PREREQUISITE REVIEW 2.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, find an equation of the line containing P and Q.

1. $P(2, 1),\ Q(2, 4)$
2. $P(2, 2),\ Q(-5, 2)$

In Exercises 3–6, find the limit.

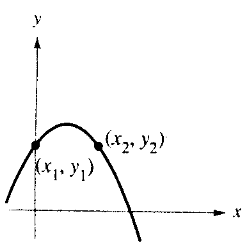
3. $\displaystyle\lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$
4. $\displaystyle\lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$
5. $\displaystyle\lim_{\Delta x \to 0} \frac{1}{x(x + \Delta x)}$
6. $\displaystyle\lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$

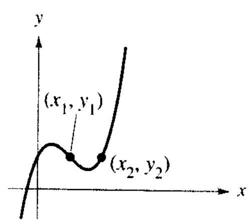
In Exercises 7–10, find the domain of the function.

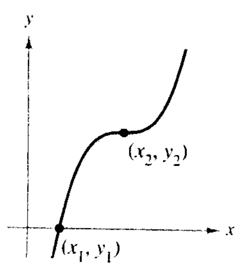
7. $f(x) = \dfrac{1}{x - 1}$
8. $f(x) = \dfrac{1}{5}x^3 - 2x^2 + \dfrac{1}{3}x - 1$
9. $f(x) = \dfrac{6x}{x^3 + x}$
10. $f(x) = \dfrac{x^2 - 2x - 24}{x^2 + x - 12}$

EXERCISES 2.1

In Exercises 1–4, trace the graph and sketch the tangent lines at (x_1, y_1) and (x_2, y_2).

1.


2.


3.


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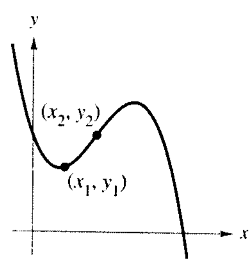
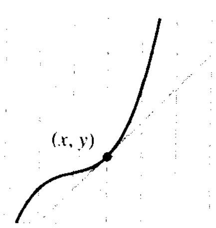


In Exercises 5–10, estimate the slope of the graph at the point (x, y). (Each square on the grid is 1 unit by 1 unit.)

5.


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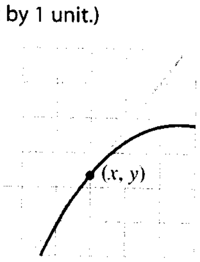

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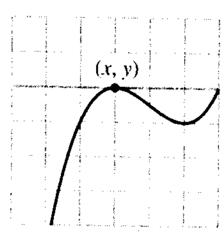

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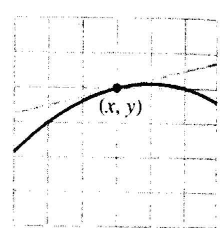
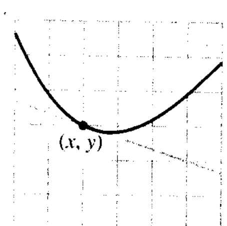
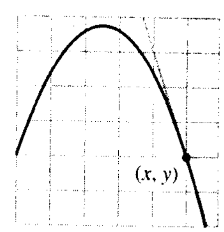
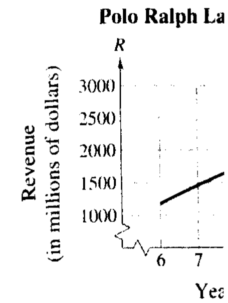


9.


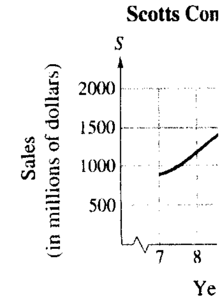
10.


11. ***Revenue*** The graph (on page 91) represents the revenue R (in millions of dollars per year) for Polo Ralph Lauren from 1996 through 2002, where $t = 6$ corresponds to 1996. Estimate the slopes of the graph for the years 1997, 2000, and 2002. *(Source: Polo Ralph Lauren Corp.)*

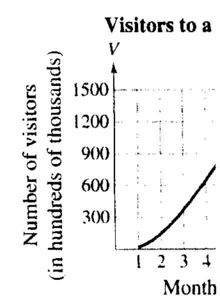
Figure for 11



12. ***Sales*** The graph rep dollars per year) for S 2003, where $t = 7$ corre of the graph for the yea *Scotts Company)*

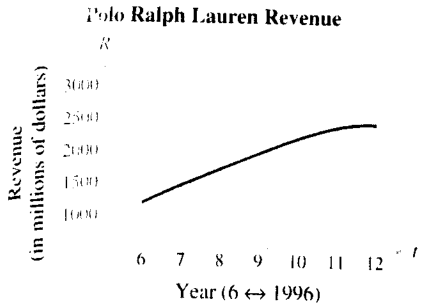


13. ***Consumer Trends*** visitors V to a national a one-year period, wl Estimate the slopes of

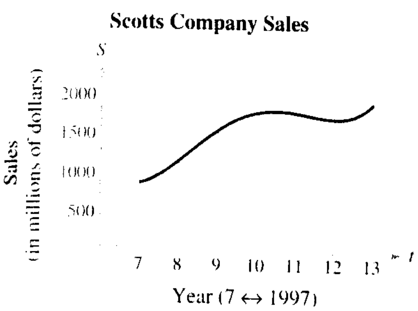


14. ***Athletics*** Two long side begin a 10,000-m $s = f(t)$ and $s = g(t)$, thousands of meters a

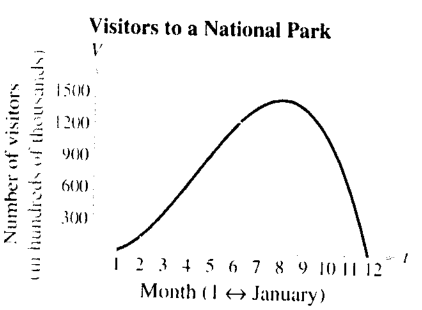
Figure for 11



12. ***Sales*** The graph represents the sales S (in millions of dollars per year) for Scotts Company from 1997 through 2003, where $t = 7$ corresponds to 1997. Estimate the slopes of the graph for the years 1998, 2001, and 2003.

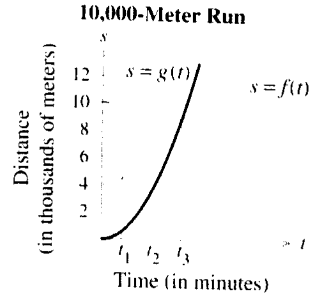


13. ***Consumer Trends*** The graph shows the number of visitors V to a national park in hundreds of thousands during a one-year period, where $t = 1$ corresponds to January. Estimate the slopes of the graph at $t = 1$, 8, and 12.



14. ***Athletics*** Two long distance runners starting out side by side begin a 10,000-meter run. Their distances are given by $s = f(t)$ and $s = g(t)$, respectively, where s is measured in thousands of meters and t is measured in minutes.

Figure for 14



(a) Which runner is running faster at t_1?

(b) What conclusion can you make regarding their rates at t_2?

(c) What conclusion can you make regarding their rates at t_3?

(d) Which runner finishes the race first? Explain.

In Exercises 15–26, use the limit definition to find the derivative of the function.

15. $f(x) = 3$
16. $f(x) = -4$
17. $f(x) = -5x + 3$
18. $f(x) = \frac{1}{2}x + 5$
19. $f(x) = x^2 - 4$
20. $f(x) = 1 - x^2$
21. $h(t) = \sqrt{t - 1}$
22. $f(x) = \sqrt{x + 2}$
23. $f(t) = t^3 - 12t$
24. $f(t) = t^3 + t^2$
25. $f(x) = \dfrac{1}{x + 2}$
26. $g(s) = \dfrac{1}{s - 1}$

In Exercises 27–36, find the slope of the tangent line to the graph of f at the given point.

27. $f(x) = 6 - 2x$; $(2, 2)$
28. $f(x) = 2x + 4$; $(1, 6)$
29. $f(x) = -1$; $(0, -1)$
30. $f(x) = 6$; $(-2, 6)$
31. $f(x) = x^2 - 2$; $(2, 2)$
32. $f(x) = x^2 + 2x + 1$; $(-3, 4)$
33. $f(x) = x^3 - x$; $(2, 6)$
34. $f(x) = x^3 + 2x$; $(1, 3)$
35. $f(x) = \sqrt{1 - 2x}$; $(-4, 3)$
36. $f(x) = \sqrt{2x - 2}$; $(9, 4)$

In Exercises 37–44, find an equation of the tangent line to the graph of f at the given point. Then verify your result by sketching the graph of f and the tangent line.

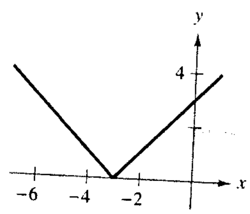
37. $f(x) = \frac{1}{2}x^2$; $(2, 2)$
38. $f(x) = -x^2$; $(-1, -1)$
39. $f(x) = (x - 1)^2$; $(-2, 9)$
40. $f(x) = 2x^2 - 1$; $(0, -1)$
41. $f(x) = \sqrt{x} + 1$; $(4, 3)$
42. $f(x) = \sqrt{x + 2}$; $(7, 3)$
43. $f(x) = \dfrac{1}{x}$; $(1, 1)$
44. $f(x) = \dfrac{1}{x - 1}$; $(2, 1)$

In Exercises 45–48, find an equation of the line that is tangent to the graph of f and parallel to the given line.

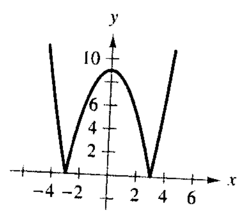
Function	*Line*
45. $f(x) = -\frac{1}{4}x^2$	$x + y = 0$
46. $f(x) = x^2 + 1$	$2x + y = 0$
47. $f(x) = -\frac{1}{2}x^3$	$6x + y + 4 = 0$
48. $f(x) = x^2 - x$	$x + 2y - 6 = 0$

In Exercises 49–56, describe the x-values at which the function is differentiable. Explain your reasoning.

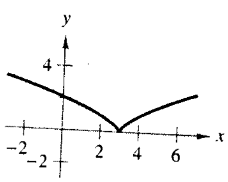
49. $y = |x + 3|$



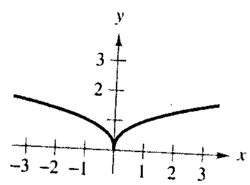
50. $y = |x^2 - 9|$



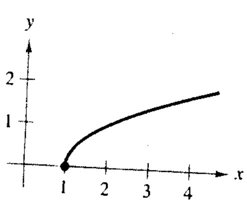
51. $y = (x - 3)^{2/3}$



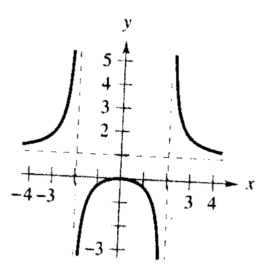
52. $y = x^{2/5}$



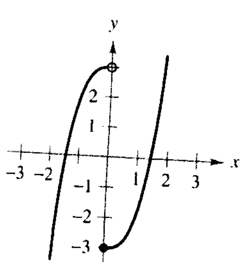
53. $y = \sqrt{x - 1}$



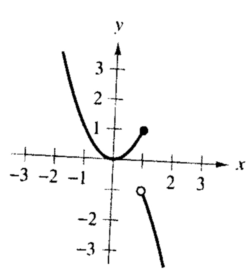
54. $y = \dfrac{x^2}{x^2 - 4}$



55. $y = \begin{cases} x^3 + 3, & x < 0 \\ x^3 - 3, & x \geq 0 \end{cases}$



56. $y = \begin{cases} x^2, & x \leq 1 \\ -x^2, & x > 1 \end{cases}$



Graphical, Numerical, and Analytic Analysis In Exercises 57–60, use a graphing utility to graph f on the interval $[-2, 2]$. Complete the table by graphically estimating the slopes of the graph at the given points. Then evaluate the slopes analytically and compare your results with those obtained graphically.

x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$									
$f'(x)$									

57. $f(x) = \frac{1}{4}x^3$

58. $f(x) = \frac{1}{2}x^2$

59. $f(x) = -\frac{1}{2}x^3$

60. $f(x) = -\frac{3}{2}x^2$

In Exercises 61–64, find the derivative of the given function f. Then use a graphing utility to graph f and its derivative in the same viewing window. What does the x-intercept of the derivative indicate about the graph of f?

61. $f(x) = x^2 - 4x$ **62.** $f(x) = 2 + 6x - x^2$

63. $f(x) = x^3 - 3x$ **64.** $f(x) = x^3 - 6x^2$

65. ***Think About It*** Sketch a graph of a function whose derivative is always negative.

66. ***Think About It*** Sketch a graph of a function whose derivative is always positive.

67. ***Writing*** Use a graphing utility to graph the two functions $f(x) = x^2 + 1$ and $g(x) = |x| + 1$ in the same viewing window. Use the *zoom* and *trace* features to analyze the graphs near the point $(0, 1)$. What do you observe? Which function is differentiable at this point? Write a short paragraph describing the geometric significance of differentiability at a point.

True or False? In Exercises 68–71, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

68. The slope of the graph of $y = x^2$ is different at every point on the graph of f.

69. If a function is continuous at a point, then it is differentiable at that point.

70. If a function is differentiable at a point, then it is continuous at that point.

71. A tangent line to a graph can intersect the graph at more than one point.

2.2

The Constant Ru

In Section 2.1, you foun
even for simple functio
differentiation. These ru
use of limits.

The Constant Rule

The derivative of a c

$$\frac{d}{dx}[c] = 0,$$

PROOF Let $f(x) = c$.
write

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x +$$

So,

$$\frac{d}{dx}[c] = 0.$$

STUDY TIP

Note in Figure 2.12
slope of a horizontal

EXAMPLE 1

(a) $\frac{d}{dx}[7] = 0$

(c) If $y = 2$, then $\frac{dy}{dx} =$

TRY IT 1

Find the derivative o

(a) $f(x) = -2$ (

PREREQUISITE REVIEW 2.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, evaluate each expression when $x = 2$.

1. (a) $2x^2$ (b) $(2x)^2$ (c) $2x^{-2}$

2. (a) $\dfrac{1}{(3x)^2}$ (b) $\dfrac{1}{4x^3}$ (c) $\dfrac{(2x)^{-3}}{4x^{-2}}$

In Exercises 3–6, simplify the expression.

3. $4(3)x^3 + 2(2)x$

4. $\frac{1}{2}(3)x^2 - \frac{3}{2}x^{1/2}$

5. $\left(\frac{1}{4}\right)x^{-3/4}$

6. $\frac{1}{3}(3)x^2 - 2\left(\frac{1}{2}\right)x^{-1/2} + \frac{1}{3}x^{-2/3}$

In Exercises 7–10, solve the equation.

7. $3x^2 + 2x = 0$

8. $x^3 - x = 0$

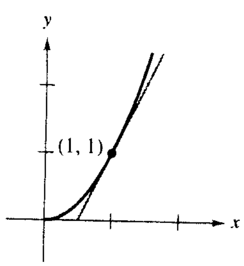
9. $x^2 + 8x - 20 = 0$

10. $x^2 - 10x - 24 = 0$

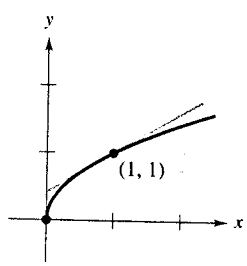
EXERCISES 2.2

In Exercises 1–4, find the slope of the tangent line to $y = x^n$ at the point $(1, 1)$.

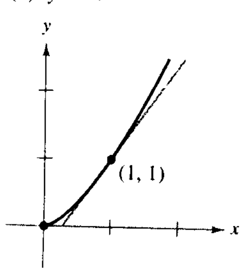
1. (a) $y = x^2$



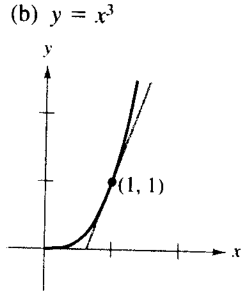
(b) $y = x^{1/2}$



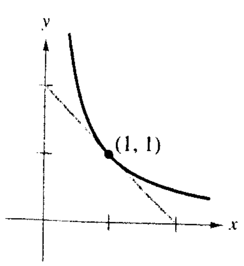
2. (a) $y = x^{3/2}$



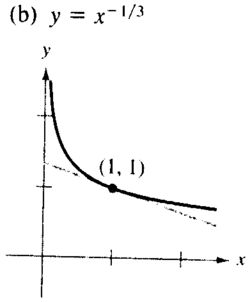
(b) $y = x^3$



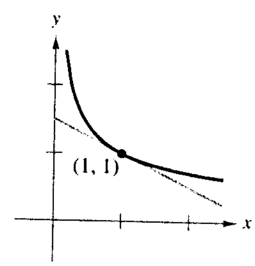
3. (a) $y = x^{-1}$



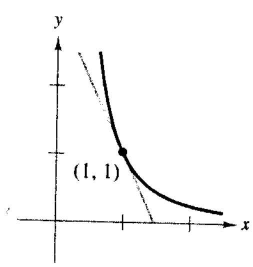
(b) $y = x^{-1/3}$



4. (a) $y = x^{-1/2}$



(b) $y = x^{-2}$



In Exercises 5–20, find the derivative of the function.

5. $y = 3$

6. $f(x) = -2$

7. $f(x) = 4x + 1$

8. $g(x) = 3x - 1$

9. $g(x) = x^2 + 4x - 1$

10. $y = t^2 + 2t - 3$

11. $f(t) = -3t^2 + 2t - 4$

12. $y = x^3 - 9x^2 + 2$

13. $s(t) = t^3 - 2t + 4$

14. $y = 2x^3 - x^2 + 3x - 1$

15. $y = 4t^{4/3}$

16. $h(x) = x^{5/2}$

17. $f(x) = 4\sqrt{x}$

18. $g(x) = 4\sqrt[3]{x} + 2$

19. $y = 4x^{-2} + 2x^2$

20. $s(t) = 4t^{-1} + 1$

In Exercises 21–26, use Ex

derivative.

Function *Rewrite*

21. $y = \dfrac{1}{4x^3}$

22. $y = \dfrac{2}{3x^2}$

23. $y = \dfrac{1}{(4x)^3}$

24. $y = \dfrac{\pi}{(3x)^2}$

25. $y = \dfrac{\sqrt{x}}{x}$

26. $y = \dfrac{4x}{x^{-3}}$

In Exercises 27–32, find the v

at the given point.

Function

27. $f(x) = \dfrac{1}{x}$

28. $f(t) = 4 - \dfrac{4}{3t}$

29. $f(x) = -\frac{1}{2}x(1 + x^2)$

30. $y = 3x\left(x^2 - \dfrac{2}{x}\right)$

31. $y = (2x + 1)^2$

32. $f(x) = 3(5 - x)^2$

In Exercises 33–46, find $f'(x)$

33. $f(x) = x^2 - \dfrac{4}{x} - 3x^{-}$

34. $f(x) = x^2 - 3x - 3x^{-}$

35. $f(x) = x^2 - 2x - \dfrac{2}{x^4}$

37. $f(x) = x(x^2 + 1)$

39. $f(x) = (x + 4)(2x^2 -$

40. $f(x) = (3x^2 - 5x)(x^2$

41. $f(x) = \dfrac{2x^3 - 4x^2 + 3}{x^2}$

43. $f(x) = \dfrac{4x^3 - 3x^2 + 2}{x^2}$

44. $f(x) = \dfrac{-6x^3 + 3x^2 -}{x}$

45. $f(x) = x^{4/5} + x$

In Exercises 21–26, use Example 6 as a model to find the derivative.

Function	*Rewrite*	*Differentiate*	*Simplify*
21. $y = \dfrac{1}{4x^3}$			
22. $y = \dfrac{2}{3x^2}$			
23. $y = \dfrac{1}{(4x)^3}$			
24. $y = \dfrac{\pi}{(3x)^2}$			
25. $y = \dfrac{\sqrt{x}}{x}$			
26. $y = \dfrac{4x}{x^{-3}}$			

In Exercises 27–32, find the value of the derivative of the function at the given point.

Function	*Point*
27. $f(x) = \dfrac{1}{x}$	$(1, 1)$
28. $f(t) = 4 - \dfrac{4}{3t}$	$\left(\dfrac{1}{2}, \dfrac{4}{3}\right)$
29. $f(x) = -\frac{1}{2}x(1 + x^2)$	$(1, -1)$
30. $y = 3x\left(x^2 - \dfrac{2}{x}\right)$	$(2, 18)$
31. $y = (2x + 1)^2$	$(0, 1)$
32. $f(x) = 3(5 - x)^2$	$(5, 0)$

In Exercises 33–46, find $f'(x)$.

33. $f(x) = x^2 - \dfrac{4}{x} - 3x^{-2}$

34. $f(x) = x^2 - 3x - 3x^{-2} + 5x^{-3}$

35. $f(x) = x^2 - 2x - \dfrac{2}{x^4}$

36. $f(x) = x^2 + 4x + \dfrac{1}{x}$

37. $f(x) = x(x^2 + 1)$

38. $f(x) = (x^2 + 2x)(x + 1)$

39. $f(x) = (x + 4)(2x^2 - 1)$

40. $f(x) = (3x^2 - 5x)(x^2 + 2)$

41. $f(x) = \dfrac{2x^3 - 4x^2 + 3}{x^2}$

42. $f(x) = \dfrac{2x^2 - 3x + 1}{x}$

43. $f(x) = \dfrac{4x^3 - 3x^2 + 2x + 5}{x^2}$

44. $f(x) = \dfrac{-6x^3 + 3x^2 - 2x + 1}{x}$

45. $f(x) = x^{4/5} + x$

46. $f(x) = x^{1/3} - 1$

In Exercises 47–50, find an equation of the tangent line to the graph of the function at the given point.

Function	*Point*
47. $y = -2x^4 + 5x^2 - 3$	$(1, 0)$
48. $y = x^3 + x$	$(-1, -2)$
49. $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$	$(1, 2)$
50. $f(x) = \dfrac{1}{\sqrt[3]{x^2}} - x$	$(-1, 2)$

In Exercises 51–54, determine the point(s), if any, at which the graph of the function has a horizontal tangent line.

51. $y = -x^4 + 3x^2 - 1$

52. $y = x^3 + 3x^2$

53. $y = \frac{1}{2}x^2 + 5x$

54. $y = x^2 + 2x$

In Exercises 55 and 56,

(a) Sketch the graphs of f, g, and h on the same set of coordinate axes.

(b) Find $f'(1)$, $g'(1)$, and $h'(1)$.

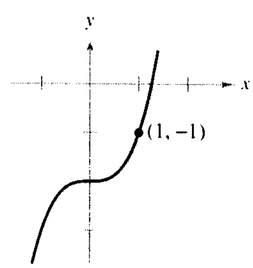
(c) Sketch the graph of the tangent line to each graph when $x = 1$.

55. $f(x) = x^3$
$g(x) = x^3 + 3$
$h(x) = x^3 - 2$

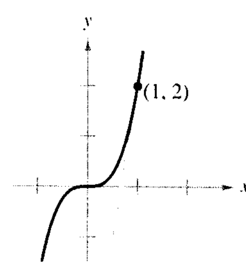
56. $f(x) = \sqrt{x}$
$g(x) = \sqrt{x} + 4$
$h(x) = \sqrt{x} - 2$

57. Use the Constant Rule, the Constant Multiple Rule, and the Sum Rule to find $h'(1)$ given that $f'(1) = 3$.

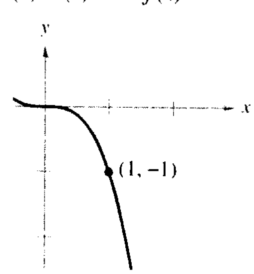
(a) $h(x) = f(x) - 2$



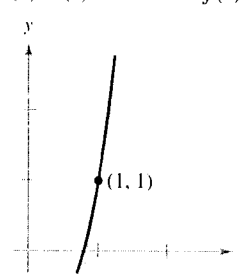
(b) $h(x) = 2f(x)$



(c) $h(x) = -f(x)$



(d) $h(x) = -1 + 2f(x)$

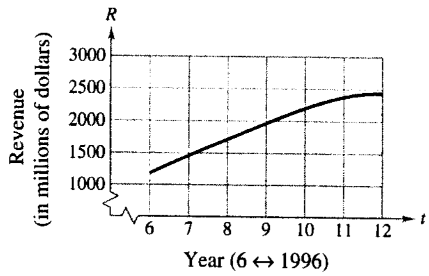


58. *Revenue* The revenue R (in millions of dollars per year) for Polo Ralph Lauren from 1996 through 2002 can be modeled by

$$R = -1.17879t^4 + 38.3641t^3 - 469.994t^2 + 2820.22t - 5577.7$$

where $t = 6$ corresponds to 1996. *(Source: Polo Ralph Lauren Corp.)*

Polo Ralph Lauren Revenue



(a) Find the slopes of the graph for the years 1997, 2000, and 2002.

(b) Compare your results with those obtained in Exercise 11 in Section 2.1.

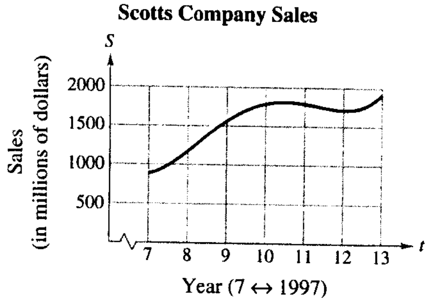
(c) What are the units for the slope of the graph? Interpret the slope of the graph in the context of the problem.

59. *Sales* The sales S (in millions of dollars per year) for Scotts Company from 1997 through 2003 can be modeled by

$$S = 8.70947t^4 - 341.0927t^3 + 4885.752t^2 - 30{,}118.17t + 68{,}395.3$$

where $t = 7$ corresponds to 1997. *(Source: Scotts Company)*

Scotts Company Sales



(a) Find the slopes of the graph for the years 1998, 2001, and 2003.

(b) Compare your results with those obtained in Exercise 12 in Section 2.1.

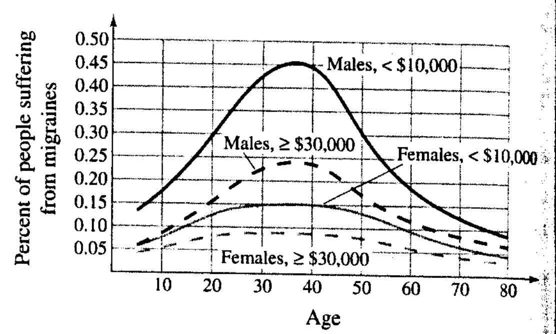
(c) What are the units for the slope of the graph? Interpret the slope of the graph in the context of the problem.

60. *Cost* The variable cost for manufacturing an electrical component is \$7.75 per unit, and the fixed cost is \$500. Write the cost C as a function of x, the number of units produced. Show that the derivative of this cost function is a constant and is equal to the variable cost.

61. *Profit* A college club raises funds by selling candy bars for \$1.00 each. The club pays \$0.60 for each candy bar and has annual fixed costs of \$250. Write the profit P as a function of x, the number of candy bars sold. Show that the derivative of the profit function is a constant and is equal to the profit on each candy bar sold.

62. *Psychology: Migraine Prevalence* The graph illustrates the prevalence of migraine headaches in males and females in selected income groups. *(Source: Adapted from Sue/Sue/Sue, Understanding Abnormal Behavior, Seventh Edition)*

Prevalence of Migraine Headaches



(a) Write a short paragraph describing your general observations about the prevalence of migraines in females and males with respect to age group and income bracket.

(b) Describe the graph of the derivative of each curve, and explain the significance of each derivative. Include an explanation of the units of the derivatives, and indicate the time intervals in which the derivatives would be positive and negative.

In Exercises 63 and 64, use a graphing utility to graph f and f' over the given interval. Determine any points at which the graph of f has horizontal tangents.

	Function	*Interval*
63.	$f(x) = 4.1x^3 - 12x^2 + 2.5x$	$[0, 3]$
64.	$f(x) = x^3 - 1.4x^2 - 0.96x + 1.44$	$[-2, 2]$

True or False? In Exercises 65 and 66, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

65. If $f'(x) = g'(x)$, then $f(x) = g(x)$.

66. If $f(x) = g(x) + c$, then $f'(x) = g'(x)$.

2.3

Average Rate of

In Sections 2.1 and 2.2,

1. **Slope** The derivative a point $(x, f(x))$.

2. **Rate of Change** The of $f(x)$ with respect t

In this section, you will change. A few are veloc rates, production rates, involve change with resp variable with respect to

When determining you must be careful to change. The distinction distinction between the the slope of the tangent

Definition of Avera

If $y = f(x)$, then the the interval $[a, b]$ is

Average rate of

Note that $f(a)$ is the interval, $f(b)$ is the interval, and $b - a$ i

STUDY TIP

In real-life problems change. The units fo measured in miles a *miles per hour.*

PREREQUISITE REVIEW 2.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, evaluate the expression.

1. $\dfrac{-63-(-105)}{21-7}$
2. $\dfrac{-37-54}{16-3}$

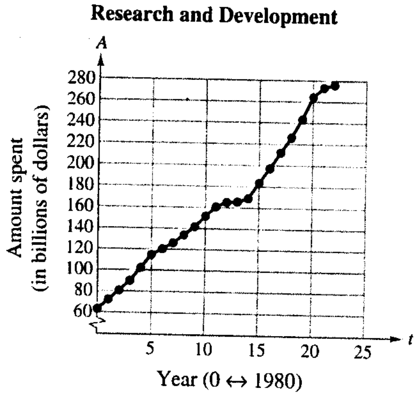
In Exercises 3–10, find the derivative of the function.

3. $y = 4x^2 - 2x + 7$
4. $y = -3t^3 + 2t^2 - 8$
5. $s = -16t^2 + 24t + 30$
6. $y = -16x^2 + 54x + 70$
7. $A = \frac{1}{10}(-2r^3 + 3r^2 + 5r)$
8. $y = \frac{1}{9}(6x^3 - 18x^2 + 63x - 15)$
9. $y = 12x - \dfrac{x^2}{5000}$
10. $y = 138 + 74x - \dfrac{x^3}{10{,}000}$

EXERCISES 2.3

1. ***Research and Development*** The graph shows the amounts A (in billions of dollars per year) spent on R&D in the United States from 1980 through 2002. Approximate the average rate of change of A during each period. *(Source: U.S. National Science Foundation)*

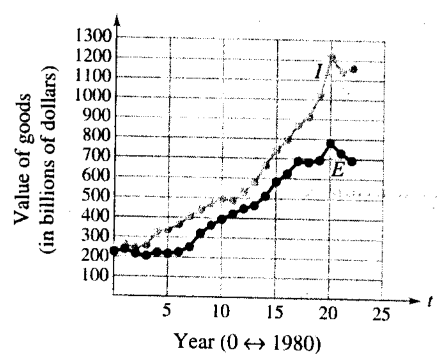
(a) 1980–1985 (b) 1985–1990 (c) 1990–1995
(d) 1995–2000 (e) 1980–2002 (f) 1990–2002



2. ***Trade Deficit*** The graph shows the values I (in billions of dollars per year) of goods imported to the United States and the value E (in billions of dollars per year) of goods exported from the United States from 1980 through 2002. Approximate each indicated average rate of change. *(Source: U.S. International Trade Administration)*

(a) Imports: 1980–1990 (b) Exports: 1980–1990
(c) Imports: 1990–2000 (d) Exports: 1990–2000
(e) Imports: 1980–2002 (f) Exports: 1980–2002

Figure for 2



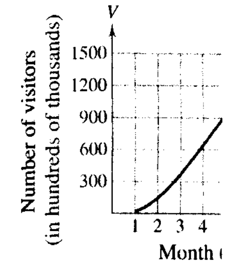
In Exercises 3–8, sketch the graph of the function and find its average rate of change on the interval. Compare this rate with the instantaneous rates of change at the endpoints of the interval.

3. $f(t) = 2t + 7; [1, 2]$
4. $h(x) = 1 - x; [0, 1]$
5. $h(x) = x^2 - 4x + 2; [-2, 2]$
6. $f(x) = x^2 - 6x - 1; [-1, 3]$
7. $f(x) = \dfrac{1}{x}; [1, 4]$
8. $f(x) = \dfrac{1}{\sqrt{x}}; [1, 4]$

In Exercises 9 and 10, use a graphing utility to graph the function and find its average rate of change on the interval. Compare this rate with the instantaneous rates of change at the endpoints of the interval.

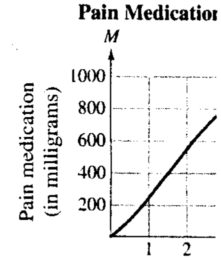
9. $g(x) = x^4 - x^2 + 2; [1, 3]$
10. $g(x) = x^3 - 1; [-1, 1]$

11. ***Consumer Trends*** visitors V to a nation during a one-year peric



(a) Estimate the rate [9, 12] and explain
(b) Over what interv approximately equ Explain your reasc

12. ***Medicine*** The grap milligrams of a pain hours after a 1000-mi given.



(a) Estimate the one-h rate of change is tl
(b) Over what interv approximately equ Explain your reasc

13. ***Medicine*** The effec a pain-killing drug t h given by

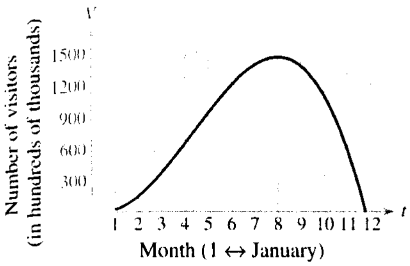
$E = \frac{1}{27}(9t + 3t^2 -$

Find the average rate interval and compare of change at the endpo

(a) $[0, 1]$ (b) $[1,$

11. ***Consumer Trends*** The graph shows the number of visitors V to a national park in hundreds of thousands during a one-year period, where $t = 1$ represents January.

Visitors to a National Park

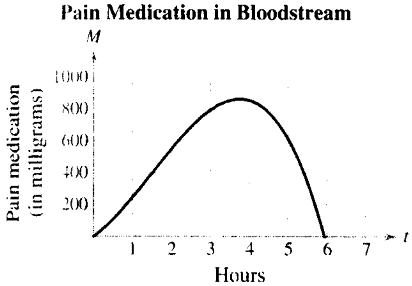


(a) Estimate the rate of change of V over the interval $[9, 12]$ and explain your results.

(b) Over what interval is the average rate of change approximately equal to the rate of change at $t = 8$? Explain your reasoning.

12. ***Medicine*** The graph shows the estimated number of milligrams of a pain medication M in the bloodstream t hours after a 1000-milligram dose of the drug has been given.

Pain Medication in Bloodstream



(a) Estimate the one-hour interval over which the average rate of change is the greatest.

(b) Over what interval is the average rate of change approximately equal to the rate of change at $t = 4$? Explain your reasoning.

13. ***Medicine*** The effectiveness E (on a scale from 0 to 1) of a pain-killing drug t hours after entering the bloodstream is given by

$$E = \frac{1}{27}(9t + 3t^2 - t^3), \quad 0 \le t \le 4.5.$$

Find the average rate of change of E on each indicated interval and compare this rate with the instantaneous rates of change at the endpoints of the interval.

(a) $[0, 1]$ (b) $[1, 2]$ (c) $[2, 3]$ (d) $[3, 4]$

14. ***Chemistry: Wind Chill*** At 0° Celsius, the heat loss H (in kilocalories per square meter per hour) from a person's body can be modeled by

$$H = 33(10\sqrt{v} - v + 10.45)$$

where v is the wind speed (in meters per second).

(a) Find $\frac{dH}{dv}$ and interpret its meaning in this situation.

(b) Find the rates of change of H when $v = 2$ and when $v = 5$.

15. ***Velocity*** The height s (in feet) at time t (in seconds) of a silver dollar dropped from the top of the Washington Monument is given by

$$s = -16t^2 + 555.$$

(a) Find the average velocity on the interval $[2, 3]$.

(b) Find the instantaneous velocities when $t = 2$ and when $t = 3$.

(c) How long will it take the dollar to hit the ground?

(d) Find the velocity of the dollar when it hits the ground.

16. ***Physics: Velocity*** A racecar travels northward on a straight, level track at a constant speed, traveling 0.750 kilometer in 20.0 seconds. The return trip over the same track is made in 25.0 seconds.

(a) What is the average velocity of the car in meters per second for the first leg of the run?

(b) What is the average velocity for the total trip?

Marginal Cost In Exercises 17–20, find the marginal cost for producing x units. (The cost is measured in dollars.)

17. $C = 4500 + 1.47x$

18. $C = 104{,}000 + 7200x$

19. $C = 55{,}000 + 470x - 0.25x^2, \quad 0 \le x \le 940$

20. $C = 100(9 + 3\sqrt{x})$

Marginal Revenue In Exercises 21–24, find the marginal revenue for producing x units. (The revenue is measured in dollars.)

21. $R = 50x - 0.5x^2$

22. $R = 30x - x^2$

23. $R = -6x^3 + 8x^2 + 200x$

24. $R = 50(20x - x^{3/2})$

Marginal Profit In Exercises 25–28, find the marginal profit for producing x units. (The profit is measured in dollars.)

25. $P = -2x^2 + 72x - 145$

26. $P = -0.25x^2 + 2000x - 1{,}250{,}000$

27. $P = -0.00025x^2 + 12.2x - 25{,}000$

28. $P = -0.5x^3 + 30x^2 - 164.25x - 1000$

29. *Marginal Cost* The cost (in dollars) of producing x units of a product is given by

$$C = 3.6\sqrt{x} + 500.$$

(a) Find the additional cost when the production increases from 9 to 10 units.

(b) Find the marginal cost when $x = 9$.

(c) Compare the results of parts (a) and (b).

30. *Marginal Revenue* The revenue (in dollars) from renting x apartments can be modeled by

$$R = 2x(900 + 32x - x^2).$$

(a) Find the additional revenue when the number of rentals is increased from 14 to 15.

(b) Find the marginal revenue when $x = 14$.

(c) Compare the results of parts (a) and (b).

31. *Marginal Profit* The profit (in dollars) from selling x units of calculus textbooks is given by

$$P = -0.05x^2 + 20x - 1000.$$

(a) Find the additional profit when the sales increase from 150 to 151 units.

(b) Find the marginal profit when $x = 150$.

(c) Compare the results of parts (a) and (b).

32. *Population Growth* The population of a developing rural area has been growing according to the model

$$P = 22t^2 + 52t + 10{,}000$$

where t is time in years, with $t = 0$ corresponding to 1990.

(a) Evaluate P for $t = 0$, 10, 15, 20, and 25. Explain these values.

(b) Determine the population growth rate, dP/dt.

(c) Evaluate dP/dt for the same values as in part (a). Explain your results.

33. *Health* The temperature of a person during an illness is given by $T = -0.0375t^2 + 0.3t + 100.4$, where t is time in hours since the person started to show signs of a fever.

(a) Use a graphing utility to graph the function. Be sure to choose an appropriate window.

(b) Do the slopes of the tangent lines appear to be positive or negative? What does this tell you?

(c) Evaluate the function for $t = 0$, 4, 8, and 12.

(d) Find dT/dt and explain its meaning in this situation.

(e) Evaluate dT/dt for $t = 0$, 4, 8, and 12.

34. *Marginal Profit* The profit (in dollars) from selling x units of a product is given by

$$P = 36{,}000 + 2048\sqrt{x} - \frac{1}{8x^2}, \quad 150 \le x \le 275.$$

Find the marginal profit for each of the following sales.

(a) $x = 150$ (b) $x = 175$ (c) $x = 200$

(d) $x = 225$ (e) $x = 250$ (f) $x = 275$

35. *Profit* The monthly demand function and cost function for x newspapers at a newsstand are given by $p = 5 - 0.001x$ and $C = 35 + 1.5x$.

(a) Find the monthly revenue R as a function of x.

(b) Find the monthly profit P as a function of x.

(c) Complete the table.

x	600	1200	1800	2400	3000
dR/dx					
dP/dx					
P					

36. *Economics* Use the table to answer the questions below.

Quantity produced and sold (Q)	Price (p)	Total revenue (TR)	Marginal revenue (MR)
0	160	0	—
2	140	280	130
4	120	480	90
6	100	600	50
8	80	640	10
10	60	600	−30

(a) Use the *regression* feature of a graphing utility to find a quadratic model that relates the total revenue (TR) to the quantity produced and sold (Q).

(b) Using derivatives, find a model for marginal revenue from the model you found in part (a).

(c) Calculate the marginal revenue for all values of Q using your model in part (b), and compare these values with the actual values given. How good is your model?

(*Source: Adapted from Taylor, Economics, Fourth Edition*)

37. *Marginal Profit* When a glass of lemonade at a lemonade stand was \$0.75, 400 glasses were sold. When the price was lowered to \$0.50, 500 glasses were sold. Assume that the demand function is linear and that the variable and fixed costs are \$0.05 and \$20, respectively.

(a) Find the profit P as a function of x, the number of glasses of lemonade sold.

(b) Use a graphing utility to graph P, and comment about the slopes of P when $x = 200$ and when $x = 400$.

(c) Find the marginal profits when 200 glasses of lemonade are sold and when 400 glasses of lemonade are sold.

38. *Marginal Profit*
baseball game was \$6
When the price was ra
sold. Assume that the d
variable and fixed cost
and \$85,000, respectiv

(a) Find the profit P
tickets sold.

(b) Use a graphing uti
the slopes of P whe

(c) Find the marginal
and when 36,000 t

39. *Marginal Profit* I
decreased to 30,000 w
would this change the

40. *Marginal Cost* Th
modeled by $C = v(x)$
cost and k represents t
cost is independent of

41. *Profit* The demand
$p = 50/\sqrt{x}$ for $1 \le$
given by $C = 0.5x +$

Find the marginal pro
(c) $x = 2500$, and (d)

If you were in charge
what price would you

42. *Inventory Managem*
a manufacturer is gi
where Q is the order si
Find the change in an
350 to 351, and comp
change when $Q = 35$

43. *Center of Populati*
ulation of the United S
ward. Use the figure t
at which the center c
during the given perio

(a) From 1790 to 190

(c) From 1790 to 200

38. ***Marginal Profit*** When the admission price for a baseball game was \$6 per ticket, 36,000 tickets were sold. When the price was raised to \$7, only 33,000 tickets were sold. Assume that the demand function is linear and that the variable and fixed costs for the ballpark owners are \$0.20 and \$85,000, respectively.

(a) Find the profit P as a function of x, the number of tickets sold.

(b) Use a graphing utility to graph P, and comment about the slopes of P when $x = 18{,}000$ and when $x = 36{,}000$.

(c) Find the marginal profits when 18,000 tickets are sold and when 36,000 tickets are sold.

39. ***Marginal Profit*** In Exercise 38, suppose ticket sales decreased to 30,000 when the price increased to \$7. How would this change the answers?

40. ***Marginal Cost*** The cost C of producing x units is modeled by $C = v(x) + k$, where v represents the variable cost and k represents the fixed cost. Show that the marginal cost is independent of the fixed cost.

41. ***Profit*** The demand function for a product is given by $p = 50/\sqrt{x}$ for $1 \le x \le 8000$, and the cost function is given by $C = 0.5x + 500$ for $0 \le x \le 8000$.

Find the marginal profits for (a) $x = 900$, (b) $x = 1600$, (c) $x = 2500$, and (d) $x = 3600$.

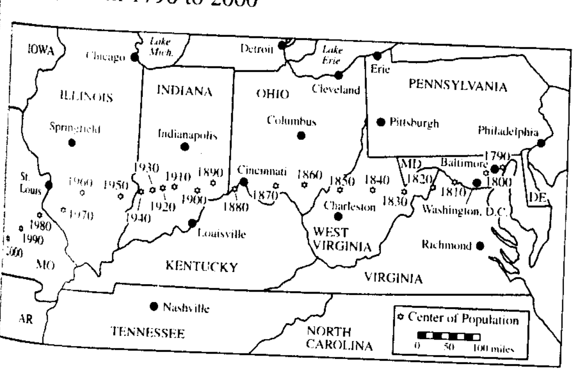
If you were in charge of setting the price for this product, what price would you set? Explain your reasoning.

42. ***Inventory Management*** The annual inventory cost for a manufacturer is given by $C = 1{,}008{,}000/Q + 6.3Q$, where Q is the order size when the inventory is replenished. Find the change in annual cost when Q is increased from 350 to 351, and compare this with the instantaneous rate of change when $Q = 350$.

43. ***Center of Population*** Since 1790, the center of population of the United States has been gradually moving westward. Use the figure to estimate the rate (in miles per year) at which the center of population was moving *westward* during the given period. (Source: U.S. Census Bureau)

(a) From 1790 to 1900 (b) From 1900 to 2000

(c) From 1790 to 2000



44. ***Consumer Awareness*** A car is driven 15,000 miles a year and gets x miles per gallon. The average fuel cost is \$1.30 per gallon. Find the annual cost C of fuel as a function of x and use this function to complete the table.

x	10	15	20	25	30	35	40
C							
dC/dx							

Who would benefit more from a 1 mile per gallon increase in fuel efficiency—the driver who gets 15 miles per gallon or the driver who gets 35 miles per gallon? Explain.

45. ***Writing*** The number N of gallons of regular unleaded gasoline sold by a gasoline station at a price of p dollars per gallon is given by $N = f(p)$.

(a) Describe the meaning of $f'(1.479)$.

(b) Is $f'(1.479)$ usually positive or negative? Explain.

46. Consider the function given by $f(x) = \dfrac{4}{x}$, $0 < x \le 5$.

(a) Use a graphing utility to graph f and f' in the same viewing window.

(b) Complete the table.

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3	4	5
$f(x)$								
$f'(x)$								

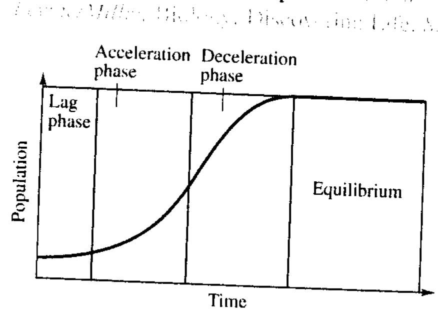
(c) Find the average rate of change of f over the intervals determined by consecutive x-values in the table.

In Exercises 47 and 48, use a graphing utility to graph f and f'. Then determine the points (if any) at which f has a horizontal tangent.

47. $f(x) = \frac{1}{4}x^3$, $-2 \le x \le 2$

48. $f(x) = x^4 - 12x^3 + 52x^2 - 96x + 64$, $1 \le x \le 5$

49. ***Biology*** Many populations in nature exhibit logistic growth, which consists of four phases, as shown in the figure. Describe the rate of growth of the population in each phase, and give possible reasons as to why the rates might be changing from phase to phase. (Source: Adapted from Levine/Miller, Biology: Discovering Life, Second Edition)



PREREQUISITE REVIEW 2.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–10, simplify the expression.

1. $(x^2 + 1)(2) + (2x + 7)(2x)$
2. $(2x - x^3)(8x) + (4x^2)(2 - 3x^2)$
3. $x(4)(x^2 + 2)^3(2x) + (x^2 + 4)(1)$
4. $x^2(2)(2x + 1)(2) + (2x + 1)^4(2x)$
5. $\dfrac{(2x + 7)(5) - (5x + 6)(2)}{(2x + 7)^2}$
6. $\dfrac{(x^2 - 4)(2x + 1) - (x^2 + x)(2x)}{(x^2 - 4)^2}$
7. $\dfrac{(x^2 + 1)(2) - (2x + 1)(2x)}{(x^2 + 1)^2}$
8. $\dfrac{(1 - x^4)(4) - (4x - 1)(-4x^3)}{(1 - x^4)^2}$
9. $(x^{-1} + x)(2) + (2x - 3)(-x^{-2} + 1)$
10. $\dfrac{(1 - x^{-1})(1) - (x - 4)(x^{-2})}{(1 - x^{-1})^2}$

In Exercises 11–14, find $f'(2)$.

11. $f(x) = 3x^2 - x + 4$
12. $f(x) = -x^3 + x^2 + 8x$
13. $f(x) = \dfrac{1}{x}$
14. $f(x) = x^2 - \dfrac{1}{x^2}$

EXERCISES 2.4

In Exercises 1–14, find the value of the derivative of the function at the given point.

	Function	*Point*
1.	$f(x) = x^2(3x^3 - 1)$	$(1, 2)$
2.	$f(x) = (x^2 + 1)(2x + 5)$	$(-1, 6)$
3.	$f(x) = \frac{1}{3}(2x^3 - 4)$	$(0, -\frac{4}{3})$
4.	$f(x) = \frac{1}{7}(5 - 6x^2)$	$(1, -\frac{1}{7})$
5.	$g(x) = (x^2 - 4x + 3)(x - 2)$	$(4, 6)$
6.	$g(x) = (x^2 - 2x + 1)(x^3 - 1)$	$(1, 0)$
7.	$h(x) = \dfrac{x}{x - 5}$	$(6, 6)$
8.	$h(x) = \dfrac{x^2}{x + 3}$	$\left(-1, \dfrac{1}{2}\right)$
9.	$f(t) = \dfrac{2t^2 - 3}{3t + 1}$	$\left(3, \dfrac{3}{2}\right)$
10.	$f(x) = \dfrac{3x}{x^2 + 4}$	$\left(-1, -\dfrac{3}{5}\right)$
11.	$g(x) = \dfrac{2x + 1}{x - 5}$	$(6, 13)$
12.	$f(x) = \dfrac{x + 1}{x - 1}$	$(2, 3)$
13.	$f(t) = \dfrac{t^2 - 1}{t + 4}$	$(1, 0)$
14.	$g(x) = \dfrac{4x - 5}{x^2 - 1}$	$(0, 5)$

In Exercises 15–22, find the derivative of the function. Use Example 7 as a model.

	Function	*Rewrite*	*Differentiate*	*Simplify*
15.	$y = \dfrac{x^2 + 2x}{x}$			
16.	$y = \dfrac{4x^{3/2}}{x}$			
17.	$y = \dfrac{7}{3x^3}$			
18.	$y = \dfrac{4}{5x^2}$			
19.	$y = \dfrac{4x^2 - 3x}{8\sqrt{x}}$			
20.	$y = \dfrac{3x^2 - 4x}{6x}$			
21.	$y = \dfrac{x^2 - 4x + 3}{x - 1}$			
22.	$y = \dfrac{x^2 - 4}{x + 2}$			

In Exercises 23–38, find the

23. $f(x) = (x^3 - 3x)(2x^2$
24. $h(t) = (t^5 - 1)(4t^2 -$
25. $g(t) = (2t^3 - 1)^2$
26. $h(p) = (p^3 - 2)^2$
27. $f(x) = \sqrt[3]{x}(\sqrt{x} + 3)$
28. $f(x) = \sqrt[3]{x}(x + 1)$
30. $f(x) = \dfrac{x^3 + 3x + 2}{x^2 - 1}$
32. $f(x) = (x^5 - 3x)\left(\dfrac{1}{x^2}\right)$
33. $f(x) = x\left(1 - \dfrac{2}{x + 1}\right)$
34. $h(t) = \dfrac{t + 2}{t^2 + 5t + 6}$
35. $g(s) = \dfrac{s^2 - 2s + 5}{\sqrt{s}}$
36. $f(x) = \dfrac{x + 1}{\sqrt{x}}$
37. $g(x) = \left(\dfrac{x - 3}{x + 4}\right)(x^2 +$
38. $f(x) = (3x^3 + 4x)(x -$

In Exercises 39–44, find an
graph of the function at t
utility to graph the functic
viewing window.

Function

39. $f(x) = (x - 1)^2(x -$
40. $h(x) = (x^2 - 1)^2$
41. $f(x) = \dfrac{x - 2}{x + 1}$
42. $f(x) = \dfrac{2x + 1}{x - 1}$
43. $f(x) = \left(\dfrac{x + 5}{x - 1}\right)(2x +$
44. $g(x) = (x + 2)\left(\dfrac{x - 5}{x + 1}\right.$

In Exercises 45–48, find the
f has a horizontal tangent.

45. $f(x) = \dfrac{x^2}{x - 1}$
47. $f(x) = \dfrac{x^4}{x^3 + 1}$

In Exercises 23–38, find the derivative of the function.

23. $f(x) = (x^3 - 3x)(2x^2 + 3x + 5)$

24. $h(t) = (t^5 - 1)(4t^2 - 7t - 3)$

25. $g(t) = (2t^3 - 1)^2$

26. $h(p) = (p^3 - 2)^2$

27. $f(x) = \sqrt[3]{x}(\sqrt{x} + 3)$

28. $f(x) = \sqrt[3]{x}(x + 1)$

29. $f(x) = \dfrac{3x - 2}{2x - 3}$

30. $f(x) = \dfrac{x^3 + 3x + 2}{x^2 - 1}$

31. $f(x) = \dfrac{3 - 2x - x^2}{x^2 - 1}$

32. $f(x) = (x^5 - 3x)\left(\dfrac{1}{x^2}\right)$

33. $f(x) = x\left(1 - \dfrac{2}{x + 1}\right)$

34. $h(t) = \dfrac{t + 2}{t^2 + 5t + 6}$

35. $g(s) = \dfrac{s^2 - 2s + 5}{\sqrt{s}}$

36. $f(x) = \dfrac{x + 1}{\sqrt{x}}$

37. $g(x) = \left(\dfrac{x - 3}{x + 4}\right)(x^2 + 2x + 1)$

38. $f(x) = (3x^3 + 4x)(x - 5)(x + 1)$

In Exercises 39–44, find an equation of the tangent line to the graph of the function at the given point. Then use a graphing utility to graph the function and the tangent line in the same viewing window.

Function	*Point*
39. $f(x) = (x - 1)^2(x - 2)$	$(0, -2)$
40. $h(x) = (x^2 - 1)^2$	$(-2, 9)$
41. $f(x) = \dfrac{x - 2}{x + 1}$	$\left(1, -\frac{1}{2}\right)$
42. $f(x) = \dfrac{2x + 1}{x - 1}$	$(2, 5)$
43. $f(x) = \left(\dfrac{x + 5}{x - 1}\right)(2x + 1)$	$(0, -5)$
44. $g(x) = (x + 2)\left(\dfrac{x - 5}{x + 1}\right)$	$(0, -10)$

In Exercises 45–48, find the point(s), if any, at which the graph of f has a horizontal tangent.

45. $f(x) = \dfrac{x^2}{x - 1}$

46. $f(x) = \dfrac{x^2}{x^2 + 1}$

47. $f(x) = \dfrac{x^4}{x^3 + 1}$

48. $f(x) = \dfrac{x^4 + 3}{x^2 + 1}$

In Exercises 49–52, use a graphing utility to graph f and f' on the interval $[-2, 2]$.

49. $f(x) = x(x + 1)$

50. $f(x) = x^2(x + 1)$

51. $f(x) = x(x + 1)(x - 1)$

52. $f(x) = x^2(x + 1)(x - 1)$

Demand In Exercises 53 and 54, use the demand function to find the rate of change in the demand x for the given price p.

53. $x = 275\left(1 - \dfrac{3p}{5p + 1}\right)$, $p = \$4$

54. $x = 300 - p - \dfrac{2p}{p + 1}$, $p = \$3$

55. ***Environment*** The model

$$f(t) = \frac{t^2 - t + 1}{t^2 + 1}$$

measures the percent of the normal level of oxygen in a pond, where t is the time (in weeks) after organic waste is dumped into the pond. Find the rates of change of f with respect to t when (a) $t = 0.5$, (b) $t = 2$, and (c) $t = 8$.

56. ***Physical Science*** The temperature T of food placed in a refrigerator is modeled by

$$T = 10\left(\frac{4t^2 + 16t + 75}{t^2 + 4t + 10}\right)$$

where t is the time (in hours). What is the initial temperature of the food? Find the rates of change of T with respect to t when (a) $t = 1$, (b) $t = 3$, (c) $t = 5$, and (d) $t = 10$.

57. ***Population Growth*** A population of bacteria is introduced into a culture. The number of bacteria P can be modeled by

$$P = 500\left(1 + \frac{4t}{50 + t^2}\right)$$

where t is the time (in hours). Find the rate of change of the population when $t = 2$.

58. ***Quality Control*** The percent P of defective parts produced by a new employee t days after the employee starts work can be modeled by

$$P = \frac{t + 1750}{50(t + 2)}.$$

Find the rates of change of P when (a) $t = 1$ and (b) $t = 10$.

59. ***Profit*** You decide to form a partnership with another business. Your business determines that the demand x for your product is inversely proportional to the square of the price for $x \geq 5$.

(a) The price is \$1000 and the demand is 16 units. Find the demand function.

(b) Your partner determines that the product costs \$250 per unit and the fixed cost is \$10,000. Find the cost function.

(c) Find the profit function and use a graphing utility to graph it. From the graph, what price would you negotiate with your partner for this product? Explain your reasoning.

60. ***Profit*** You are managing a store and have been adjusting the price of an item. You have found that you make a profit of \$50 when 10 units are sold, \$60 when 12 units are sold, and \$65 when 14 units are sold.

(a) Fit these data to the model $P = ax^2 + bx + c$.

(b) Use a graphing utility to graph P.

(c) Find the point on the graph at which the marginal profit is zero. Interpret this point in the context of the problem.

61. ***Demand Function*** Given $f(x) = x^2 + 1$, which function would most likely represent a demand function?

(a) $p = f(x)$

(b) $p = xf(x)$

(c) $p = 1/f(x)$

Explain your reasoning. Use a graphing utility to graph each function, and use each graph as part of your explanation.

62. ***Cost*** The cost of producing x units of a product is given by

$$C = x^3 - 15x^2 + 87x - 73, \qquad 4 \le x \le 9.$$

(a) Use a graphing utility to graph the marginal cost function and the average cost function, C/x, in the same viewing window.

(b) Find the point of intersection of the graphs of dC/dx and C/x. Does this point have any significance?

63. ***Inventory Replenishment*** The ordering and transportation cost C (in thousands of dollars) of the components used in manufacturing a product is given by

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x+30}\right), \qquad 1 \le x$$

where x is the order size (in hundreds). Find the rate of change of C with respect to x for each order size.

(a) $x = 10$

(b) $x = 15$

(c) $x = 20$

What do these rates of change imply about increasing the size of an order?

64. ***Sales Analysis*** The monthly sales of memberships M at a newly built fitness center are modeled by

$$M(t) = \frac{300t}{t^2 + 1} + 8$$

where t is the number of months since the center opened.

(a) Find $M'(t)$.

(b) Find $M(3)$ and $M'(3)$ and interpret the results.

(c) Find $M(24)$ and $M'(24)$ and interpret the results.

65. ***Consumer Awareness*** The prices of 1 pound of 100% ground beef in the United States from 1995 to 2002 can be modeled by

$$P = \frac{1.47 - 0.311t + 0.0173t^2}{1 - 0.206t + 0.0112t^2}$$

where t is the year, with $t = 5$ corresponding to 1995. Find dP/dt and evaluate it for $t = 5, 7, 9,$ and 11. Interpret the meaning of these values. *(Source: U.S. Bureau of Labor Statistics)*

BUSINESS CAPSULE

The Blackwood Centre for Adolescent Development, a state-sponsored secondary school for young people at risk in Victoria, Australia, joined forces in 2000 with the Centre for Executive Development (CED). With the CED providing fundraising and logistical support, the Blackwood Centre has been able to transform its program into a model of success, offering their students training and educational opportunities for entry into the business world. The CED has gained team-building skills, improved workplace assessment methods, and a stronger connection to the community.

66. ***Research Project*** Use your school's library, the Internet, or some other reference source to find information about partnerships between companies and federal, state, or local government that have benefited their communities. (One such partnership is described above.) Write a short paper about the partnership.

2.5

T

The Chain Rule

In this section, you wil
calculus—the **Chain Rul**
and adds versatility to tl
compare the functions b
Chain Rule, whereas tho

Without the Chain R

$y = x^2 + 1$

$y = x + 1$

$y = 3x + 2$

$y = \dfrac{x + 5}{x^2 + 2}$

The Chain Rule

If $y = f(u)$ is a diffe
tiable function of x,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

or, equivalently,

$$\frac{d}{dx}[f(g(x))] = f$$

Basically, the Chain
u changes du/dx times

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

times as fast as x, as i
notation for derivatives
as the Chain Rule. For i

$dy/dx = (dy/du)(d$

you can imagine that th

PREREQUISITE REVIEW 2.5

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, rewrite the expression with rational exponents.

1. $\sqrt[5]{(1-5x)^2}$
2. $\sqrt[4]{(2x-1)^3}$
3. $\dfrac{1}{\sqrt{4x^2+1}}$
4. $\dfrac{1}{\sqrt[3]{x-6}}$
5. $\dfrac{\sqrt{x}}{\sqrt[3]{1-2x}}$
6. $\dfrac{\sqrt{(3-7x)^3}}{2x}$

In Exercises 7–10, factor the expression.

7. $3x^3 - 6x^2 + 5x - 10$
8. $5x\sqrt{x} - x - 5\sqrt{x} + 1$
9. $4(x^2+1)^2 - x(x^2+1)^3$
10. $-x^5 + 3x^3 + x^2 - 3$

EXERCISES 2.5

In Exercises 1–8, identify the inside function, $u = g(x)$, and the outside function, $y = f(u)$.

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1. $y = (6x-5)^4$		
2. $y = (x^2 - 2x + 3)^3$		
3. $y = (4 - x^2)^{-1}$		
4. $y = (x^2+1)^{4/3}$		
5. $y = \sqrt{5x-2}$		
6. $y = \sqrt{9 - x^2}$		
7. $y = (3x+1)^{-1}$		
8. $y = (x+1)^{-1/2}$		

In Exercises 9–16, match the function with the rule that you would use to find the derivative *most efficiently.*

(a) Simple Power Rule
(b) Constant Rule
(c) General Power Rule
(d) Quotient Rule

9. $f(x) = \dfrac{2}{1-x^3}$
10. $f(x) = \dfrac{2x}{1-x^3}$
11. $f(x) = \sqrt[3]{8^2}$
12. $f(x) = \sqrt[3]{x^2}$
13. $f(x) = \dfrac{x^2+2}{x}$
14. $f(x) = \dfrac{x^4 - 2x + 1}{\sqrt{x}}$
15. $f(x) = \dfrac{2}{x-2}$
16. $f(x) = \dfrac{5}{x^2+1}$

In Exercises 17–34, use the General Power Rule to find the derivative of the function.

17. $y = (2x-7)^3$
18. $y = (3x^2+1)^4$
19. $g(x) = (4-2x)^3$
20. $h(t) = (1-t^2)^4$
21. $h(x) = (6x - x^3)^2$
22. $f(x) = (4x - x^2)^3$
23. $f(x) = (x^2-9)^{2/3}$
24. $f(t) = (9t+2)^{2/3}$
25. $f(t) = \sqrt{t+1}$
26. $g(x) = \sqrt{2x+3}$
27. $s(t) = \sqrt{2t^2+5t+2}$
28. $y = \sqrt[3]{3x^3+4x}$
29. $y = \sqrt[3]{9x^2+4}$
30. $y = 2\sqrt{4-x^2}$
31. $f(x) = -3\sqrt[4]{2-9x}$
32. $f(x) = (25+x^2)^{-1/2}$
33. $h(x) = (4-x^3)^{-4/3}$
34. $f(x) = (4-3x)^{-5/2}$

In Exercises 35–40, find an equation of the tangent line to the graph of f at the point $(2, f(2))$. Use a graphing utility to check your result by graphing the original function and the tangent line in the same viewing window.

35. $f(x) = 2(x^2-1)^3$
36. $f(x) = 3(9x-4)^4$
37. $f(x) = \sqrt{4x^2-7}$
38. $f(x) = x\sqrt{x^2+5}$
39. $f(x) = \sqrt{x^2-2x+1}$
40. $f(x) = (4-3x^2)^{-2/3}$

In Exercises 41–44, use a symbolic differentiation utility to find the derivative of the function. Graph the function and its derivative in the same viewing window. Describe the behavior of the function when the derivative is zero.

41. $f(x) = \dfrac{\sqrt{x}+1}{x^2+1}$
42. $f(x) = \sqrt{\dfrac{2x}{x+1}}$
43. $f(x) = \sqrt{\dfrac{x+1}{x}}$
44. $f(x) = \sqrt{x}(2-x^2)$

In Exercises 45–64, find the derivative of the function.

45. $y = \dfrac{1}{x-2}$
46. $s(t) = \dfrac{1}{t^2+3t-1}$
47. $y = -\dfrac{4}{(t+2)^2}$
48. $f(x) = \dfrac{3}{(x^3-4)^2}$
49. $f(x) = \dfrac{1}{(x^2-3x)^2}$
50. $y = \dfrac{1}{\sqrt{x+2}}$

51. $g(t) = \dfrac{1}{t^2-2}$
53. $f(x) = x(3x-9)^3$
55. $y = x\sqrt{2x+3}$
57. $y = t^2\sqrt{t-2}$
59. $f(x) = \sqrt{\dfrac{3-2x}{4x}}$
61. $f(x) = \sqrt{x^2+1} - \sqrt{\ }$
62. $y = \sqrt{x-1} + \sqrt{x+}$
63. $y = \left(\dfrac{6-5x}{x^2-1}\right)^2$

In Exercises 65–70, find an
graph of the function at th
utility to graph the functio
viewing window.

Function

65. $f(t) = \dfrac{36}{(3-t)^2}$
66. $s(x) = \dfrac{1}{\sqrt{x^2-3x+4}}$
67. $f(t) = (t^2-9)\sqrt{t+2}$
68. $y = \dfrac{2x}{\sqrt{x+1}}$
69. $f(x) = \dfrac{x+1}{\sqrt{2x-3}}$
70. $y = \dfrac{x}{\sqrt{25+x^2}}$

71. ***Compound Interest*** with an annual inter
compounded monthly.

$$A = 1000\left(1 + \frac{r}{12}\right)$$

Find the rates of cha
(a) $r = 0.08$, (b) $r = 0.$

72. ***Environment*** An en
average daily level P of
per million can be mod

$$P = 0.25\sqrt{0.5n^2 +}$$

where n is the number
thousands. Find the rat
increasing when the pop

51. $g(t) = \dfrac{1}{t^2 - 2}$

52. $g(x) = \dfrac{3}{\sqrt[3]{x^3 - 1}}$

53. $f(x) = x(3x - 9)^3$

54. $f(x) = x^3(x - 4)^2$

55. $y = x\sqrt{2x + 3}$

56. $y = t\sqrt{t + 1}$

57. $y = t^2\sqrt{t - 2}$

58. $y = \sqrt{x}(x - 2)^2$

59. $f(x) = \sqrt{\dfrac{3 - 2x}{4x}}$

60. $g(t) = \dfrac{3t^2}{\sqrt{t^2 + 2t - 1}}$

61. $f(x) = \sqrt{x^2 + 1} - \sqrt{x^2 - 1}$

62. $y = \sqrt{x - 1} + \sqrt{x + 1}$

63. $y = \left(\dfrac{6 - 5x}{x^2 - 1}\right)^2$

64. $y = \left(\dfrac{4x^2}{3 - x}\right)^3$

In Exercises 65–70, find an equation of the tangent line to the graph of the function at the given point. Then use a graphing utility to graph the function and the tangent line in the same viewing window.

Function	*Point*
65. $f(t) = \dfrac{36}{(3 - t)^2}$	$(0, 4)$
66. $s(x) = \dfrac{1}{\sqrt{x^2 - 3x + 4}}$	$\left(3, \frac{1}{2}\right)$
67. $f(t) = (t^2 - 9)\sqrt{t + 2}$	$(-1, -8)$
68. $y = \dfrac{2x}{\sqrt{x + 1}}$	$(3, 3)$
69. $f(x) = \dfrac{x + 1}{\sqrt{2x - 3}}$	$(2, 3)$
70. $y = \dfrac{x}{\sqrt{25 + x^2}}$	$(0, 0)$

71. ***Compound Interest*** You deposit \$1000 in an account with an annual interest rate of r (in decimal form) compounded monthly. At the end of 5 years, the balance is

$$A = 1000\left(1 + \frac{r}{12}\right)^{60}.$$

Find the rates of change of A with respect to r when (a) $r = 0.08$, (b) $r = 0.10$, and (c) $r = 0.12$.

72. ***Environment*** An environmental study indicates that the average daily level P of a certain pollutant in the air in parts per million can be modeled by the equation

$$P = 0.25\sqrt{0.5n^2 + 5n + 25}$$

where n is the number of residents of the community in thousands. Find the rate at which the level of pollutant is increasing when the population of the community is 12,000.

73. ***Biology*** The number N of bacteria in a culture after t days is modeled by

$$N = 400\left[1 - \frac{3}{(t^2 + 2)^2}\right].$$

Complete the table. What can you conclude?

t	0	1	2	3	4
dN/dt					

74. ***Depreciation*** The value V of a machine t years after it is purchased is inversely proportional to the square root of $t + 1$. The initial value of the machine is \$10,000.

(a) Write V as a function of t.

(b) Find the rate of depreciation when $t = 1$.

(c) Find the rate of depreciation when $t = 3$.

75. ***Depreciation*** Repeat Exercise 74 given that the value of the machine t years after it is purchased is inversely proportional to the cube root of $t + 1$.

76. ***Credit Card Rate*** The average annual rate r (in percent form) for commercial bank credit cards from 1994 through 2002 can be modeled by

$$r = \sqrt{-0.14239t^4 + 3.939t^3 - 39.0835t^2 + 161.037t + 22.13}$$

where $t = 4$ corresponds to 1994.

(a) Find the derivative of this model. Which differentiation rule(s) did you use?

(b) Use a graphing utility to graph the derivative. Use the interval $4 \le t \le 12$.

(c) Use the *trace* feature to find the years during which the finance rate was changing the most.

(d) Use the *trace* feature to find the years during which the finance rate was changing the least.

True or False? In Exercises 77 and 78, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

77. If $y = (1 - x)^{1/2}$, then $y' = \frac{1}{2}(1 - x)^{-1/2}$.

78. If y is a differentiable function of u, u is a differentiable function of v, and v is a differentiable function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}.$$

PREREQUISITE REVIEW 2.6

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, solve the equation.

1. $-16t^2 + 24t = 0$

2. $-16t^2 + 80t + 224 = 0$

3. $-16t^2 + 128t + 320 = 0$

4. $-16t^2 + 9t + 1440 = 0$

In Exercises 5–8, find dy/dx.

5. $y = x^2(2x + 7)$

6. $y = (x^2 + 3x)(2x^2 - 5)$

7. $y = \dfrac{x^2}{2x + 7}$

8. $y = \dfrac{x^2 + 3x}{2x^2 - 5}$

In Exercises 9 and 10, find the domain and range of f.

9. $f(x) = x^2 - 4$

10. $f(x) = \sqrt{x - 7}$

EXERCISES 2.6

In Exercises 1–14, find the second derivative of the function.

1. $f(x) = 5 - 4x$

2. $f(x) = 3x - 1$

3. $f(x) = x^2 + 7x - 4$

4. $f(x) = 3x^2 + 4x$

5. $g(t) = \frac{1}{3}t^3 - 4t^2 + 2t$

6. $f(x) = 4(x^2 - 1)^2$

7. $f(t) = \dfrac{3}{4t^2}$

8. $g(t) = t^{-1/3}$

9. $f(x) = 3(2 - x^2)^3$

10. $f(x) = x\sqrt[3]{x}$

11. $f(x) = \dfrac{x + 1}{x - 1}$

12. $g(t) = -\dfrac{4}{(t + 2)^2}$

13. $y = x^2(x^2 + 4x + 8)$

14. $h(s) = s^3(s^2 - 2s + 1)$

In Exercises 15–20, find the third derivative of the function.

15. $f(x) = x^5 - 3x^4$

16. $f(x) = x^4 - 2x^3$

17. $f(x) = 5x(x + 4)^3$

18. $f(x) = (x - 1)^2$

19. $f(x) = \dfrac{3}{16x^2}$

20. $f(x) = \dfrac{1}{x}$

In Exercises 21–26, find the given value.

	Function	*Value*
21.	$g(t) = 5t^4 + 10t^2 + 3$	$g''(2)$
22.	$f(x) = 9 - x^2$	$f''(-\sqrt{5})$
23.	$f(x) = \sqrt{4 - x}$	$f'''(-5)$
24.	$f(t) = \sqrt{2t + 3}$	$f'''(\frac{1}{2})$
25.	$f(x) = x^2(3x^2 + 3x - 4)$	$f'''(-2)$
26.	$g(x) = 2x^3(x^2 - 5x + 4)$	$g'''(0)$

In Exercises 27–32, find the higher-order derivative.

	Given	*Derivative*
27.	$f'(x) = 2x^2$	$f''(x)$
28.	$f''(x) = 20x^3 - 36x^2$	$f'''(x)$
29.	$f''(x) = (2x - 2)/x$	$f'''(x)$
30.	$f'''(x) = 2\sqrt{x - 1}$	$f^{(4)}(x)$
31.	$f^{(4)}(x) = (x + 1)^2$	$f^{(6)}(x)$
32.	$f(x) = x^3 - 2x$	$f''(x)$

In Exercises 33–40, find the second derivative and solve th equation $f''(x) = 0$.

33. $f(x) = x^3 - 9x^2 + 27x - 27$

34. $f(x) = 3x^3 - 9x + 1$

35. $f(x) = (x + 3)(x - 4)(x + 5)$

36. $f(x) = (x + 2)(x - 2)(x + 3)(x - 3)$

37. $f(x) = x\sqrt{x^2 - 1}$

38. $f(x) = x\sqrt{4 - x^2}$

39. $f(x) = \dfrac{x}{x^2 + 3}$

40. $f(x) = \dfrac{x}{x^2 + 1}$

41. ***Velocity and Acceleration*** A ball is propelled straigh upward from ground level with an initial velocity of 14 feet per second.

(a) Write the position function of the ball.

(b) Write the velocity and acceleration functions.

(c) When is the ball at its highest point? How high is thi point?

(d) How fast is the ball traveling when it hits the ground How is this speed related to the initial velocity?

42. ***Velocity and Acceleration*** A brick becomes dislodged from the top of the Empire State Building (at a height of 1250 feet) and falls to the sidewalk below.
   (a) Write the position function of the brick.
   (b) Write the velocity and acceleration functions.
   (c) How long does it take the brick to hit the sidewalk?
   (d) How fast is the brick traveling when it hits the sidewalk?

43. ***Velocity and Acceleration*** The velocity (in feet per second) of an automobile starting from rest is modeled by

$$\frac{ds}{dt} = \frac{90t}{t + 10}.$$

Create a table showing the velocity and acceleration at 10-second intervals during the first minute of travel. What can you conclude?

44. ***Stopping Distance*** A car is traveling at a rate of 66 feet per second (45 miles per hour) when the brakes are applied. The position function for the car is given by $s = -8.25t^2 + 66t$, where s is measured in feet and t is measured in seconds. Create a table showing the position, velocity, and acceleration for each given value of t. What can you conclude?

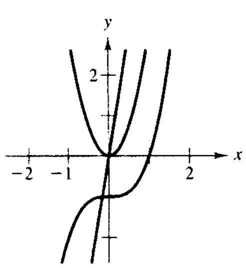
In Exercises 45 and 46, use a graphing utility to graph f, f', and f'' in the same viewing window. What is the relationship among the degree of f and the degrees of its successive derivatives? In general, what is the relationship among the degree of a polynomial function and the degrees of its successive derivatives?

45. $f(x) = x^2 - 6x + 6$

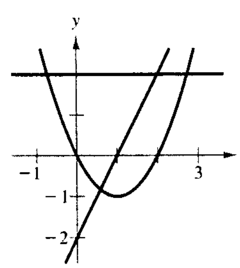
46. $f(x) = 3x^3 - 9x$

In Exercises 47 and 48, the graphs of f, f', and f'' are shown on the same set of coordinate axes. Which is which? Explain your reasoning.

47.



48.



49. ***Data Analysis*** The table shows the median prices y (in thousands of dollars) of new privately owned U.S. homes in the South for 1995 to 2002. *(Source: U.S. Census Bureau)*

t	5	6	7	8	9
y	124.5	126.2	129.6	135.8	145.9

t	10	11	12
y	148.0	155.4	163.4

A model for the data is

$$y = -0.0828t^3 + 2.443t^2 - 17.06t + 158.7$$

where t is the year, with $t = 5$ corresponding to 1995.
   (a) Use a graphing utility to graph the model and plot the data in the same viewing window.
   (b) Find the first and second derivatives of the function.
   (c) Show that the price of homes was increasing from 1995 to 2002.
   (d) Find the year when the price was increasing at the greatest rate.
   (e) Explain the relationship among your answers for parts (b), (c), and (d).

50. ***Projectile Motion*** An object is thrown upward from the top of a 64-foot building with an initial velocity of 48 feet per second.
   (a) Write the position function of the object.
   (b) Find the velocity and acceleration functions.
   (c) When will the object hit the ground?
   (d) When is the velocity of the object zero?
   (e) How high does the object go?
   (f) Use a graphing utility to graph the position, velocity, and acceleration functions in the same viewing window. Write a short paragraph that describes the relationship among these functions.

True or False? In Exercises 51–56, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

51. If $y = f(x)g(x)$, then $y' = f'(x)g'(x)$.

52. If $y = (x + 1)(x + 2)(x + 3)(x + 4)$, then $\frac{d^5y}{dx^5} = 0$.

53. If $f'(c)$ and $g'(c)$ are zero and $h(x) = f(x)g(x)$, then $h'(c) = 0$.

54. If $f(x)$ is an nth-degree polynomial, then $f^{(n+1)}(x) = 0$.

55. The second derivative represents the rate of change of the first derivative.

56. If the velocity of an object is constant, then its acceleration is zero.

57. ***Finding a Pattern*** Develop a general rule for $[xf(x)]^{(n)}$ where f is a differentiable function of x.

2.7

Implicit and Expli

So far in the text, functic in the **explicit form** y = given in terms of the otl

$y = 3x - 5,\quad s =$

are each written in expl and w, explicitly. Many implied by a given equa

EXAMPLE 1 F

Find dy/dx for the equa

$xy = 1.$

SOLUTION In this equ to find dy/dx is first to

$$xy = 1$$
$$y = \frac{1}{x}$$
$$= x^{-1}$$
$$\frac{dy}{dx} = -x^{-2}$$
$$= -\frac{1}{x^2}$$

TRY IT 1

Find dy/dx for the eq

The procedure sh write the given functio you are unable to solve dy/dx in the equation

$x^2 - 2y^3 + 4y =$

where it is very difficu can use a procedure ca

PREREQUISITE REVIEW 2.7

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, solve the equation for y.

1. $x - \dfrac{y}{x} = 2$

2. $\dfrac{4}{x-3} = \dfrac{1}{y}$

3. $xy - x + 6y = 6$

4. $12 + 3y = 4x^2 + x^2y$

5. $x^2 + y^2 = 5$

6. $x = \pm\sqrt{6 - y^2}$

In Exercises 7–10, evaluate the expression at the given point.

7. $\dfrac{3x^2 - 4}{3y^2}$, $(2, 1)$

8. $\dfrac{x^2 - 2}{1 - y}$, $(0, -3)$

9. $\dfrac{5x}{3y^2 - 12y + 5}$, $(-1, 2)$

10. $\dfrac{1}{y^2 - 2xy + x^2}$, $(4, 3)$

EXERCISES 2.7

In Exercises 1–12, find dy/dx.

1. $5xy = 1$

2. $\frac{1}{2}x^2 - y = 6x$

3. $y^2 = 1 - x^2$, $0 \le x \le 1$

4. $4x^2y - \dfrac{3}{y} = 0$

5. $x^2y^2 - 4y = 1$

6. $xy^2 + 4xy = 10$

7. $4y^2 - xy = 2$

8. $2xy^3 - x^2y = 2$

9. $\dfrac{2y - x}{y^2 - 3} = 5$

10. $\dfrac{xy - y^2}{y - x} = 1$

11. $\dfrac{x + y}{2x - y} = 1$

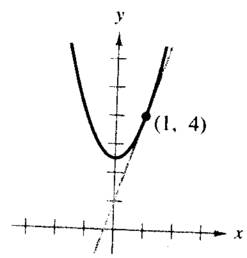
12. $\dfrac{2x + y}{x - 5y} = 1$

In Exercises 13–24, find dy/dx by implicit differentiation and evaluate the derivative at the given point.

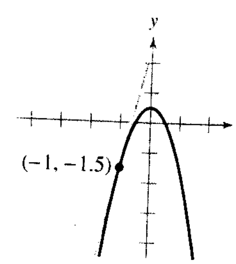
	Equation	*Point*
13.	$x^2 + y^2 = 49$	$(0, 7)$
14.	$x^2 - y^2 = 16$	$(4, 0)$
15.	$y + xy = 4$	$(-5, -1)$
16.	$x^2 - y^3 = 3$	$(2, 1)$
17.	$x^3 - xy + y^2 = 4$	$(0, -2)$
18.	$x^2y + y^2x = -2$	$(2, -1)$
19.	$x^3y^3 - y = x$	$(0, 0)$
20.	$x^3 + y^3 = 2xy$	$(1, 1)$
21.	$x^{1/2} + y^{1/2} = 9$	$(16, 25)$
22.	$\sqrt{xy} = x - 2y$	$(4, 1)$
23.	$x^{2/3} + y^{2/3} = 5$	$(8, 1)$
24.	$(x + y)^3 = x^3 + y^3$	$(-1, 1)$

In Exercises 25–30, find the slope of the graph at the given point.

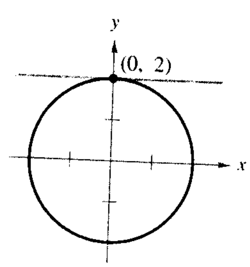
25. $3x^2 - 2y + 5 = 0$



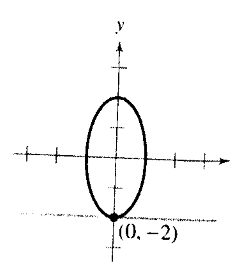
26. $4x^2 + 2y - 1 = 0$



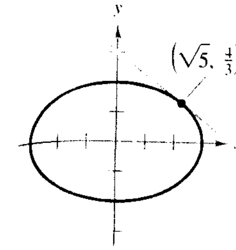
27. $x^2 + y^2 = 4$



28. $4x^2 + y^2 = 4$

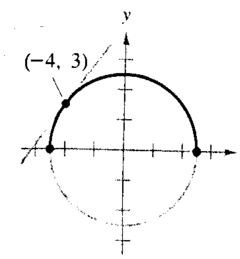


29. $4x^2 + 9y^2 = 36$

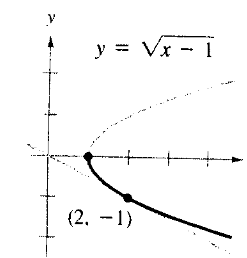


In Exercises 31–34, find *dy* explicit functions are shown results are equivalent. Use t the tangent line at the labe analytically by evaluating *dy*

31. $x^2 + y^2 = 25$



33. $x - y^2 - 1 = 0$

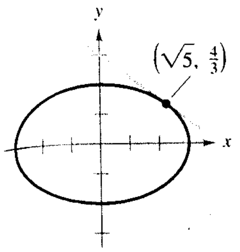


In Exercises 35–40, find equ graph at the given points. U equation and the tangent lin

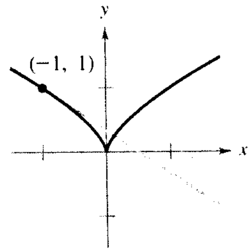
Equation

35. $x^2 + y^2 = 169$

29. $4x^2 + 9y^2 = 36$

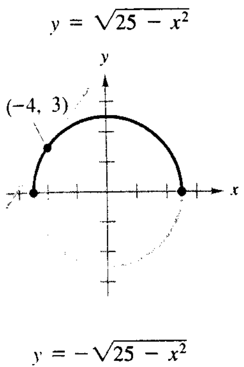


30. $x^2 - y^3 = 0$

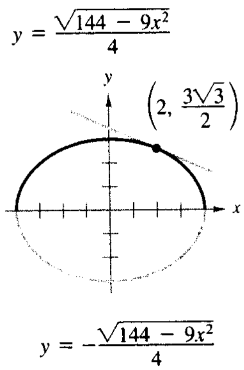


In Exercises 31–34, find dy/dx implicitly and explicitly (the explicit functions are shown on the graph) and show that the results are equivalent. Use the graph to estimate the slope of the tangent line at the labeled point. Then verify your result analytically by evaluating dy/dx at the point.

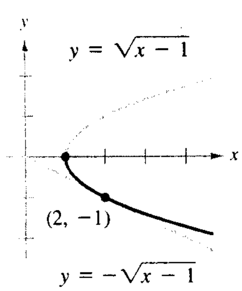
31. $x^2 + y^2 = 25$



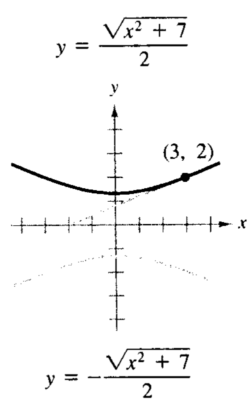
32. $9x^2 + 16y^2 = 144$



33. $x - y^2 - 1 = 0$



34. $4y^2 - x^2 = 7$



In Exercises 35–40, find equations of the tangent lines to the graph at the given points. Use a graphing utility to graph the equation and the tangent lines in the same viewing window.

Equation	*Points*
35. $x^2 + y^2 = 169$	$(5, 12)$ and $(-12, 5)$
36. $x^2 + y^2 = 9$	$(0, 3)$ and $(2, \sqrt{5})$
37. $y^2 = 5x^3$	$(1, \sqrt{5})$ and $(1, -\sqrt{5})$
38. $4xy + x^2 = 5$	$(1, 1)$ and $(5, -1)$
39. $x^3 + y^3 = 8$	$(0, 2)$ and $(2, 0)$
40. $y^2 = \dfrac{x^3}{4 - x}$	$(2, 2)$ and $(2, -2)$

Demand In Exercises 41–44, find the rate of change of x with respect to p.

41. $p = 0.006x^4 + 0.02x^2 + 10, \quad x \ge 0$

42. $p = 0.002x^4 + 0.01x^2 + 5, \quad x \ge 0$

43. $p = \sqrt{\dfrac{200 - x}{2x}}, \quad 0 < x \le 200$

44. $p = \sqrt{\dfrac{500 - x}{2x}}, \quad 0 < x \le 500$

45. ***Production*** Let x represent the units of labor and y the capital invested in a manufacturing process. When 135,540 units are produced, the relationship between labor and capital can be modeled by $100x^{0.75}y^{0.25} = 135{,}540$.

(a) Find the rate of change of y with respect to x when $x = 1500$ and $y = 1000$.

(b) The model used in the problem is called the *Cobb-Douglas production function*. Graph the model on a graphing utility and describe the relationship between labor and capital.

46. ***Health: U.S. AIDS Epidemic*** The numbers (in millions) of cases y of AIDS reported in the years 1994 to 2001 can be modeled by

$$y^2 + 4436 = -4.2460t^4 + 146.821t^3 - 1728.00t^2 + 7456.6t$$

where $t = 4$ corresponds to 1994. *(Source: U.S. Centers for Disease Control and Prevention)*

(a) Use a graphing utility to graph the model and describe the results.

(b) Use the graph to determine the year during which the number of reported cases decreasing most rapidly.

(c) Complete the table to confirm your estimate.

t	4	5	6	7	8	9	10	11
y								
y'								

PREREQUISITE REVIEW 2.8

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, write a formula for the given quantity.

1. Area of a circle

2. Volume of a sphere

3. Surface area of a cube

4. Volume of a cube

5. Volume of a cone

6. Area of a triangle

In Exercises 7–10, find dy/dx by implicit differentiation.

7. $x^2 + y^2 = 9$

8. $3xy - x^2 = 6$

9. $x^2 + 2y + xy = 12$

10. $x + xy^2 - y^2 = xy$

EXERCISES 2.8

In Exercises 1–4, find the given values of dy/dt and dx/dt.

Equation	*Find*	*Given*
1. $y = x^2 - \sqrt{x}$	(a) $\dfrac{dy}{dt}$	$x = 4, \dfrac{dx}{dt} = 8$
	(b) $\dfrac{dx}{dt}$	$x = 16, \dfrac{dy}{dt} = 12$
2. $y = x^2 - 3x$	(a) $\dfrac{dy}{dt}$	$x = 3, \dfrac{dx}{dt} = 2$
	(b) $\dfrac{dx}{dt}$	$x = 1, \dfrac{dy}{dt} = 5$
3. $xy = 4$	(a) $\dfrac{dy}{dt}$	$x = 8, \dfrac{dx}{dt} = 10$
	(b) $\dfrac{dx}{dt}$	$x = 1, \dfrac{dy}{dt} = -6$
4. $x^2 + y^2 = 25$	(a) $\dfrac{dy}{dt}$	$x = 3, y = 4, \dfrac{dx}{dt} = 8$
	(b) $\dfrac{dx}{dt}$	$x = 4, y = 3, \dfrac{dy}{dt} = -2$

5. ***Area*** The radius r of a circle is increasing at a rate of 2 inches per minute. Find the rates of change of the area when (a) $r = 6$ inches and (b) $r = 24$ inches.

6. ***Volume*** The radius r of a sphere is increasing at a rate of 2 inches per minute. Find the rates of change of the volume when (a) $r = 6$ inches and (b) $r = 24$ inches.

7. ***Area*** Let A be the area of a circle of radius r that is changing with respect to time. If dr/dt is constant, is dA/dt constant? Explain your reasoning.

8. ***Volume*** Let V be the volume of a sphere of radius r that is changing with respect to time. If dr/dt is constant, is dV/dt constant? Explain your reasoning.

9. ***Volume*** A spherical balloon is inflated with gas at a rate of 20 cubic feet per minute. How fast is the radius of the balloon changing at the instant the radius is (a) 1 foot and (b) 2 feet?

10. ***Volume*** The radius r of a right circular cone is increasing at a rate of 2 inches per minute. The height h of the cone is related to the radius by $h = 3r$. Find the rates of change of the volume when (a) $r = 6$ inches and (b) $r = 24$ inches.

11. ***Cost, Revenue, and Profit*** A company that manufactures sport supplements calculates that its costs and revenue can be modeled by the equations

$$C = 125{,}000 + 0.75x \quad \text{and} \quad R = 250x - \frac{1}{10}x^2$$

where x is the number of units of sport supplements produced in 1 week. If production in one particular week is 1000 units and is increasing at a rate of 150 units per week, find:

(a) the rate at which the cost is changing.

(b) the rate at which the revenue is changing.

(c) the rate at which the profit is changing.

12. ***Cost, Revenue, and Profit*** A company that manufactures pet toys calculates that its costs and revenue can be modeled by the equations

$$C = 75{,}000 + 1.05x \quad \text{and} \quad R = 500x - \frac{x^2}{25}$$

where x is the number of toys produced in 1 week. If production in one particular week is 5000 toys and is increasing at a rate of 250 toys per week, find:

(a) the rate at which the cost is changing.

(b) the rate at which the revenue is changing.

(c) the rate at which the profit is changing.

13. ***Expanding Cube*** All
a rate of 3 centimeters pe
changing when each edg
centimeters?

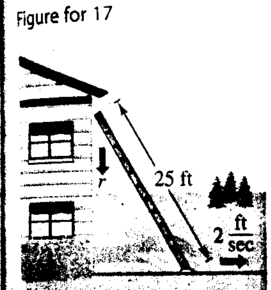
14. ***Expanding Cube*** All
at a rate of 3 centimeters p
area changing when eac
(b) 10 centimeters?

15. ***Moving Point*** A poi
$y = x^2$ such that dx/dt i
dy/dt for each value of x
(a) $x = -3$ (b) $x =$

16. ***Moving Point*** A poi
$y = 1/(1 + x^2)$ such that
Find dy/dt for each valu
(a) $x = -2$ (b) $x =$

17. ***Speed*** A 25-foot ladd
figure). The base of the
house at a rate of 2 feet
the ladder moving down
(b) 15 feet, and (c) 24 fe

Figure for 17



18. ***Speed*** A boat is pull
winch is 12 feet above th
winch pulls the rope at a
speed of the boat when
to the speed of the boat
dock?

19. ***Air Traffic Control***
airplanes at the same alt
fly at right angles to ea
from the point and has
other is 200 miles fro
600 miles per hour.

(a) At what rate is t
changing?

(b) How much time do
the airplanes on a d

13. ***Expanding Cube*** All edges of a cube are expanding at a rate of 3 centimeters per second. How fast is the volume changing when each edge is (a) 1 centimeter and (b) 10 centimeters?

14. ***Expanding Cube*** All edges of a cube are expanding at a rate of 3 centimeters per second. How fast is the surface area changing when each edge is (a) 1 centimeter and (b) 10 centimeters?

15. ***Moving Point*** A point is moving along the graph of $y = x^2$ such that dx/dt is 2 centimeters per minute. Find dy/dt for each value of x.

(a) $x = -3$ (b) $x = 0$ (c) $x = 1$ (d) $x = 3$

16. ***Moving Point*** A point is moving along the graph of $y = 1/(1 + x^2)$ such that dx/dt is 2 centimeters per minute. Find dy/dt for each value of x.

(a) $x = -2$ (b) $x = 2$ (c) $x = 0$ (d) $x = 10$

17. ***Speed*** A 25-foot ladder is leaning against a house (see figure). The base of the ladder is pulled away from the house at a rate of 2 feet per second. How fast is the top of the ladder moving down the wall when the base is (a) 7 feet, (b) 15 feet, and (c) 24 feet from the house?

Figure for 17

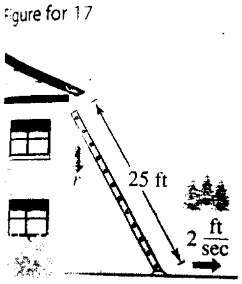
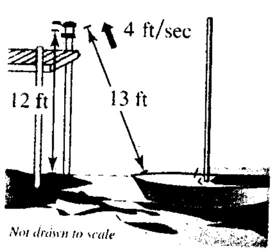


Figure for 18



18. ***Speed*** A boat is pulled by a winch on a dock, and the winch is 12 feet above the deck of the boat (see figure). The winch pulls the rope at a rate of 4 feet per second. Find the speed of the boat when 13 feet of rope is out. What happens to the speed of the boat as it gets closer and closer to the dock?

19. ***Air Traffic Control*** An air traffic controller spots two airplanes at the same altitude converging to a point as they fly at right angles to each other. One airplane is 150 miles from the point and has a speed of 450 miles per hour. The other is 200 miles from the point and has a speed of 600 miles per hour.

(a) At what rate is the distance between the planes changing?

(b) How much time does the controller have to get one of the airplanes on a different flight path?

20. ***Speed*** An airplane flying at an altitude of 6 miles passes directly over a radar antenna (see figure). When the airplane is 10 miles away ($s = 10$), the radar detects that the distance s is changing at a rate of 240 miles per hour. What is the speed of the airplane?

Figure for 20

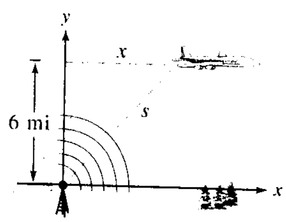
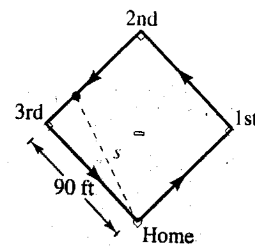


Figure for 21



21. ***Athletics*** A (square) baseball diamond has sides that are 90 feet long (see figure). A player 26 feet from third base is running at a speed of 30 feet per second. At what rate is the player's distance from home plate changing?

22. ***Advertising Costs*** A retail sporting goods store estimates that weekly sales S and weekly advertising costs x are related by the equation $S = 2250 + 50x + 0.35x^2$. The current weekly advertising costs are \$1500, and these costs are increasing at a rate of \$125 per week. Find the current rate of change of weekly sales.

23. ***Environment*** An accident at an oil drilling platform is causing a circular oil slick. The slick is 0.08 foot thick, and when the radius is 750 feet, the radius of the slick is increasing at the rate of 0.5 foot per minute. At what rate (in cubic feet per minute) is oil flowing from the site of the accident?

24. ***Profit*** A company is increasing the production of a product at the rate of 25 units per week. The demand and cost functions for the product are given by $p = 50 - 0.01x$ and $C = 4000 + 40x - 0.02x^2$. Find the rate of change of the profit with respect to time when the weekly sales are $x = 800$ units. Use a graphing utility to graph the profit function, and use the *zoom* and *trace* features of the graphing utility to verify your result.

25. ***Sales*** The profit for a product is increasing at a rate of \$6384 per week. The demand and cost functions for the product are given by $p = 6000 - 0.4x^2$ and $C = 2400x + 5200$. Find the rate of change of sales with respect to time when the weekly sales are $x = 44$ units.

26. ***Cost*** The annual cost (in millions of dollars) for a government agency to seize $p\%$ of an illegal drug is given by

$$C = \frac{528p}{100 - p}, \quad 0 \le p < 100.$$

The agency's goal is to increase p by 5% per year. Find the rates of change of the cost when (a) $p = 30\%$ and (b) $p = 60\%$. Use a graphing utility to graph C. What happens to the graph of C as p approaches 100?

73. $h(x) = [x^2(2x + 3)]^3$

74. $f(x) = [(x - 2)(x + 4)]^2$

75. $f(x) = x^2(x - 1)^5$

76. $f(s) = s^3(s^2 - 1)^{5/2}$

77. $h(t) = \dfrac{\sqrt{3t + 1}}{(1 - 3t)^2}$

78. $g(x) = \dfrac{(3x + 1)^2}{(x^2 + 1)^2}$

79. ***Physical Science*** The temperature T (in degrees Fahrenheit) of food placed in a freezer can be modeled by

$$T = \frac{1300}{t^2 + 2t + 25}$$

where t is the time (in hours).

(a) Find the rates of change of T when $t = 1$, $t = 3$, $t = 5$, and $t = 10$.

(b) Graph the model on a graphing utility and describe the rate at which the temperature is changing.

80. ***Forestry*** According to the *Doyle Log Rule*, the volume V (in board-feet) of a log of length L (feet) and diameter D (inches) at the small end is

$$V = \left(\frac{D - 4}{4}\right)^2 L.$$

Find the rates at which the volume is changing with respect to D for a 12-foot-long log whose smallest diameter is (a) 8 inches, (b) 16 inches, (c) 24 inches, and (d) 36 inches.

In Exercises 81–88, find the given derivative.

81. Given $f(x) = 3x^2 + 7x + 1$, find $f''(x)$.

82. Given $f'(x) = 5x^4 - 6x^2 + 2x$, find $f'''(x)$.

83. Given $f'''(x) = -\dfrac{6}{x^4}$, find $f^{(5)}(x)$.

84. Given $f(x) = \sqrt{x}$, find $f^{(4)}(x)$.

85. Given $f'(x) = 7x^{5/2}$, find $f''(x)$.

86. Given $f(x) = x^2 + \dfrac{3}{x}$, find $f''(x)$.

87. Given $f'(x) = 6\sqrt[3]{x}$, find $f'''(x)$.

88. Given $f''(x) = 20x^4 - \dfrac{2}{x^3}$, find $f^{(5)}(x)$.

89. ***Athletics*** A person dives from a 30-foot platform with an initial velocity of 5 feet per second (upward).

(a) Find the position function of the diver.

(b) How long will it take for the diver to hit the water?

(c) What is the diver's velocity at impact?

(d) What is the diver's acceleration at impact?

90. ***Velocity and Acceleration*** The position function of a particle is given by

$$s = \frac{1}{t^2 + 2t + 1}$$

where s is the height (in feet) and t is the time (in seconds). Find the velocity and acceleration functions.

In Exercises 91–94, use implicit differentiation to find dy/dx.

91. $x^2 + 3xy + y^3 = 10$

92. $x^2 + 9xy + y^2 = 0$

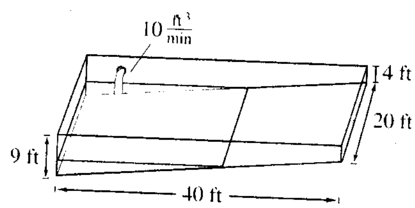
93. $y^2 - x^2 + 8x - 9y - 1 = 0$

94. $y^2 + x^2 - 6y - 2x - 5 = 0$

In Exercises 95–98, use implicit differentiation to find an equation of the tangent line at the given point.

	Equation	*Point*
95.	$y^2 = x - y$	$(2, 1)$
96.	$2\sqrt[3]{x} + 3\sqrt{y} = 10$	$(8, 4)$
97.	$y^2 - 2x = xy$	$(1, 2)$
98.	$y^3 - 2x^2y + 3xy^2 = -1$	$(0, -1)$

99. ***Water Level*** A swimming pool is 40 feet long, 20 feet wide, 4 feet deep at the shallow end, and 9 feet deep at the deep end (see figure). Water is being pumped into the pool at the rate of 10 cubic feet per minute. How fast is the water level rising when there is 4 feet of water in the deep end?



100. ***Profit*** The demand and cost functions for a product can be modeled by

$$p = 211 - 0.002x$$

and

$$C = 30x + 1{,}500{,}000$$

where x is the number of units produced.

(a) Write the profit function for this product.

(b) Find the marginal profit when 80,000 units are produced.

(c) Graph the profit function on a graphing utility and use the graph to determine the price you would charge for the product. Explain your reasoning.