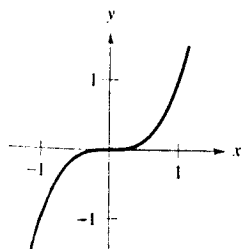
15. $f(x) = \frac{x - 2}{x + 4}$

16. $f(x) = \frac{x^2}{1 - x}$

In Exercises 17–20, find the domain and range of the function. Use interval notation to write your result.

17. $f(x) = x^3$

18. $f(x) = \sqrt{2x - 3}$



In Exercises 21–24, evaluate the function at the specified values of the independent variable. Simplify the result.

21. $f(x) = 2x - 3$

(a) $f(0)$

(b) $f(-3)$

(c) $f(x - 1)$

(d) $f(x + \Delta x)$

22. $f(x) = x^2 - 2x + 2$

(a) $f(\frac{1}{2})$

(b) $f(-1)$

(c) $f(c + 2)$

(d) $f(x + \Delta x)$

23. $g(x) = 1/x$

(a) $g(2)$

(b) $g(\frac{1}{3})$

(c) $g(x + 4)$

(d) $g(x + \Delta x) - g(x)$

24. $f(x) = |x| + 4$

(a) $f(2)$

(b) $f(-2)$

(c) $f(x + 2)$

(d) $f(x + \Delta x) - f(x)$

In Exercises 25–30, evaluate the difference quotient and simplify the result.

25. $f(x) = x^2 - 4x + 1$

26. $h(x) = x^2 - x + 1$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\frac{h(2 + \Delta x) - h(2)}{\Delta x}$$

27. $g(x) = \sqrt{x+3}$

$$\frac{g(x+\Delta x) - g(x)}{\Delta x}$$

29. $f(x) = \frac{1}{x-2}$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

28. $f(x) = \frac{1}{\sqrt{x-1}}$

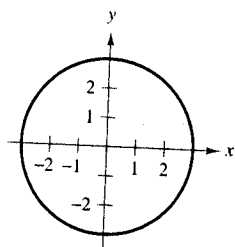
$$\frac{f(x) - f(2)}{x-2}$$

30. $f(x) = \frac{1}{x+4}$

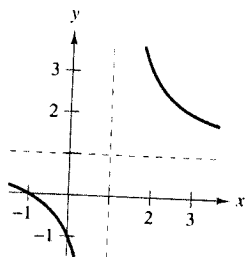
$$\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

In Exercises 31–34, use the vertical line test to determine whether y is a function of x .

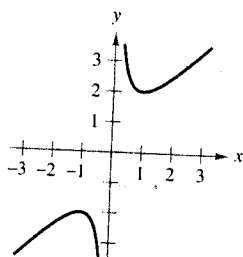
31. $x^2 + y^2 = 9$



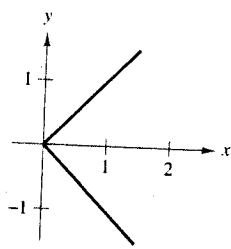
32. $x - xy + y + 1 = 0$



33. $x^2 = xy - 1$



34. $x = |y|$



In Exercises 35–40, find (a) $f(x) + g(x)$, (b) $f(x) \cdot g(x)$, (c) $f(x)/g(x)$, (d) $f(g(x))$, and (e) $g(f(x))$ if defined.

35. $f(x) = 2x - 5$

$g(x) = 5$

36. $f(x) = 2x - 5$

$g(x) = 2 - x$

37. $f(x) = x^2 + 1$

$g(x) = x - 1$

38. $f(x) = x^2 + 5$

$g(x) = \sqrt{1-x}$

39. $f(x) = \frac{1}{x}$

$g(x) = \frac{1}{x^2}$

40. $f(x) = \frac{x}{x+1}$

$g(x) = x^3$

41. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, find the composite functions.

(a) $f(g(1))$

(b) $g(f(1))$

(c) $g(f(0))$

(d) $f(g(-4))$

(e) $f(g(x))$

(f) $g(f(x))$

42. Given $f(x) = 1/x$ and $g(x) = x^2 - 1$, find the composite functions.

(a) $f(g(2))$

(b) $g(f(2))$

(c) $f(g(1/\sqrt{2}))$

(d) $g(f(1/\sqrt{2}))$

(e) $f(g(x))$

(f) $g(f(x))$

In Exercises 43–46, select a function from (a) $f(x) = x$, (b) $g(x) = cx^2$, (c) $h(x) = c\sqrt{|x|}$, and (d) $r(x) = c/x$ and determine the value of the constant c such that the function fits the data in the table.

x	-4	-1	0	1	4
y	-32	-2	0	-2	-32

x	-4	-1	0	1	4
y	-1	$-\frac{1}{4}$	0	$\frac{1}{4}$	1

x	-4	-1	0	1	4
y	-8	-32	Undefined	32	8

x	-4	-1	0	1	4
y	6	3	0	3	6

In Exercises 47–50, show that f and g are inverse functions by showing that $f(g(x)) = x$ and $g(f(x)) = x$. Then sketch the graphs of f and g on the same coordinate axes.

47. $f(x) = 5x + 1$

$g(x) = \frac{x-1}{5}$

48. $f(x) = \frac{1}{x}$

$g(x) = \frac{1}{x}$

49. $f(x) = 9 - x^2, x \geq 0$

$g(x) = \sqrt{9-x}, x \leq 9$

50. $f(x) = 1 - x^3$

$g(x) = \sqrt[3]{1-x}$

In Exercises 51–58, find the inverse function of f . Then sketch the graphs of f and f^{-1} on the same coordinate axes.

51. $f(x) = 2x - 3$

52. $f(x) = 6 - 3x$

53. $f(x) = x^5$

54. $f(x) = x^3 + 1$

55. $f(x) = \sqrt{9-x^2}, 0 \leq x \leq 3$

56. $f(x) = \sqrt{x^2-4}, x \geq 2$

57. $f(x) = x^{2/3}, x \geq 0$

58. $f(x) = x^{3/5}$



In Exercises 59–64, use a graphing utility to graph the function. Then use the horizontal line test to determine whether the function is one-to-one. If it is, find its inverse function.

59. $f(x) = 3 - 7x$

60. $f(x) = \sqrt{x-2}$

61. $f(x) = x^2$

62. $f(x) = x^4$

63. $f(x) = |x-2|$

64. Use the graph of $f(x) =$ each function.

(a) $y = \sqrt{x} + 2$

(b) $y = -\sqrt{x}$

(c) $y = \sqrt{x-2}$

(d) $y = \sqrt{x+3}$

(e) $y = \sqrt{x-4}$

(f) $y = 2\sqrt{x}$

65. Use the graph of $f(x) =$ each function.

(a) $y = |x| + 3$

(b) $y = -\frac{1}{2}|x|$

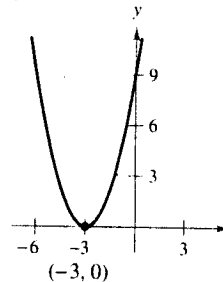
(c) $y = |x-2|$

(d) $y = |x+1| - 1$

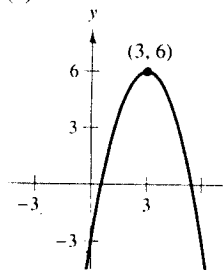
(e) $y = 2|x|$

67. Use the graph of $f(x) =$ functions whose graphs

(a)



(c)



68. **Real Estate** Express terms of x , the number acre is valued at \$250,000.

he comp 63. $f(x) = |x - 2|$

64. $f(x) = 3$

65. Use the graph of $f(x) = \sqrt{x}$ below to sketch the graph of each function.

(a) $y = \sqrt{x} + 2$

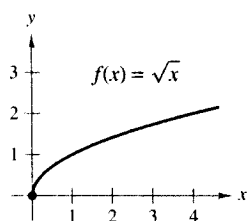
(b) $y = -\sqrt{x}$

(c) $y = \sqrt{x - 2}$

(d) $y = \sqrt{x + 3}$

(e) $y = \sqrt{x - 4}$

(f) $y = 2\sqrt{x}$



66. Use the graph of $f(x) = |x|$ below to sketch the graph of each function.

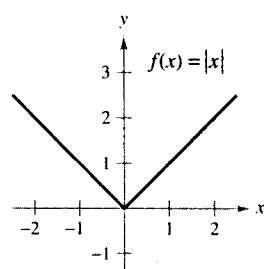
(a) $y = |x| + 3$

(b) $y = -\frac{1}{2}|x|$

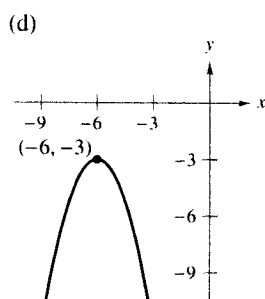
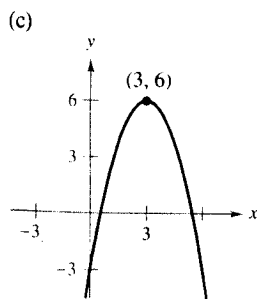
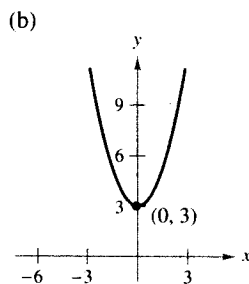
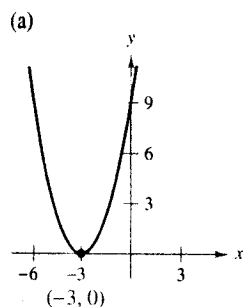
(c) $y = |x - 2|$

(d) $y = |x + 1| - 1$

(e) $y = 2|x|$



67. Use the graph of $f(x) = x^2$ to find a formula for each of the functions whose graphs are shown.



69. **Owning a Business** You own two restaurants. From 1998 to 2004, the sales R_1 (in thousands of dollars) for one restaurant can be modeled by

$$R_1 = 480 - 8t - 0.8t^2, \quad t = 0, 1, 2, 3, 4, 5, 6$$

where $t = 0$ represents 1998. During the same seven-year period, the sales R_2 (in thousands of dollars) for the second restaurant can be modeled by

$$R_2 = 254 + 0.78t, \quad t = 0, 1, 2, 3, 4, 5, 6.$$

Write a function that represents the total sales for the two restaurants. Use a graphing utility to graph the total sales function.

70. **Cost** The inventor of a new game believes that the variable cost for producing the game is \$0.95 per unit. The fixed cost is \$6000.

(a) Express the total cost C as a function of x , the number of games sold.

(b) Find a formula for the average cost per unit $\bar{C} = C/x$.

(c) The selling price for each game is \$1.69. How many units must be sold before the average cost per unit falls below the selling price?

71. **Demand** The demand function for a commodity is

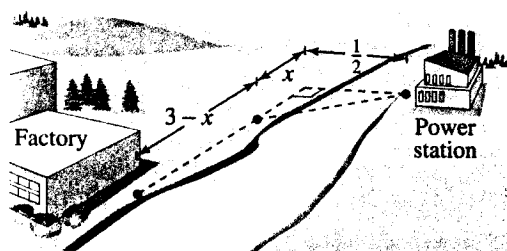
$$p = \frac{14.75}{1 + 0.01x}, \quad x \geq 0$$

where p is the price per unit and x is the number of units sold.

(a) Find x as a function of p .

(b) Find the number of units sold when the price is \$10.

72. **Cost** A power station is on one side of a river that is $\frac{1}{2}$ mile wide. A factory is 3 miles downstream on the other side of the river (see figure). It costs \$10/ft to run the power lines on land and \$15/ft to run them under water. Express the cost C of running the lines from the power station to the factory as a function of x .



73. **Cost** The weekly cost of producing x units in a manufacturing process is given by the function

$$C(x) = 70x + 375.$$

The number of units produced in t hours is given by $x(t) = 40t$. Find and interpret $C(x(t))$.

68. **Real Estate** Express the value V of a real estate firm in terms of x , the number of acres of property owned. Each acre is valued at \$2500 and other company assets total \$750,000.

74. **Market Equilibrium** The supply function for a product relates the number of units x that producers are willing to supply for a given price per unit p . The supply and demand functions for a market are

$$p = \frac{2}{5}x + 4 \quad \text{Supply}$$

$$p = -\frac{16}{25}x + 30 \quad \text{Demand}$$

- Use a graphing utility to graph the supply and demand functions in the same viewing window.
 - Use the *trace* feature of the graphing utility to find the *equilibrium point* for the market.
 - For what values of x does the demand exceed the supply?
 - For what values of x does the supply exceed the demand?
75. **Profit** A radio manufacturer charges \$90 per unit for units that cost \$60 to produce. To encourage large orders from distributors, the manufacturer will reduce the price by \$0.01 per unit for each unit in excess of 100 units. (For example, an order of 101 units would have a price of \$89.99 per unit, and an order of 102 units would have a price of \$89.98 per unit.) This price reduction is discontinued when the price per unit drops to \$75.

- Express the price per unit p as a function of the order size x .
 - Express the profit P as a function of the order size x .
76. **Cost, Revenue, and Profit** A company invests \$98,000 for equipment to produce a new product. Each unit of the product costs \$12.30 and is sold for \$17.98. Let x be the number of units produced and sold.

- Write the total cost C as a function of x .
- Write the revenue R as a function of x .
- Write the profit P as a function of x .

77. **Revenue** For groups of 80 or more people, a charter bus company determines the rate r (in dollars per person) according to the formula

$$r = 8 - 0.05(n - 80), \quad n \geq 80$$

where n is the number of people.

- Express the revenue R for the bus company as a function of n .
- Complete the table.

n	90	100	110	120	130	140	150
R							

- Criticize the formula for the rate. Would you use this formula? Explain your reasoning.

78. **Medicine** The temperature of a patient after being given a fever-reducing drug is given by

$$F(t) = 98 + \frac{3}{t+1}$$

where F is the temperature in degrees Fahrenheit and t is the time in hours since the drug was administered. Use a graphing utility to graph the function. Be sure to choose an appropriate viewing window. For what values of t do you think this function would be valid? Explain.

- In Exercises 79–86, use a graphing utility to graph the function. Then use the *zoom* and *trace* features to find the zeros of the function. Is the function one-to-one?

79. $f(x) = 9x - 4x^2$

80. $f(x) = 2\left(3x^2 - \frac{6}{x}\right)$

81. $g(t) = \frac{t+3}{1-t}$

82. $h(x) = 6x^3 - 12x^2 + 4$

83. $f(x) = \frac{4-x^2}{x}$

84. $g(x) = \left|\frac{1}{2}x^2 - 4\right|$

85. $g(x) = x^2\sqrt{x^2-4}$

86. $f(x) = \frac{\sqrt{x^2-16}}{x^2}$

BUSINESS CAPSULE



CardSenders is a home-based greeting card service for businesses. Phyllis and Robert Boverie bought the company in 1990, which has expanded into the United Kingdom, Canada, Asia, and Mexico. Currently there are over 200 licensees and consultants. Start-up costs run from \$995.00 for consultants and \$6,900.00 for licensees.

87. **Research Project** Use your school's library, the Internet, or some other reference source to find information about the start-up costs of beginning a business, such as the example above. Write a short paper about the company.

The Limit of a Function

In everyday language, people talk about the limit of one's endurance. On other occasions may be

Consider a spring attached to a wall. To determine how much weight could attach increasing each weight w , as shown in Figure 1.1, then it is said that the limit is much like the limit

$$\lim_{x \rightarrow c} f(x) = L$$

which is read as "the limit of $f(x)$ as x approaches c is L ."

EXAMPLE 1 Finding a Limit

Find the limit: $\lim_{x \rightarrow 1} (x^2 - 1)$

SOLUTION Let $f(x) = x^2 - 1$. Then $f(x)$ approaches 0 as x approaches 1.

$$\lim_{x \rightarrow 1} (x^2 - 1) = 0$$

The table yields the same result. As x gets closer and closer to 1, $f(x)$ gets closer and closer to 0.

x approaches

x	0.900	0.990
$f(x)$	1.810	1.980

$f(x)$ approaches

TRY IT 1

Find the limit: $\lim_{x \rightarrow 1} (2x - 1)$

**PREREQUISITE
REVIEW 1.5**

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, evaluate the expression and simplify.

1. $f(x) = x^2 - 3x + 3$

(a) $f(-1)$ (b) $f(c)$ (c) $f(x + h)$

2. $f(x) = \begin{cases} 2x - 2, & x < 1 \\ 3x + 1, & x \geq 1 \end{cases}$

(a) $f(-1)$ (b) $f(3)$ (c) $f(t^2 + 1)$

3. $f(x) = x^2 - 2x + 2$ $\frac{f(1+h) - f(1)}{h}$

4. $f(x) = 4x$ $\frac{f(2+h) - f(2)}{h}$

In Exercises 5–8, find the domain and range of the function and sketch its graph.

5. $h(x) = -\frac{5}{x}$

6. $g(x) = \sqrt{25 - x^2}$

7. $f(x) = |x - 3|$

8. $f(x) = \frac{|x|}{x}$

In Exercises 9 and 10, determine whether y is a function of x .

9. $9x^2 + 4y^2 = 49$

10. $2x^2y + 8x = 7y$

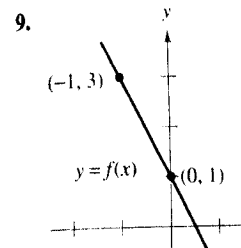
7. $\lim_{x \rightarrow 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$

x	-0.5	-0.1
$f(x)$		

8. $\lim_{x \rightarrow 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{2x}$

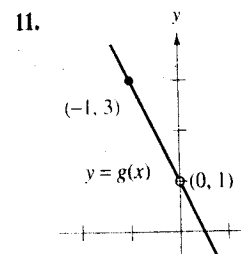
x	0.5	0.1
$f(x)$		

In Exercises 9–12, use the graph.



(a) $\lim_{x \rightarrow 0} f(x)$

(b) $\lim_{x \rightarrow -1} f(x)$



(a) $\lim_{x \rightarrow 0} g(x)$

(b) $\lim_{x \rightarrow -1} g(x)$

In Exercises 13 and 14,

(b) $f(x)g(x)$, and (c) $f(x)/g(x)$

13. $\lim_{x \rightarrow c} f(x) = 3$

$\lim_{x \rightarrow c} g(x) = 9$

In Exercises 15 and 16, find

(c) $[f(x)]^2$ as x approaches:

15. $\lim_{x \rightarrow c} f(x) = 16$

EXERCISES 1.5

In Exercises 1–8, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.

1. $\lim_{x \rightarrow 2} (5x + 4)$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				?			

2. $\lim_{x \rightarrow 2} (x^2 - 3x + 1)$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				?			

3. $\lim_{x \rightarrow 2} \frac{x-2}{x^2-4}$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				?			

4. $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x - 2}$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				?			

5. $\lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$				?			

6. $\lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$				?			

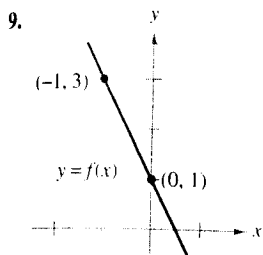
$$7. \lim_{x \rightarrow 0^-} \frac{1}{x+4} - \frac{1}{4}$$

x	-0.5	-0.1	-0.01	-0.001	0
$f(x)$					?

$$8. \lim_{x \rightarrow 0^+} \frac{1}{2+x} - \frac{1}{2}$$

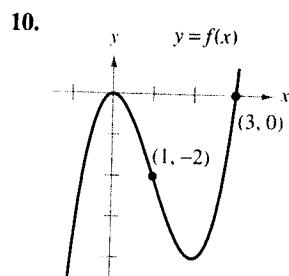
x	0.5	0.1	0.01	0.001	0
$f(x)$					?

In Exercises 9–12, use the graph to find the limit (if it exists).



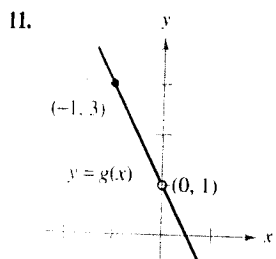
(a) $\lim_{x \rightarrow 0} f(x)$

(b) $\lim_{x \rightarrow -1} f(x)$



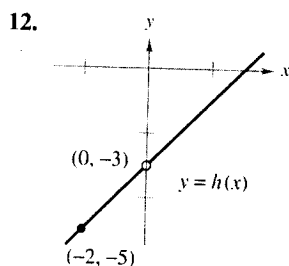
(a) $\lim_{x \rightarrow 1} f(x)$

(b) $\lim_{x \rightarrow 3} f(x)$



(a) $\lim_{x \rightarrow 0} g(x)$

(b) $\lim_{x \rightarrow -1} g(x)$



(a) $\lim_{x \rightarrow -2} h(x)$

(b) $\lim_{x \rightarrow 0} h(x)$

In Exercises 13 and 14, find the limit of (a) $f(x) + g(x)$, (b) $f(x)g(x)$, and (c) $f(x)/g(x)$ as x approaches c .

13. $\lim_{x \rightarrow c} f(x) = 3$

$\lim_{x \rightarrow c} g(x) = 9$

14. $\lim_{x \rightarrow c} f(x) = \frac{3}{2}$

$\lim_{x \rightarrow c} g(x) = \frac{1}{2}$

In Exercises 15 and 16, find the limit of (a) $\sqrt{f(x)}$, (b) $[3f(x)]$, and (c) $[f(x)]^2$ as x approaches c .

15. $\lim_{x \rightarrow c} f(x) = 16$

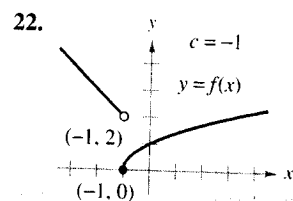
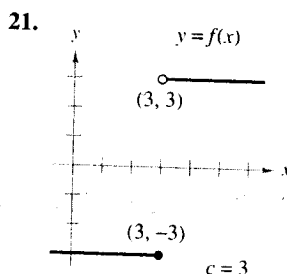
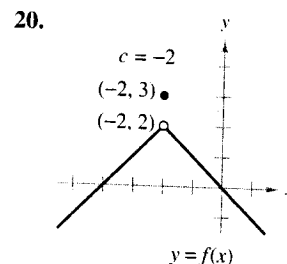
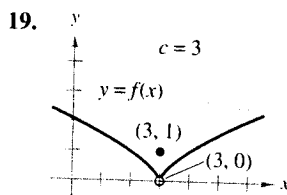
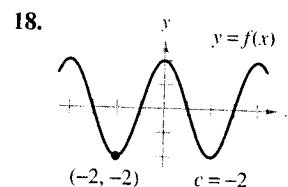
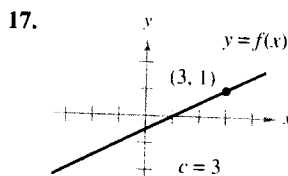
16. $\lim_{x \rightarrow c} f(x) = 9$

In Exercises 17–22, use the graph to find the limit (if it exists).

(a) $\lim_{x \rightarrow c} f(x)$

(b) $\lim_{x \rightarrow c} f(x)$

(c) $\lim_{x \rightarrow c} f(x)$



In Exercises 23–40, find the limit.

23. $\lim_{x \rightarrow 2} x^4$

24. $\lim_{x \rightarrow -2} x^3$

25. $\lim_{x \rightarrow -3} (3x + 2)$

26. $\lim_{x \rightarrow 0} (2x - 3)$

27. $\lim_{x \rightarrow 1} (1 - x^2)$

28. $\lim_{x \rightarrow 2} (-x^2 + x - 2)$

29. $\lim_{x \rightarrow 3} \sqrt{x + 1}$

30. $\lim_{x \rightarrow 4} \sqrt[3]{x + 4}$

31. $\lim_{x \rightarrow -3} \frac{2}{x + 2}$

32. $\lim_{x \rightarrow -2} \frac{3x + 1}{2 - x}$

33. $\lim_{x \rightarrow -2} \frac{x^2 - 1}{2x}$

34. $\lim_{x \rightarrow -1} \frac{4x - 5}{3 - x}$

35. $\lim_{x \rightarrow 7} \frac{5x}{x + 2}$

36. $\lim_{x \rightarrow 3} \frac{\sqrt{x + 1}}{x - 4}$

37. $\lim_{x \rightarrow 3} \frac{\sqrt{x + 1} - 1}{x}$

38. $\lim_{x \rightarrow 5} \frac{\sqrt{x + 4} - 2}{x}$

39. $\lim_{x \rightarrow 1} \frac{1}{x + 4} - \frac{1}{4}$

40. $\lim_{x \rightarrow 2} \frac{1}{x + 2} - \frac{1}{2}$

In Exercises 41–58, find the limit (if it exists).

41. $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$

42. $\lim_{x \rightarrow -1} \frac{2x^2 - x - 3}{x + 1}$

43. $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4x + 4}$

44. $\lim_{x \rightarrow 2} \frac{2 - x}{x^2 - 4}$

45. $\lim_{t \rightarrow 5} \frac{t - 5}{t^2 - 25}$

46. $\lim_{t \rightarrow 1} \frac{t^2 + t - 2}{t^2 - 1}$

47. $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$

48. $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$

49. $\lim_{x \rightarrow -2} \frac{|x + 2|}{x + 2}$

50. $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$

51. $\lim_{x \rightarrow 3} f(x)$, where $f(x) = \begin{cases} \frac{1}{3}x - 2, & x \leq 3 \\ -2x + 5, & x > 3 \end{cases}$

52. $\lim_{s \rightarrow 1} f(s)$, where $f(s) = \begin{cases} s, & s \leq 1 \\ 1 - s, & s > 1 \end{cases}$

53. $\lim_{\Delta x \rightarrow 0} \frac{2(x + \Delta x) - 2x}{\Delta x}$

54. $\lim_{\Delta x \rightarrow 0} \frac{4(x + \Delta x) - 5 - (4x - 5)}{\Delta x}$

55. $\lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + 2 + \Delta x} - \sqrt{x + 2}}{\Delta x}$

56. $\lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x}$

57. $\lim_{\Delta t \rightarrow 0} \frac{(t + \Delta t)^2 - 5(t + \Delta t) - (t^2 - 5t)}{\Delta t}$

58. $\lim_{\Delta t \rightarrow 0} \frac{(t + \Delta t)^2 - 4(t + \Delta t) + 2 - (t^2 - 4t + 2)}{\Delta t}$

Graphical, Numerical, and Analytic Analysis In Exercises 59–62, use a graphing utility to graph the function and estimate the limit. Use a table to reinforce your conclusion. Then find the limit by analytic methods.

59. $\lim_{x \rightarrow 1^-} \frac{2}{x^2 - 1}$

60. $\lim_{x \rightarrow 1^+} \frac{5}{1 - x}$

61. $\lim_{x \rightarrow -2^-} \frac{1}{x + 2}$

62. $\lim_{x \rightarrow 0^-} \frac{x + 1}{x}$

Graphical, Numerical, and Analytic Analysis In Exercises 63–66, use a graphing utility to estimate the limit (if it exists).

63. $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 4x + 4}$

64. $\lim_{x \rightarrow 1} \frac{x^2 + 6x - 7}{x^3 - x^2 + 2x - 2}$

65. $\lim_{x \rightarrow -4} \frac{x^3 + 4x^2 + x + 4}{2x^2 + 7x - 4}$

66. $\lim_{x \rightarrow -2} \frac{4x^3 + 7x^2 + x + 6}{3x^2 - x - 14}$

67. The limit of

$$f(x) = (1 + x)^{1/x}$$

is a natural base for many business applications, as you will see in Section 4.2.

$$\lim_{x \rightarrow 0} (1 + x)^{1/x} = e \approx 2.718$$

(a) Show the reasonableness of this limit by completing the table.

x	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01
$f(x)$							

(b) Use a graphing utility to graph f and to confirm the answer in part (a).

(c) Find the domain and range of the function.

68. Find $\lim_{x \rightarrow 0} f(x)$, given

$$4 - x^2 \leq f(x) \leq 4 + x^2, \text{ for all } x.$$

69. **Environment** The cost (in dollars) of removing $p\%$ of the pollutants from the water in a small lake is given by

$$C = \frac{25,000p}{100 - p}, \quad 0 \leq p < 100$$

where C is the cost and p is the percent of pollutants.

(a) Find the cost of removing 50% of the pollutants.

(b) What percent of the pollutants can be removed for \$100,000?

(c) Evaluate $\lim_{p \rightarrow 100^-} C$. Explain your results.

70. **Compound Interest** You deposit \$1000 in an account that is compounded quarterly at an annual rate of r (in decimal form). The balance A after 10 years is

$$A = 1000 \left(1 + \frac{r}{4} \right)^{40}$$

Does the limit of A exist as the interest rate approaches 6%? If so, what is the limit?

Compound Interest Consider a certificate of deposit that pays 10% (annual percentage rate) on an initial deposit of \$500. The balance A after 10 years is

$$A = 500(1 + 0.1x)^{10/x}$$

where x is the length of the compounding period (in years).

(a) Use a graphing utility to graph A , where $0 \leq x \leq 1$.

(b) Use the *zoom* and *trace* features to estimate the balance for quarterly compounding and daily compounding.

(c) Use the *zoom* and *trace* features to estimate

$$\lim_{x \rightarrow 0^+} A.$$

What do you think this limit represents? Explain your reasoning.

Continuity

In mathematics, the term everyday use. To say that interruption in the graph no holes, jumps, or gaps. tion eluded mathematici.

1800s that a precise defi Before looking at thi in Figure 1.60. This figu not continuous.

1. At $x = c_1$, $f(c_1)$ is no

2. At $x = c_2$, $\lim_{x \rightarrow c_2} f(x)$ d

3. At $x = c_3$, $f(c_3) \neq \lim_{x \rightarrow c_3} f(x)$

At all other points in th implies that the function

Definition of Continuity

Let c be a number in domain contains the point c if the follow

1. $f(c)$ is defined.

2. $\lim_{x \rightarrow c} f(x)$ exists.

3. $\lim_{x \rightarrow c} f(x) = f(c)$.

If f is continuous at on an open interval

Roughly, you can s on the interval can be t from the paper, as show

PREREQUISITE
REVIEW 1.6

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, simplify the expression.

$$1. \frac{x^2 + 6x + 8}{x^2 - 6x - 16}$$

$$2. \frac{x^2 - 5x - 6}{x^2 - 9x + 18}$$

$$3. \frac{2x^2 - 2x - 12}{4x^2 - 24x + 36}$$

$$4. \frac{x^3 - 16x}{x^3 + 2x^2 - 8x}$$

In Exercises 5–8, solve for x .

$$5. x^2 + 7x = 0$$

$$6. x^2 + 4x - 5 = 0$$

$$7. 3x^2 + 8x + 4 = 0$$

$$8. x^3 + 5x^2 - 24x = 0$$

In Exercises 9 and 10, find the limit.

$$9. \lim_{x \rightarrow 3} (2x^2 - 3x + 4)$$

$$10. \lim_{x \rightarrow -2} (3x^3 - 8x + 7)$$

EXERCISES 1.6

In Exercises 1–10, determine whether the function is continuous on the entire real line. Explain your reasoning.

$$1. f(x) = 5x^3 - x^2 + 2$$

$$2. f(x) = (x^2 - 1)^3$$

$$3. f(x) = \frac{1}{x^2 - 4}$$

$$4. f(x) = \frac{1}{9 - x^2}$$

$$5. f(x) = \frac{1}{4 + x^2}$$

$$6. f(x) = \frac{3x}{x^2 + 1}$$

$$7. f(x) = \frac{2x - 1}{x^2 - 8x + 15}$$

$$8. f(x) = \frac{x + 4}{x^2 - 6x + 5}$$

$$9. g(x) = \frac{x^2 - 4x + 4}{x^2 - 4}$$

$$10. g(x) = \frac{x^2 - 9x + 20}{x^2 - 16}$$

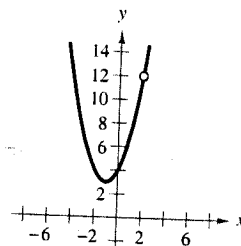
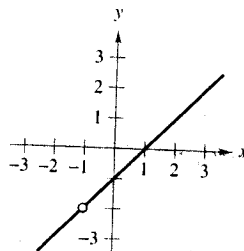
In Exercises 11–34, describe the interval(s) on which the function is continuous.

$$11. f(x) = \frac{x^2 - 1}{x}$$

$$12. f(x) = \frac{1}{x^2 - 4}$$

$$13. f(x) = \frac{x^2 - 1}{x + 1}$$

$$14. f(x) = \frac{x^3 - 8}{x - 2}$$



$$15. f(x) = x^2 - 2x + 1$$

$$16. f(x) = 3 - 2x - x^2$$

$$17. f(x) = \frac{x}{x^2 - 1}$$

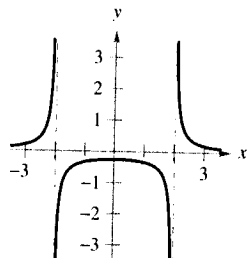
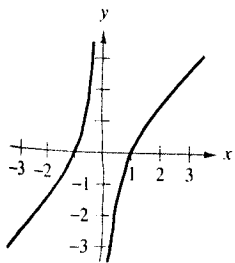
$$18. f(x) = \frac{x - 3}{x^2 - 9}$$

$$19. f(x) = \frac{x}{x^2 + 1}$$

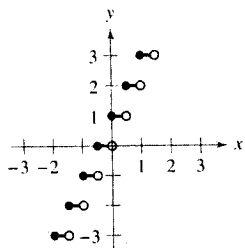
$$20. f(x) = \frac{1}{x^2 + 1}$$

$$21. f(x) = \frac{x - 5}{x^2 - 9x + 20}$$

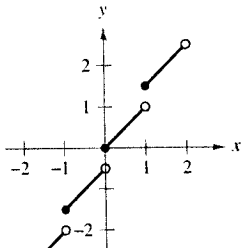
$$22. f(x) = \frac{x - 1}{x^2 + x - 2}$$



23. $f(x) = \lfloor 2x \rfloor + 1$



24. $f(x) = \frac{\lfloor x \rfloor}{2} + x$



25. $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2, & x \geq 1 \end{cases}$

26. $f(x) = \begin{cases} 3 + x, & x \leq 2 \\ x^2 + 1, & x > 2 \end{cases}$

27. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \leq 2 \\ 3 - x, & x > 2 \end{cases}$

28. $f(x) = \begin{cases} x^2 - 4, & x \leq 0 \\ 3x + 1, & x > 0 \end{cases}$

29. $f(x) = \frac{|x + 1|}{x + 1}$

30. $f(x) = \frac{|4 - x|}{4 - x}$

31. $f(x) = \lfloor x - 1 \rfloor$

32. $f(x) = x - \lfloor x \rfloor$

33. $h(x) = f(g(x)), f(x) = \frac{1}{\sqrt{x}}, g(x) = x - 1, x > 1$

34. $h(x) = f(g(x)), f(x) = \frac{1}{x - 1}, g(x) = x^2 + 5$

In Exercises 35–38, discuss the continuity of the function on the closed interval. If there are any discontinuities, determine whether they are removable.

Function	Interval
35. $f(x) = x^2 - 4x - 5$	$[-1, 5]$

36. $f(x) = \frac{5}{x^2 + 1}$	$[-2, 2]$
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37. $f(x) = \frac{1}{x - 2}$	$[1, 4]$
------------------------------	----------

38. $f(x) = \frac{x}{x^2 - 4x + 3}$	$[0, 4]$
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In Exercises 39–44, sketch the graph of the function and describe the interval(s) on which the function is continuous.

39. $f(x) = \frac{x^2 - 16}{x - 4}$

40. $f(x) = \frac{2x^2 + x}{x}$

41. $f(x) = \frac{x^3 + x}{x}$

42. $f(x) = \frac{x - 3}{4x^2 - 12x}$

43. $f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$

44. $f(x) = \begin{cases} x^2 - 4, & x \leq 0 \\ 2x + 4, & x > 0 \end{cases}$

In Exercises 45 and 46, find the constant a (Exercise 45) and the constants a and b (Exercise 46) such that the function is continuous on the entire real line.

45. $f(x) = \begin{cases} x^3, & x \leq 2 \\ ax^2, & x > 2 \end{cases}$

46. $f(x) = \begin{cases} 2, & x \leq -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \geq 3 \end{cases}$

In Exercises 47–52, use a graphing utility to graph the function. Use the graph to determine any x -values at which the function is not continuous.

47. $h(x) = \frac{1}{x^2 - x - 2}$

48. $k(x) = \frac{x - 4}{x^2 - 5x + 4}$

49. $f(x) = \begin{cases} 2x - 4, & x \leq 3 \\ x^2 - 2x, & x > 3 \end{cases}$

50. $f(x) = \begin{cases} 3x - 1, & x \leq 1 \\ x + 1, & x > 1 \end{cases}$

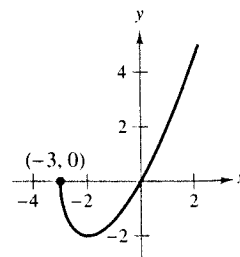
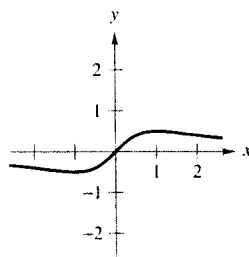
51. $f(x) = x - 2\lfloor x \rfloor$

52. $f(x) = \lfloor 2x - 1 \rfloor$

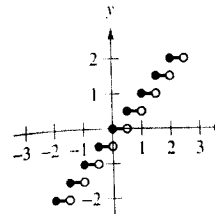
In Exercises 53–56, describe the interval(s) on which the function is continuous.

53. $f(x) = \frac{x}{x^2 + 1}$

54. $f(x) = x\sqrt{x + 3}$



55. $f(x) = \frac{1}{2}\lfloor 2x \rfloor$



Writing In Exercises 57–61, the function on the interval appear to be continuous. In fact, the function is continuous on $[-4, 4]$. The importance of examining the function graphically.

57. $f(x) = \frac{x^2 + x}{x}$

58. $f(x) = \frac{x^3 - 8}{x - 2}$

59. Compound Interest An account that pays 6% annual interest. A is the amount in the account after t years.

$$A = 7500(1.015)^{6t}$$

(a) Sketch the graph of A versus t . Explain your reasoning.

(b) What is the balance after 10 years?

60. Environmental Costs The cost of removing x percent of the smokestack of a factory is given by

$$C = \frac{2x}{100 - x}$$

(a) What is the initial cost of removing 0% of the smokestack?

(b) Use a graphing utility to graph the function. Describe the function's continuity.

(c) Find the cost of removing 90% of the smokestack.

61. Consumer Awareness The cost of sending an overnight package is \$9.80 for the first pound or fraction thereof. Create a model for the cost of sending a package weighing x pounds. Graph the function, and describe the interval(s) on which the function is continuous.