PREREQUISITE REVIEW 3.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, solve the equation.

1. $x^2 = 8x$

2. $15x = \frac{5}{8}x^2$

3. $\frac{x^2 - 25}{x^3} = 0$

4. $\frac{2x}{\sqrt{1 - x^2}} = 0$

In Exercises 5–8, find the domain of the expression.

5. $\frac{x + 3}{x - 3}$

6. $\frac{2}{\sqrt{1 - x}}$

7. $\frac{2x + 1}{x^2 - 3x - 10}$

8. $\frac{3x}{\sqrt{9 - 3x^2}}$

In Exercises 9–12, evaluate the expression when $x = -2$, 0, and 2.

9. $-2(x + 1)(x - 1)$

10. $4(2x + 1)(2x - 1)$

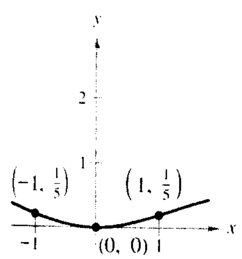
11. $\frac{2x + 1}{(x - 1)^2}$

12. $\frac{-2(x + 1)}{(x - 4)^2}$

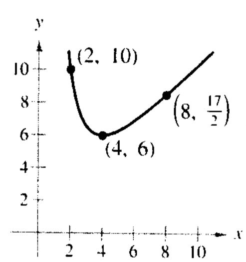
EXERCISES 3.1

In Exercises 1–4, evaluate the derivative of the function at the indicated points on the graph.

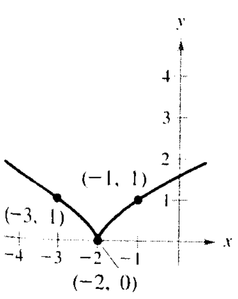
1. $f(x) = \frac{x^2}{x^2 + 4}$



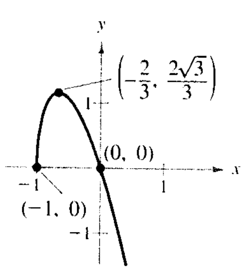
2. $f(x) = x + \frac{32}{x^2}$



3. $f(x) = (x + 2)^{2/3}$

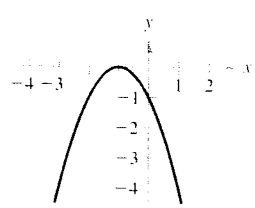


4. $f(x) = -3x\sqrt{x + 1}$

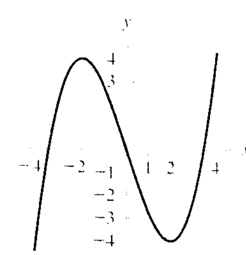


In Exercises 5–8, use the derivative to identify the open intervals on which the function is increasing or decreasing. Verify your result with the graph of the function.

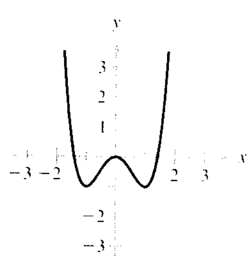
5. $f(x) = -(x + 1)^2$



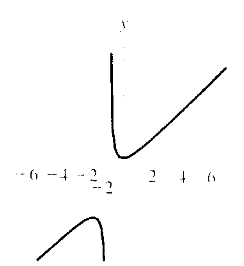
6. $f(x) = \frac{x^3}{4} - 3x$



7. $f(x) = x^4 - 2x^2$



8. $f(x) = \frac{x^2}{x + 1}$



In Exercises 9–18, find the critical numbers and the open intervals on which the function is increasing or decreasing. Sketch the graph of the function.

9. $f(x) = 2x - 3$ **10.** $f(x) = 5 - 3x$

11. $g(x) = -(x - 1)^2$ **12.** $g(x) = (x + 2)^2$

13. $y = x^2 - 5x$ **14.** $y = -x^2 + 2x$

15. $y = x^3 - 6x^2$ **16.** $y = (x - 2)^3$

17. $f(x) = \sqrt{x^2 - 1}$ **18.** $f(x) = \sqrt{4 - x^2}$

In Exercises 19–28, find the critical numbers and the open intervals on which the function is increasing or decreasing. Then use a graphing utility to graph the function.

19. $f(x) = -2x^2 + 4x + 3$ **20.** $f(x) = x^2 + 8x + 10$

21. $y = 3x^3 + 12x^2 + 15x$ **22.** $y = x^3 - 3x + 2$

23. $f(x) = x\sqrt{x + 1}$ **24.** $h(x) = x\sqrt[3]{x - 1}$

25. $f(x) = x^4 - 2x^3$ **26.** $f(x) = \frac{1}{4}x^4 - 2x^2$

27. $f(x) = \dfrac{x}{x^2 + 4}$ **28.** $f(x) = \dfrac{x^2}{x^2 + 4}$

In Exercises 29–34, find the critical numbers and the open intervals on which the function is increasing or decreasing. (*Hint:* Check for discontinuities.) Sketch the graph of the function.

29. $f(x) = \dfrac{2x}{16 - x^2}$

30. $f(x) = \dfrac{x}{x + 1}$

31. $y = \begin{cases} 4 - x^2, & x \le 0 \\ -2x, & x > 0 \end{cases}$

32. $y = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$

33. $y = \begin{cases} 3x + 1, & x \le 1 \\ 5 - x^2, & x > 1 \end{cases}$

34. $y = \begin{cases} -x^3 + 1, & x \le 0 \\ -x^2 + 2x, & x > 0 \end{cases}$

35. ***Cost*** The ordering and transportation cost C (in hundreds of dollars) for an automobile dealership is modeled by

$$C = 10\left(\frac{1}{x} + \frac{x}{x + 3}\right), \quad 1 \le x$$

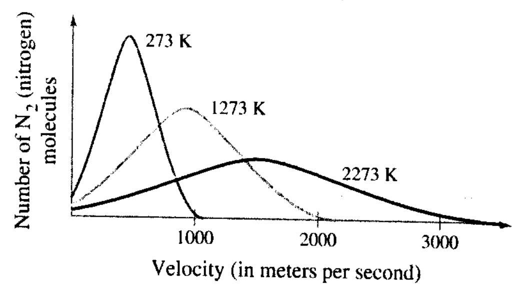
where x is the number of automobiles ordered.

(a) Find the intervals on which C is increasing or decreasing.

(b) Use a graphing utility to graph the cost function.

(c) Use the *trace* feature to determine the order sizes for which the cost is \$900. Assuming that the revenue function is increasing for $x \ge 0$, which order size would you use? Explain your reasoning.

36. ***Chemistry: Molecular Velocity*** Plots of the relative numbers of N_2 (nitrogen) molecules that have a given velocity at each of three temperatures (in degrees Kelvin) are shown in the figure. Identify the differences in the average velocities (indicated by the peaks of the curves) for the three temperatures, and describe the intervals on which the velocity is increasing and decreasing for each of the three temperatures. (*Source: Adapted from Zumdahl, Chemistry, Sixth Edition*)



Position Function In Exercises 37 and 38, the position function gives the height s (in feet) of a ball, where the time t is measured in seconds. Find the time interval on which the ball is rising and the interval on which it is falling.

37. $s = 96t - 16t^2, \quad 0 \le t \le 6$

38. $s = -16t^2 + 64t, \quad 0 \le t \le 4$

39. ***Law Degrees*** The number y of law degrees conferred in the United States from 1970 to 2000 can be modeled by

$$y = 2.743t^3 - 171.55t^2 + 3462.3t + 15{,}265, \quad 0 \le t \le 30$$

where t is the time in years, with $t = 0$ corresponding to 1970. (*Source: U.S. National Center for Education Statistics*)

(a) Use a graphing utility to graph the model. Then graphically estimate the years during which the model is increasing and the years during which it is decreasing.

(b) Use the test for increasing and decreasing functions to verify the result of part (a).

40. ***Profit*** The profit P made by a cinema from selling x bags of popcorn can be modeled by

$$P = 2.36x - \frac{x^2}{25{,}000} - 3500, \quad 0 \le x \le 50{,}000.$$

(a) Find the intervals on which P is increasing and decreasing.

(b) If you owned the cinema, what price would you charge to obtain a maximum profit for popcorn? Explain your reasoning.

3.2

Relative Extrema

You have used the deriv
increasing or decreasing
function changes from in
function has a **relative e**
ative extrema of a functi
the function. For instan
extrema—the left point
minimum.

Definition of Relati

Let f be a function d

1. $f(c)$ is a **relative**
containing c such

2. $f(c)$ is a **relative**
containing c such

If $f(c)$ is a relative e
occur at $x = c$.

For a continuous
numbers of the function

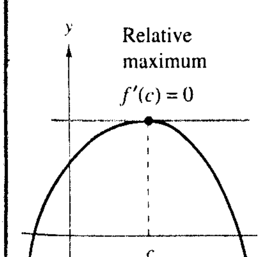


FIGURE 3.11

Occurrence of Rela

If f has a relative m
critical number of f.

PREREQUISITE REVIEW 3.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, solve the equation $f'(x) = 0$.

1. $f(x) = 4x^4 - 2x^2 + 1$

2. $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 10x$

3. $f(x) = 5x^{4/5} - 4x$

4. $f(x) = \frac{1}{2}x^2 - 3x^{5/3}$

5. $f(x) = \dfrac{x+4}{x^2+1}$

6. $f(x) = \dfrac{x-1}{x^2+4}$

In Exercises 7–10, use $g(x) = -x^5 - 2x^4 + 4x^3 + 2x - 1$ to determine the sign of the derivative.

7. $g'(-4)$

8. $g'(0)$

9. $g'(1)$

10. $g'(3)$

In Exercises 11 and 12, decide whether the function is increasing or decreasing on the given interval.

11. $f(x) = 2x^2 - 11x - 6,\quad (3, 6)$

12. $f(x) = x^3 + 2x^2 - 4x - 8,\quad (-2, 0)$

EXERCISES 3.2

In Exercises 1–4, use a table similar to that in Example 1 to find all relative extrema of the function.

1. $f(x) = -2x^2 + 4x + 3$

2. $f(x) = x^2 + 8x + 10$

3. $f(x) = x^2 - 6x$

4. $f(x) = -4x^2 + 4x + 1$

In Exercises 5–12, find all relative extrema of the function.

5. $g(x) = 6x^3 - 15x^2 + 12x$

6. $g(x) = \frac{1}{5}x^5 - x$

7. $h(x) = -(x + 4)^3$

8. $h(x) = 2(x - 3)^3$

9. $f(x) = x^3 - 6x^2 + 15$

10. $f(x) = x^4 - 32x + 4$

11. $f(x) = x^4 - 2x^3$

12. $f(x) = x^4 - 12x^3$

In Exercises 13–18, use a graphing utility to graph the function. Then find all relative extrema of the function.

13. $f(x) = (x - 1)^{2/3}$

14. $f(t) = (t - 1)^{1/3}$

15. $g(t) = t - \dfrac{1}{2t^2}$

16. $f(x) = x + \dfrac{1}{x}$

17. $f(x) = \dfrac{x}{x+1}$

18. $h(x) = \dfrac{4}{x^2+1}$

In Exercises 19–26, find the absolute extrema of the function on the closed interval.

	Function	Interval
19.	$f(x) = 2(3 - x)$	$[-1, 2]$
20.	$f(x) = \frac{1}{3}(2x + 5)$	$[0, 5]$
21.	$f(x) = 5 - 2x^2$	$[0, 3]$
22.	$f(x) = x^2 + 2x - 4$	$[-1, 1]$
23.	$f(x) = x^3 - 3x^2$	$[-1, 3]$
24.	$f(x) = x^3 - 12x$	$[0, 4]$
25.	$h(s) = \dfrac{1}{3-s}$	$[0, 2]$
26.	$h(t) = \dfrac{t}{t-2}$	$[3, 5]$

In Exercises 27–30, find the absolute extrema of the function on the closed interval. Use a graphing utility to verify your results.

	Function	Interval
27.	$f(x) = 3x^{2/3} - 2x$	$[-1, 2]$
28.	$g(t) = \dfrac{t^2}{t^2+3}$	$[-1, 1]$
29.	$h(t) = (t - 1)^{2/3}$	$[-7, 2]$
30.	$g(x) = 4\left(1 + \dfrac{1}{x} + \dfrac{1}{x^2}\right)$	$[-4, 5]$

In Exercises 31–34, use a graphing utility to find graphically the absolute extrema of the function on the closed interval.

	Function	Interval
31.	$f(x) = 0.4x^3 - 1.8x^2 + x - 3$	$[0, 5]$
32.	$f(x) = 3.2x^5 + 5x^3 - 3.5x$	$[0, 1]$
33.	$f(x) = \frac{4}{3}x\sqrt{3 - x}$	$[0, 3]$
34.	$f(x) = 4\sqrt{x} - 2x + 1$	$[0, 6]$

In Exercises 35–38, find the absolute extrema of the function on the interval $[0, \infty)$.

35. $f(x) = \dfrac{4x}{x^2 + 1}$

36. $f(x) = \dfrac{8}{x + 1}$

37. $f(x) = \dfrac{2x}{x^2 + 4}$

38. $f(x) = 8 - \dfrac{4x}{x^2 + 1}$

In Exercises 39 and 40, find the maximum value of $|f''(x)|$ on the closed interval. (You will use this skill in Section 6.5 to estimate the error in the Trapezoidal Rule.)

Function	*Interval*
39. $f(x) = x^3(3x^2 - 10)$	$[0, 1]$
40. $f(x) = \dfrac{1}{x^2 + 1}$	$[0, 3]$

In Exercises 41 and 42, find the maximum value of $|f^{(4)}(x)|$ on the closed interval. (You will use this skill in Section 6.5 to estimate the error in Simpson's Rule.) Use a graphing utility to verify your answer.

Function	*Interval*
41. $f(x) = 15x^4 - \left(\dfrac{2x - 1}{2}\right)^6$	$[0, 1]$
42. $f(x) = \dfrac{1}{x^2}$	$[1, 2]$

43. ***Cost*** A retailer has determined the cost C for ordering and storing x units of a product to be modeled by

$$C = 3x + \frac{20{,}000}{x}, \quad 0 < x \leq 200.$$

The delivery truck can bring at most 200 units per order. Find the order size that will minimize the cost. Use a graphing utility to verify your result.

44. ***Profit*** The quantity demanded x for a product is inversely proportional to the cube of the price p for $p > 1$. When the price is \$10 per unit, the quantity demanded is eight units. The initial cost is \$100 and the cost per unit is \$4. What price will yield a maximum profit?

45. ***Profit*** When soft drinks were sold for \$0.80 per can at football games, approximately 6000 cans were sold. When the price was raised to \$1.00 per can, the quantity demanded dropped to 5600. The initial cost is \$5000 and the cost per unit is \$0.40. Assuming that the demand function is linear, use the *table* feature of a graphing utility to determine the price that will yield a maximum profit.

46. ***Medical Science*** Coughing forces the trachea (windpipe) to contract, which in turn affects the velocity of the air through the trachea. The velocity of the air during coughing can be modeled by

$$v = k(R - r)r^2, \quad 0 \leq r < R$$

where k is a constant, R is the normal radius of the trachea, and r is the radius during coughing. What radius r will produce the maximum air velocity?

47. ***Population*** The resident population P (in millions) of the United States from 1790 to 2000 can be modeled by

$$P = 0.00000583t^3 + 0.005003t^2 + 0.13775t + 4.658, \quad -10 \leq t \leq 200$$

where $t = 0$ corresponds to 1800. *(Source: U.S. Census Bureau)*

(a) Make a conjecture about the maximum and minimum populations in the U.S. from 1790 to 2000.

(b) Analytically find the maximum and minimum populations over the interval.

(c) Write a brief paragraph comparing your conjecture with your results in part (b).

48. ***Biology: Fertility Rates*** The graph of the United States fertility rate shows the number of births per 1000 women in their lifetime according to the birth rate in that particular year. *(Source: U.S. National Center for Health Statistics)*

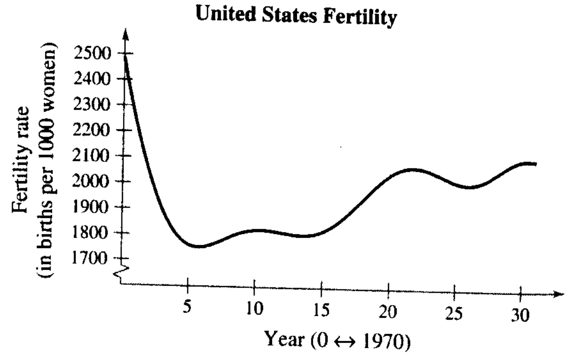
(a) Around what year was the fertility rate the highest, and to how many births per 1000 women did this rate correspond?

(b) During which time periods was the fertility rate increasing most rapidly? Most slowly?

(c) During which time periods was the fertility rate decreasing most rapidly? Most slowly?

(d) Give some possible real-life reasons for fluctuations in the fertility rate.

United States Fertility



3.3

Concavity

You already know that l… decreases is helpful in … locating the intervals on … graph of f is curving u… upward or downward is … function.

Definition of Conca…

Let f be differentiab…

1. **concave upward**

2. **concave downw…**

From Figure 3.20, … concavity.

1. A curve that is conc…

2. A curve that is conc…

This visual test for con… determine concavity wi… that you can use the se… same way that you use … increasing or decreasin…

Test for Concavity

Let f be a function …

1. If $f''(x) > 0$ for

2. If $f''(x) < 0$ for

PREREQUISITE REVIEW 3.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, find the second derivative of the function.

1. $f(x) = 4x^4 - 9x^3 + 5x - 1$
2. $g(s) = (s^2 - 1)(s^2 - 3s + 2)$
3. $g(x) = (x^2 + 1)^4$
4. $f(x) = (x - 3)^{4/3}$
5. $h(x) = \dfrac{4x + 3}{5x - 1}$
6. $f(x) = \dfrac{2x - 1}{3x + 2}$

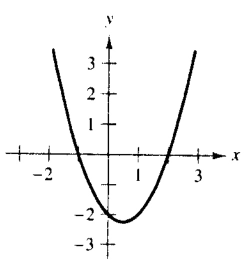
In Exercises 7–10, find the critical numbers of the function.

7. $f(x) = 5x^3 - 5x + 11$
8. $f(x) = x^4 - 4x^3 - 10$
9. $g(t) = \dfrac{16 + t^2}{t}$
10. $h(x) = \dfrac{x^4 - 50x^2}{8}$

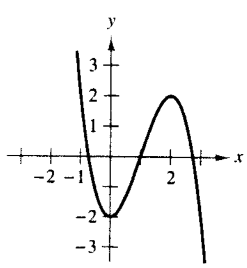
EXERCISES 3.3

In Exercises 1–8, analytically find the intervals on which the graph is concave upward and those on which it is concave downward. Verify your results using the graph of the function.

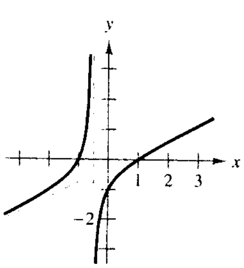
1. $y = x^2 - x - 2$



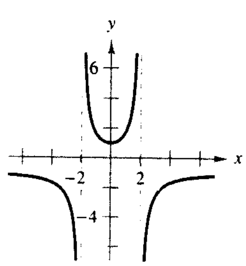
2. $y = -x^3 + 3x^2 - 2$



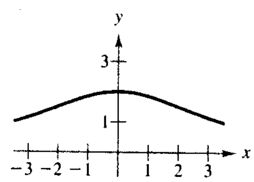
3. $f(x) = \dfrac{x^2 - 1}{2x + 1}$



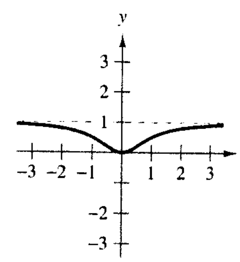
4. $f(x) = \dfrac{x^2 + 4}{4 - x^2}$



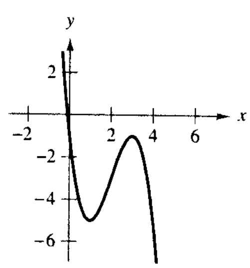
5. $f(x) = \dfrac{24}{x^2 + 12}$



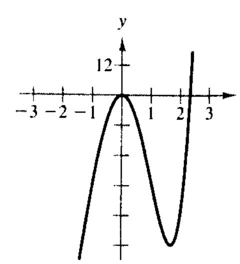
6. $f(x) = \dfrac{x^2}{x^2 + 1}$



7. $y = -x^3 + 6x^2 - 9x - 1$



8. $y = x^5 + 5x^4 - 40x^2$



In Exercises 9–18, find all re
the Second-Derivative Test v

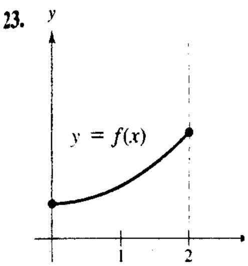
9. $f(x) = 6x - x^2$
11. $f(x) = x^3 - 5x^2 + 7x$
13. $f(x) = x^{2/3} - 3$
15. $f(x) = \sqrt{x^2 + 1}$
17. $f(x) = \dfrac{x}{x - 1}$

In Exercises 19–22, use a gr
all relative extrema of the fu

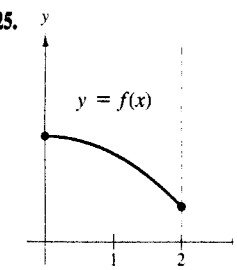
19. $f(x) = \frac{1}{2}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x$
20. $f(x) = -\frac{1}{3}x^5 - \frac{1}{2}x^4 + .$
21. $f(x) = 5 + 3x^2 - x^3$
22. $f(x) = 3x^3 + 5x^2 - 2$

In Exercises 23–26, state t
interval (0, 2).

23.



25.



In Exercises 27–34, find the
the function.

27. $f(x) = x^3 - 9x^2 + 24$
28. $f(x) = x(6 - x)^2$
29. $f(x) = (x - 1)^3(x - 5$
30. $f(x) = x^4 - 18x^2 + 5$
31. $g(x) = 2x^4 - 8x^3 + 1$
32. $f(x) = -4x^3 - 8x^2 +$
33. $h(x) = (x - 2)^3(x - 1$
34. $f(t) = (1 - t)(t - 4)($

In Exercises 9–18, find all relative extrema of the function. Use the Second-Derivative Test when applicable.

9. $f(x) = 6x - x^2$

10. $f(x) = (x - 5)^2$

11. $f(x) = x^3 - 5x^2 + 7x$

12. $f(x) = x^4 - 4x^3 + 2$

13. $f(x) = x^{2/3} - 3$

14. $f(x) = x + \dfrac{4}{x}$

15. $f(x) = \sqrt{x^2 + 1}$

16. $f(x) = \sqrt{4 - x^2}$

17. $f(x) = \dfrac{x}{x - 1}$

18. $f(x) = \dfrac{x}{x^2 - 1}$

In Exercises 19–22, use a graphing utility to estimate graphically all relative extrema of the function.

19. $f(x) = \frac{1}{2}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2$

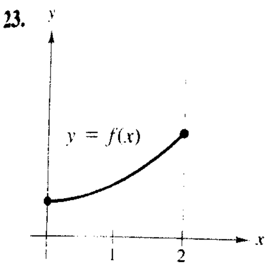
20. $f(x) = -\frac{1}{3}x^5 - \frac{1}{2}x^4 + x$

21. $f(x) = 5 + 3x^2 - x^3$

22. $f(x) = 3x^3 + 5x^2 - 2$

In Exercises 23–26, state the signs of $f'(x)$ and $f''(x)$ on the interval $(0, 2)$.

23.



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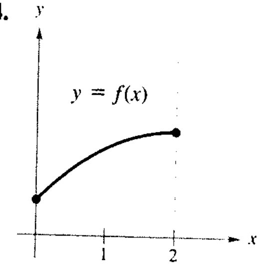

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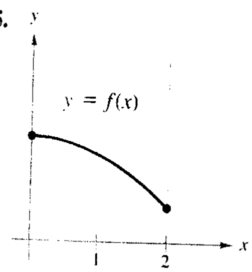

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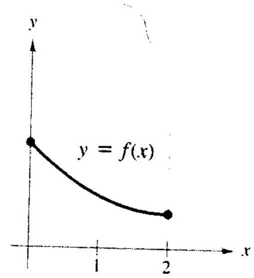


In Exercises 27–34, find the point(s) of inflection of the graph of the function.

27. $f(x) = x^3 - 9x^2 + 24x - 18$

28. $f(x) = x(6 - x)^2$

29. $f(x) = (x - 1)^3(x - 5)$

30. $f(x) = x^4 - 18x^2 + 5$

31. $g(x) = 2x^4 - 8x^3 + 12x^2 + 12x$

32. $f(x) = -4x^3 - 8x^2 + 32$

33. $h(x) = (x - 2)^3(x - 1)$

34. $f(t) = (1 - t)(t - 4)(t^2 - 4)$

In Exercises 35–46, use a graphing utility to graph the function and identify all relative extrema and points of inflection.

35. $f(x) = x^3 - 12x$

36. $f(x) = x^3 - 3x$

37. $f(x) = x^3 - 6x^2 + 12x$

38. $f(x) = x^3 - \frac{3}{2}x^2 - 6x$

39. $f(x) = \frac{1}{4}x^4 - 2x^2$

40. $f(x) = 2x^4 - 8x + 3$

41. $g(x) = (x - 2)(x + 1)^2$

42. $g(x) = (x - 6)(x + 2)^3$

43. $g(x) = x\sqrt{x + 3}$

44. $g(x) = x\sqrt{9 - x}$

45. $f(x) = \dfrac{4}{1 + x^2}$

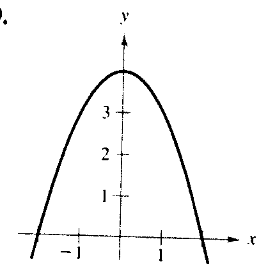
46. $f(x) = \dfrac{2}{x^2 - 1}$

In Exercises 47 and 48, sketch a graph of a function f having the given characteristics.

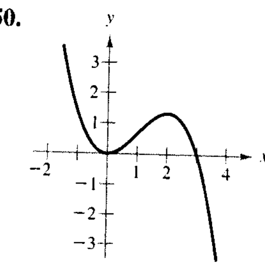
	Function	First Derivative	Second Derivative
47.	$f(2) = 0$	$f'(x) < 0,\ x < 3$	$f''(x) > 0$
	$f(4) = 0$	$f'(3) = 0$	
		$f'(x) > 0,\ x > 3$	
48.	$f(2) = 0$	$f'(x) > 0,\ x < 3$	$f''(x) > 0,\ x \neq 3$
	$f(4) = 0$	$f'(3)$ is undefined.	
		$f'(x) < 0,\ x > 3$	

In Exercises 49 and 50, use the graph to sketch the graph of f'. Find the intervals on which (a) $f'(x)$ is positive, (b) $f'(x)$ is negative, (c) f' is increasing, and (d) f' is decreasing. For each of these intervals, describe the corresponding behavior of f.

49.



50.



Point of Diminishing Returns In Exercises 51 and 52, identify the point of diminishing returns for the input-output function. For each function, R is the revenue and x is the amount spent on advertising. Use a graphing utility to verify your results.

51. $R = \dfrac{1}{50{,}000}(600x^2 - x^3), \qquad 0 \le x \le 400$

52. $R = -\frac{4}{9}(x^3 - 9x^2 - 27), \qquad 0 \le x \le 5$

Average Cost In Exercises 53 and 54, you are given the total cost of producing x units. Find the production level that minimizes the average cost per unit. Use a graphing utility to verify your results.

53. $C = 0.5x^2 + 15x + 5000$

54. $C = 0.002x^3 + 20x + 500$

Productivity In Exercises 55 and 56, consider a college student who works from 7 P.M. to 11 P.M. assembling mechanical components. The number N of components assembled after t hours is given by the function. At what time is the student assembling components at the greatest rate?

55. $N = -0.12t^3 + 0.54t^2 + 8.22t, \quad 0 \le t \le 4$

56. $N = \dfrac{20t^2}{4 + t^2}, \quad 0 \le t \le 4$

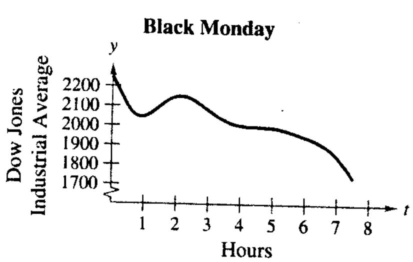
Sales Growth In Exercises 57 and 58, find the time t in years when the annual sales x of a new product are increasing at the greatest rate. Use a graphing utility to verify your results.

57. $x = \dfrac{10{,}000t^2}{9 + t^2}$ **58.** $x = \dfrac{500{,}000t^2}{36 + t^2}$

In Exercises 59–62, use a graphing utility to graph f, f', and f'' in the same viewing window. Graphically locate the relative extrema and points of inflection of the graph.

	Function	*Interval*
59.	$f(x) = \frac{1}{2}x^3 - x^2 + 3x - 5$	$[0, 3]$
60.	$f(x) = -\frac{1}{20}x^5 - \frac{1}{12}x^2 - \frac{1}{3}x + 1$	$[-2, 2]$
61.	$f(x) = \dfrac{2}{x^2 + 1}$	$[-3, 3]$
62.	$f(x) = \dfrac{x^2}{x^2 + 1}$	$[-3, 3]$

63. ***Dow Jones Industrial Average*** The graph shows the Dow Jones Industrial Average y on Black Monday, October 19, 1987, where $t = 0$ corresponds to 8:30 A.M., when the market opens, and $t = 7.5$ corresponds to 4 P.M., the closing time. *(Source: Wall Street Journal)*



(a) Estimate the relative extrema and absolute extrema of the graph. Interpret your results in the context of the problem.

(b) Estimate the point of inflection of the graph on the interval $[3, 6]$. Interpret your result in the context of the problem.

64. ***Think About It*** Let S represent monthly sales of a new model of MP3 player. Write a statement describing S' and S'' for each of the following.

(a) The rate of change of sales is increasing.

(b) Sales are increasing, but at a greater rate.

(c) The rate of change of sales is steady.

(d) Sales are steady.

(e) Sales are declining, but at a lower rate.

(f) Sales have bottomed out and have begun to rise.

65. ***Medicine*** The spread of a virus can be modeled by

$$N = -t^3 + 12t^2, \quad 0 \le t \le 12$$

where N is the number of people infected in hundreds, and t is the time in weeks.

(a) What is the maximum number of people projected to be infected?

(b) When will the virus be spreading most rapidly?

(c) Use a graphing utility to verify your results.

BUSINESS CAPSULE

Gordon Weinberger won a baking contest two years in a row and decided to start his own pie making company. He raised money and opened Gordon's Top of the Tree Baking Company in Londonderry, NH in 1994. His pies sold well in small stores, but not in the larger markets. He was in debt, so he painted a schoolbus in psychedelic patterns and drove 1500 miles a week to visit as many supermarkets as he could. He eventually found a buyer who ordered 40 tractor-trailers full, which meant \$1 million in sales. Gordon's yearly sales hit \$5 million. In 2002, he sold the company to Mrs. Smith's Bakeries.

66. ***Research Project*** Use your school's library, the Internet, or some other reference source to research the financial history of a small company like the one above. Gather the data on the company's costs and revenues over a period of time, and use a graphing utility to graph a scatter plot of the data. Fit models to the data. Do the models appear to be concave upward or downward? Do they appear to be increasing or decreasing? Discuss the implications of your answers.

3.4

Solving Optimiz

One of the most con
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solving optimization pı

EXAMPLE 1

A manufacturer wants
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SOLUTION Because t

$V = x^2h.$

This equation is called
quantity to be optimize

$S =$ (area of ba

$108 = x^2 + 4xh.$

Because V is to be op
variable. To do this, so

$h = \dfrac{108 - x^2}{4x}$

and substitute into the

$V = x^2h = x^2\Big(\dfrac{10}{}$

Before finding which
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in the problem? Becau
is at most 108, you ca

$0 \le x \le \sqrt{108}.$

Using the techniques c
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maximum when $x = 6$

TRY IT 1

Use a graphing utilit
$0 \le x \le \sqrt{108}$ from
maximum when $x =$

PREREQUISITE REVIEW 3.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, write a formula for the written statement.

1. The sum of one number and half a second number is 12.

2. The product of one number and twice another is 24.

3. The area of a rectangle is 24 square units.

4. The distance between two points is 10 units.

In Exercises 5–10, find the critical numbers of the function.

5. $y = x^2 + 6x - 9$ **6.** $y = 2x^3 - x^2 - 4x$ **7.** $y = 5x + \dfrac{125}{x}$

8. $y = 3x + \dfrac{96}{x^2}$ **9.** $y = \dfrac{x^2 + 1}{x}$ **10.** $y = \dfrac{x}{x^2 + 9}$

EXERCISES 3.4

In Exercises 1–6, find two positive numbers satisfying the given requirements.

1. The sum is 110 and the product is a maximum.

2. The sum is S and the product is a maximum.

3. The sum of the first and twice the second is 36 and the product is a maximum.

4. The sum of the first and twice the second is 100 and the product is a maximum.

5. The product is 192 and the sum is a minimum.

6. The product is 192 and the sum of the first plus three times the second is a minimum.

7. What positive number x minimizes the sum of x and its reciprocal?

8. The difference of two numbers is 50. Find the two numbers such that their product is a minimum.

In Exercises 9 and 10, find the length and width of a rectangle that has the given perimeter and a maximum area.

9. Perimeter: 100 meters **10.** Perimeter: P units

In Exercises 11 and 12, find the length and width of the rectangle that has the given area and a minimum perimeter.

11. Area: 64 square feet **12.** Area: A square centimeters

13. ***Maximum Area*** A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?

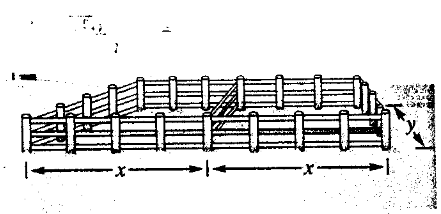


Figure for 13

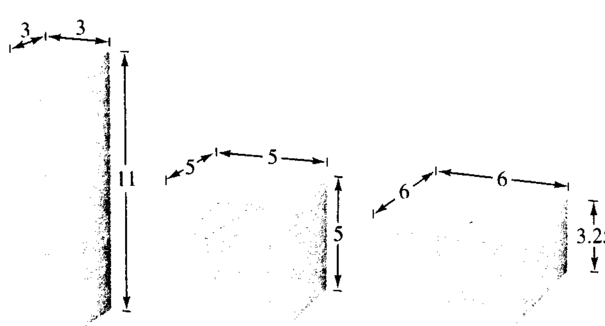
14. ***Area*** A dairy farmer plans to enclose a rectangular pasture adjacent to a river. To provide enough grass for the herd, the pasture must contain 180,000 square meters. No fencing is required along the river. What dimensions will use the smallest amount of fencing?

15. ***Maximum Volume***

(a) Verify that each of the rectangular solids shown in the figure has a surface area of 150 square inches.

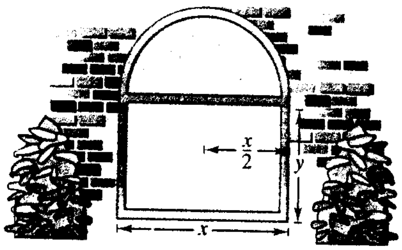
(b) Find the volume of each solid.

(c) Determine the dimensions of a rectangular solid (with a square base) of maximum volume if its surface area is 150 square inches.

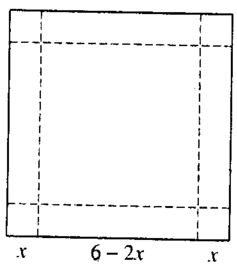


16. *Maximum Volume* Determine the dimensions of a rectangular solid (with a square base) with maximum volume if its surface area is 337.5 square centimeters.

17. *Maximum Area* A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see figure). Find the dimensions of a Norman window of maximum area if the total perimeter is 16 feet.

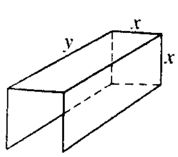


18. *Volume* An open box is to be made from a six-inch by six-inch square piece of material by cutting equal squares from the corners and turning up the sides (see figure). Find the volume of the largest box that can be made.



19. *Volume* An open box is to be made from a two-foot by three-foot rectangular piece of material by cutting equal squares from the corners and turning up the sides. Find the volume of the largest box that can be made in this manner.

20. *Minimum Surface Area* A net enclosure for golf practice is open at one end (see figure). The volume of the enclosure is $83\frac{1}{3}$ cubic meters. Find the dimensions that require the smallest amount of netting.



21. *Gardening* A home gardener estimates that if she plants 16 apple trees, the average yield will be 80 apples per tree. But because of the size of the garden, for each additional tree planted the yield will decrease by four apples per tree. How many trees should be planted to maximize the total yield of apples? What is the maximum yield?

22. *Area* A rectangular page is to contain 36 square inches of print. The margins at the top and bottom and on each side are to be $1\frac{1}{2}$ inches. Find the dimensions of the page that will minimize the amount of paper used.

23. *Area* A rectangular page is to contain 30 square inches of print. The margins at the top and bottom of the page are to be 2 inches wide. The margins on each side are to be 1 inch wide. Find the dimensions of the page such that the least amount of paper is used.

24. *Maximum Area* A rectangle is bounded by the x- and y-axes and the graph of

$$y = \frac{6 - x}{2}$$

(see figure). What length and width should the rectangle have so that its area is a maximum?

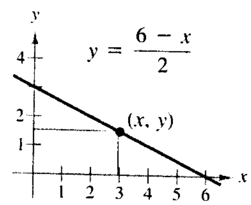


Figure for 24

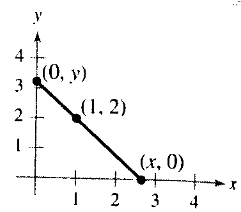


Figure for 25

25. *Minimum Length* A right triangle is formed in the first quadrant by the x- and y-axes and a line through the point $(1, 2)$ (see figure).

(a) Write the length L of the hypotenuse as a function of x.

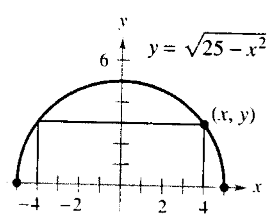
(b) Use a graphing utility to approximate x graphically such that the length of the hypotenuse is a minimum.

(c) Find the vertices of the triangle such that its area is a minimum.

26. *Maximum Area* A rectangle is bounded by the x-axis and the semicircle

$$y = \sqrt{25 - x^2}$$

(see figure). What length and width should the rectangle have so that its area is a maximum?



27. *Area* Find the dimensions of the largest rectangle that can be inscribed in a semicircle of radius r. (See Exercise 26.)

28. *Volume* You are designing a soft drink container that has the shape of a right circular cylinder. The container is supposed to hold 12 fluid ounces (1 fluid ounce is approximately 1.80469 cubic inches). Find the dimensions that will use a minimum amount of construction material.

29. *Volume* Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius r (see figure on next page).

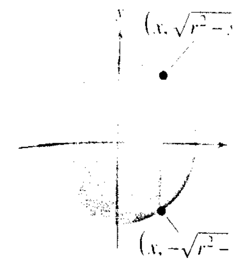


Figure for 29

30. *Maximum Volume*
circular cone that can t

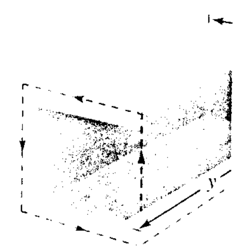
In Exercises 31 and 32, fir
function that are closest to t

Function

31. $f(x) = x^2 + 1$

32. $f(x) = x^2$

33. *Maximum Volume*
a postal service can ha
girth of 108 inches. F
with maximum volu
dimensions are x by x l



34. *Minimum Surface A*
two hemispheres to the
total volume of the soli
of the cylinder that pro

35. *Minimum Area* T
and a square is 16. Fi
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36. *Minimum Area* T
eral triangle and a squ
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37. *Minimum Time* Y
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3 miles down the coas
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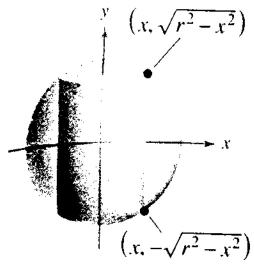


Figure for 29

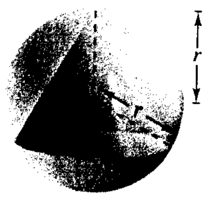


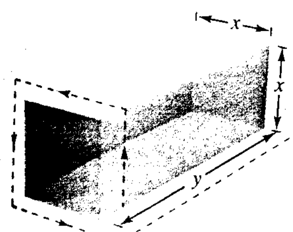
Figure for 30

30. *Maximum Volume* Find the volume of the largest right circular cone that can be inscribed in a sphere of radius r.

In Exercises 31 and 32, find the points on the graph of the function that are closest to the given point.

Function	*Point*
31. $f(x) = x^2 + 1$	$(0, 4)$
32. $f(x) = x^2$	$(2, \frac{1}{2})$

33. *Maximum Volume* A rectangular package to be sent by a postal service can have a maximum combined length and girth of 108 inches. Find the dimensions of the package with maximum volume. Assume that the package's dimensions are x by x by y (see figure).



34. *Minimum Surface Area* A solid is formed by adjoining two hemispheres to the ends of a right circular cylinder. The total volume of the solid is 12 cubic inches. Find the radius of the cylinder that produces the minimum surface area.

35. *Minimum Area* The combined perimeter of a circle and a square is 16. Find the dimensions of the circle and square that produce a minimum total area.

36. *Minimum Area* The combined perimeter of an equilateral triangle and a square is 10. Find the dimensions of the triangle and square that produce a minimum total area.

37. *Minimum Time* You are in a boat 2 miles from the nearest point on the coast. You are to go to point Q, located 3 miles down the coast and 1 mile inland (see figure). You can row at a rate of 2 miles per hour and you can walk at a rate of 4 miles per hour. Toward what point on the coast should you row in order to reach point Q in the least time?

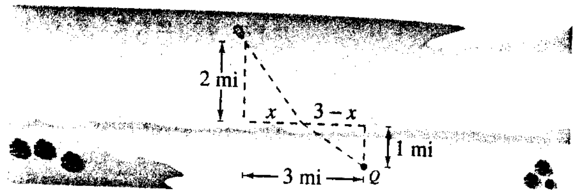
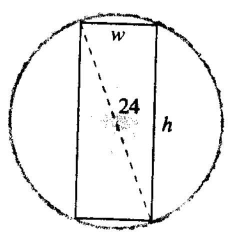


Figure for 37

38. *Maximum Area* An indoor physical fitness room consists of a rectangular region with a semicircle on each end. The perimeter of the room is to be a 200-meter running track. Find the dimensions that will make the area of the rectangular region as large as possible.

39. *Farming* A strawberry farmer will receive \$4 per bushel of strawberries during the first week of harvesting. Each week after that, the value will drop \$0.10 per bushel. The farmer estimates that there are approximately 120 bushels of strawberries in the fields, and that the crop is increasing at a rate of four bushels per week. When should the farmer harvest the strawberries to maximize their value? How many bushels of strawberries will yield the maximum value? What is the maximum value of the strawberries?

40. *Beam Strength* A wooden beam has a rectangular cross section of height h and width w (see figure). The strength S of the beam is directly proportional to its width and the square of its height. What are the dimensions of the strongest beam that can be cut from a round log of diameter 24 inches? (*Hint:* $S = kh^2w$, where k is the proportionality constant.)



41. *Maximum Area* Use a graphing utility to graph the primary equation and its first derivative to find the dimensions of the rectangle of maximum area that can be inscribed in a semicircle of radius 10.

42. *Area* Four feet of wire is to be used to form a square and a circle.

(a) Express the sum of the areas of the square and the circle as a function A of the side of the square x.

(b) What is the domain of A?

(c) Use a graphing utility to graph A on its domain.

(d) How much wire should be used for the square and how much for the circle in order to enclose the smallest total area? the greatest total area?

PREREQUISITE REVIEW 3.5

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, evaluate the expression for $x = 150$.

1. $\left|-\frac{300}{x} + 3\right|$
2. $\left|-\frac{600}{5x} + 2\right|$
3. $\left|\frac{(20x^{-1/2})/x}{-10x^{-3/2}}\right|$
4. $\left|\frac{(4000/x^2)/x}{-8000x^{-3}}\right|$

In Exercises 5–10, find the marginal revenue, marginal cost, or marginal profit.

5. $C = 650 + 1.2x + 0.003x^2$
6. $P = 0.01x^2 + 11x$
7. $R = 14x - \frac{x^2}{2000}$
8. $R = 3.4x - \frac{x^2}{1500}$
9. $P = -0.7x^2 + 7x - 50$
10. $C = 1700 + 4.2x + 0.001x^3$

EXERCISES 3.5

In Exercises 1–4, find the number of units x that produces a maximum revenue R.

1. $R = 800x - 0.2x^2$
2. $R = 48x^2 - 0.02x^3$
3. $R = 400x - x^2$
4. $R = 30x^{2/3} - 2x$

In Exercises 5–8, find the number of units x that produces the minimum average cost per unit $\bar{C}$.

5. $C = 1.25x^2 + 25x + 8000$
6. $C = 0.001x^3 + 5x + 250$
7. $C = 2x^2 + 255x + 5000$
8. $C = 0.02x^3 + 55x^2 + 1250$

In Exercises 9–12, find the price per unit p that produces the maximum profit P.

	Cost Function	*Demand Function*
9.	$C = 100 + 30x$	$p = 90 - x$
10.	$C = 0.5x + 600$	$p = \frac{60}{\sqrt{x}}$
11.	$C = 8000 + 50x + 0.03x^2$	$p = 70 - 0.001x$
12.	$C = 35x + 500$	$p = 50 - 0.1\sqrt{x}$

Average Cost In Exercises 13 and 14, use the cost function to find the production level for which the average cost is a minimum. For this production level, show that the marginal cost and average cost are equal. Use a graphing utility to graph the average cost function and verify your results.

13. $C = 2x^2 + 5x + 18$
14. $C = x^3 - 6x^2 + 13x$

15. ***Maximum Profit*** A commodity has a demand function modeled by

$$p = 100 - 0.5x^2$$

and a total cost function modeled by $C = 40x + 37.5$.

(a) What price yields a maximum profit?

(b) When the profit is maximized, what is the average cost per unit?

16. ***Maximum Profit*** How would the answer to Exercise 15 change if the marginal cost rose from \$40 per unit to \$50 per unit? In other words, rework Exercise 15 using the cost function $C = 50x + 37.5$.

Maximum Profit In Exercises 17 and 18, find the amount s of advertising that maximizes the profit P. (s and P are measured in thousands of dollars.) Find the point of diminishing returns.

17. $P = -2s^3 + 35s^2 - 100s + 200$
18. $P = -0.1s^3 + 6s^2 + 400$

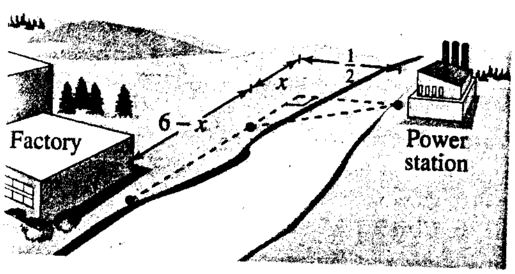
19. ***Maximum Profit*** The cost per unit of producing a type of radio is \$60. The manufacturer charges \$90 per unit for orders of 100 or less. To encourage large orders, however, the manufacturer reduces the charge by \$0.10 per radio for each order in excess of 100 units. For instance, an order of 101 radios would be \$89.90 per radio, an order of 102 radios would be \$89.80 per radio, and so on. Find the largest order the manufacturer should allow to obtain a maximum profit.

20. *Maximum Profit* A real estate office handles a 50-unit apartment complex. When the rent is \$580 per month, all units are occupied. For each \$40 increase in rent, however, an average of one unit becomes vacant. Each occupied unit requires an average of \$45 per month for service and repairs. What rent should be charged to obtain a maximum profit?

21. *Maximum Revenue* When a wholesaler sold a product at \$40 per unit, sales were 300 units per week. After a price increase of \$5, however, the average number of units sold dropped to 275 per week. Assuming that the demand function is linear, what price per unit will yield a maximum total revenue?

22. *Maximum Profit* Assume that the amount of money deposited in a bank is proportional to the square of the interest rate the bank pays on the money. Furthermore, the bank can reinvest the money at 12% simple interest. Find the interest rate the bank should pay to maximize its profit.

23. *Minimum Cost* A power station is on one side of a river that is 0.5 mile wide, and a factory is 6 miles downstream on the other side of the river (see figure). It costs \$6 per foot to run overland power lines and \$8 per foot to run underwater power lines. Write a cost function for running the power lines from the power station to the factory. Use a graphing utility to graph your function. Estimate the value of x that minimizes the cost. Explain your results.



24. *Minimum Cost* An offshore oil well is 1 mile off the coast. The oil refinery is 2 miles down the coast. Laying pipe in the ocean is twice as expensive as laying it on land. Find the most economical path for the pipe from the well to the oil refinery.

25. *Minimum Cost* A small business uses a minivan to make deliveries. The cost per hour for fuel is $C = v^2/600$, where v is the speed of the minivan (in miles per hour). The driver is paid \$10 per hour. Find the speed that minimizes the cost of a 110-mile trip. (Assume there are no costs other than fuel and wages.)

26. *Minimum Cost* Repeat Exercise 25 for a fuel cost per hour of

$$C = \frac{v^2 + 360}{720}$$

and a wage of \$8 per hour.

Elasticity In Exercises 27–32, find the price elasticity of demand for the demand function at the indicated x-value. Is the demand elastic, inelastic, or of unit elasticity at the indicated x-value? Use a graphing utility to graph the revenue function, and identify the intervals of elasticity and inelasticity.

	Demand Function	*Quantity Demanded*
27.	$p = 400 - 3x$	$x = 20$
28.	$p = 5 - 0.03x$	$x = 100$
29.	$p = 20 - 0.0002x$	$x = 30$
30.	$p = \dfrac{500}{x + 2}$	$x = 23$
31.	$p = \dfrac{100}{x^2} + 2$	$x = 10$
32.	$p = 100 - \sqrt{0.2x}$	$x = 125$

33. *Elasticity* The demand function for a product is given by $x = p^2 - 20p + 100$.

(a) Consider a price of \$2. If the price increases by 5%, what is the corresponding percent change in the quantity demanded?

(b) Average elasticity of demand is defined to be the percent change in quantity divided by the percent change in price. Use the percent in part (a) to find the average elasticity over the interval $[2, 2.1]$.

(c) Find the elasticity for a price of \$2 and compare the result with that in part (b).

(d) Find an expression for the total revenue and find the values of x and p that maximize the total revenue.

34. *Elasticity* The demand function for a product is given by $p^3 + x^3 = 9$.

(a) Find the price elasticity of demand when $x = 2$.

(b) Find the values of x and p that maximize the total revenue.

(c) For the value of x found in part (b), show that the price elasticity of demand has unit elasticity.

35. *Elasticity* The demand function for a product is given by $p = 20 - 0.02x, \quad 0 < x < 1000$.

(a) Find the price elasticity of demand when $x = 560$.

(b) Find the values of x and p that maximize the total revenue.

(c) For the value of x found in part (b), show that the price elasticity of demand has unit elasticity.

36. *Minimum Cost* Tl
of the components ι
modeled by

$C = 100\left(\frac{200}{x^2} + \right.$

where C is measured
order size in hundred
the cost. (*Hint:* Use tl

37. *Revenue* The dem

$x = 600 - 50p$

where the current pri
by lowering the price
Use price elasticity o

38. *Revenue* Repeat E

$x = 800 - 40p.$

39. *Demand* A demai
where a is a constar
other words, show th
$m\%$ decrease in the c

40. *Sales* The sales S
Lowe's for the years

$S = 201.556t^2 -$

$4 \le t \le 13$

where $t = 4$ corre

(a) During which y
sales increasing

(b) During which
lowest rate?

(c) Find the rate of
parts (a) and (b)

(d) Use a graphing
use the *zoom* ar
in parts (a), (b),

41. *Revenue* The rev
for Papa John's for
modeled by

$R = \frac{-18.0 +}{1 - 0.16t}$

where $t = 4$ corres

(a) During which y
revenue the gre

(b) During which
greatest rate? d

(c) Use a graphing
and confirm yo

36. ***Minimum Cost*** The ordering and transportation cost C of the components used in manufacturing a product is modeled by

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x + 30}\right), \quad x \geq 1$$

where C is measured in thousands of dollars and x is the order size in hundreds. Find the order size that minimizes the cost. (*Hint:* Use the *root* feature of a graphing utility.)

37. ***Revenue*** The demand for a car wash is

$$x = 600 - 50p$$

where the current price is \$5.00. Can revenue be increased by lowering the price and thus attracting more customers? Use price elasticity of demand to determine your answer.

38. ***Revenue*** Repeat Exercise 37 for a demand function of $x = 800 - 40p$.

39. ***Demand*** A demand function is modeled by $x = a/p^m$, where a is a constant and $m > 1$. Show that $\eta = -m$. In other words, show that a 1% increase in price results in an m% decrease in the quantity demanded.

40. ***Sales*** The sales S (in millions of dollars per year) for Lowe's for the years 1994 through 2003 can be modeled by

$$S = 201.556t^2 - 502.29t + 2622.8 + \frac{9286}{t},$$
$$4 \leq t \leq 13$$

where $t = 4$ corresponds to 1994.

(a) During which year, from 1994 to 2003, were Lowe's sales increasing most rapidly?

(b) During which year were the sales increasing at the lowest rate?

(c) Find the rate of increase or decrease for each year in parts (a) and (b).

(d) Use a graphing utility to graph the sales function. Then use the *zoom* and *trace* features to confirm the results in parts (a), (b), and (c).

41. ***Revenue*** The revenue R (in millions of dollars per year) for Papa John's for the years 1994 through 2003 can be modeled by

$$R = \frac{-18.0 + 24.74t}{1 - 0.16t + 0.008t^2}, \quad 4 \leq t \leq 13$$

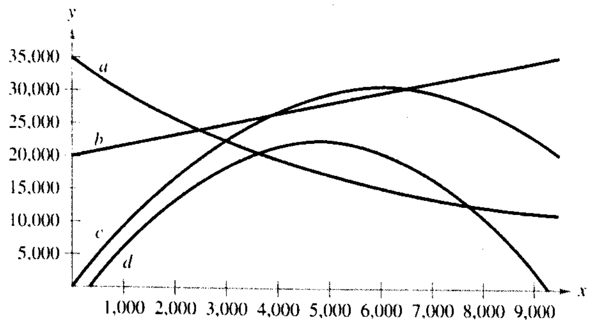
where $t = 4$ corresponds to 1994.

(a) During which year, from 1994 to 2003, was Papa John's revenue the greatest? the least?

(b) During which year was the revenue increasing at the greatest rate? decreasing at the greatest rate?

(c) Use a graphing utility to graph the revenue function, and confirm your results in parts (a) and (b).

42. Match each graph with the function it best represents—a demand function, a revenue function, a cost function, or a profit function. Explain your reasoning. (The graphs are labeled a–d.)



BUSINESS CAPSULE

Courtesy of Transperfect Translations

While graduate students, Elizabeth Elting and Phil Shawe co-founded TransPerfect Translations in 1992. They used a rented computer and a \$5000 credit card cash advance to market their service-oriented translation firm, now one of the largest in the country. Currently, they have a network of 4000 certified language specialists in North America, Europe, and Asia, which translates technical, legal, business, and marketing materials. In 2004, the company estimates its gross sales will be \$35 million.

43. ***Research Project*** Choose an innovative product like the one described above. Use your school's library, the Internet, or some other reference source to research the history of the product or service. Collect data about the revenue that the product or service has generated, and find a mathematical model of the data. Summarize your findings.

PREREQUISITE REVIEW 3.6

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–8, find the limit.

1. $\lim_{x \to 2} (x + 1)$

2. $\lim_{x \to -1} (3x + 4)$

3. $\lim_{x \to -3} \dfrac{2x^2 + x - 15}{x + 3}$

4. $\lim_{x \to 2} \dfrac{3x^2 - 8x + 4}{x - 2}$

5. $\lim_{x \to 2^+} \dfrac{x^2 - 5x + 6}{x^2 - 4}$

6. $\lim_{x \to 1^-} \dfrac{x^2 - 6x + 5}{x^2 - 1}$

7. $\lim_{x \to 0^+} \sqrt{x}$

8. $\lim_{x \to 1^+} \left(x + \sqrt{x - 1}\right)$

In Exercises 9–12, find the average cost and the marginal cost.

9. $C = 150 + 3x$

10. $C = 1900 + 1.7x + 0.002x^2$

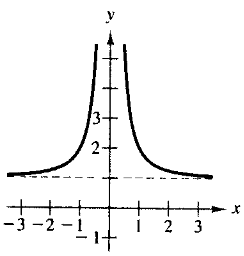
11. $C = 0.005x^2 + 0.5x + 1375$

12. $C = 760 + 0.05x$

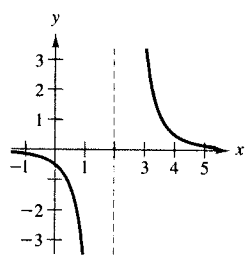
EXERCISES 3.6

In Exercises 1–8, find the vertical and horizontal asymptotes. Write the asymptotes as equations of lines.

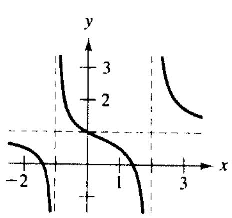
1. $f(x) = \dfrac{x^2 + 1}{x^2}$



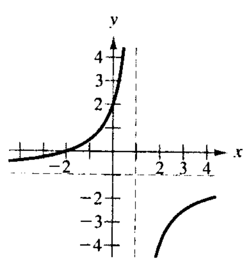
2. $f(x) = \dfrac{4}{(x - 2)^3}$



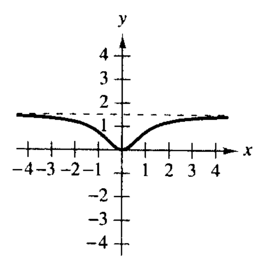
3. $f(x) = \dfrac{x^2 - 2}{x^2 - x - 2}$



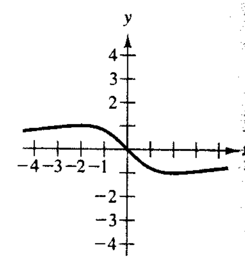
4. $f(x) = \dfrac{2 + x}{1 - x}$



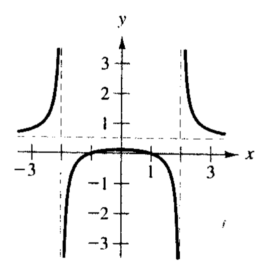
5. $f(x) = \dfrac{3x^2}{2(x^2 + 1)}$



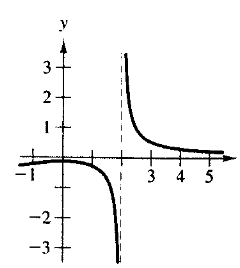
6. $f(x) = \dfrac{-4x}{x^2 + 4}$



7. $f(x) = \dfrac{x^2 - 1}{2x^2 - 8}$

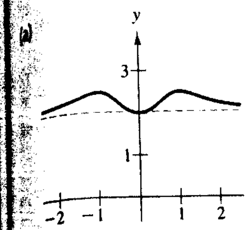


8. $f(x) = \dfrac{x^2 + 1}{x^3 - 8}$

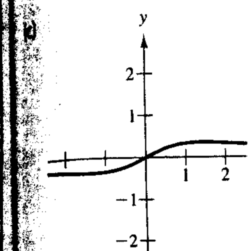


In Exercises 9–14, matcl
horizontal asymptotes as a

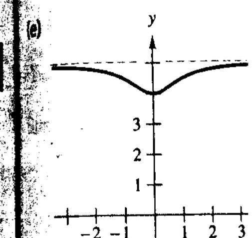
(a)



(c)



(e)



9. $f(x) = \dfrac{3x^2}{x^2 + 2}$

11. $f(x) = \dfrac{x}{x^2 + 2}$

13. $f(x) = 5 - \dfrac{1}{x^2 + 1}$

In Exercises 15–22, find tl

15. $\lim_{x \to -2^-} \dfrac{1}{(x + 2)^2}$

17. $\lim_{x \to 3^+} \dfrac{x - 4}{x - 3}$

19. $\lim_{x \to 4^-} \dfrac{x^2}{x^2 - 16}$

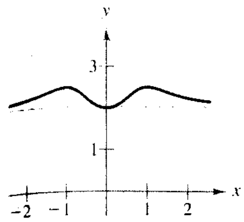
21. $\lim_{x \to 0^-} \left(1 + \dfrac{1}{x}\right)$

In Exercises 23–32, find t

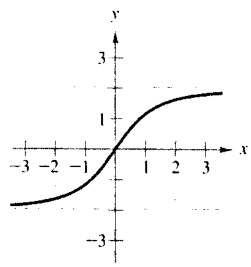
23. $\lim_{x \to \infty} \dfrac{2x - 1}{3x + 2}$

In Exercises 9–14, match the function with its graph. Use horizontal asymptotes as an aid. [The graphs are labeled (a)–(f).]

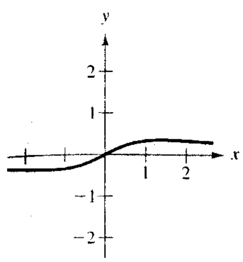
(a)



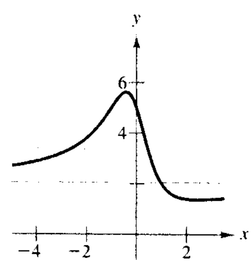
(b)



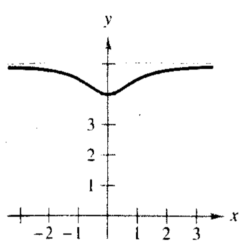
(c)



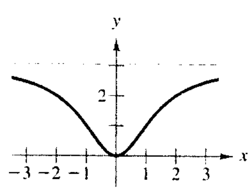
(d)



(e)



(f)



9. $f(x) = \dfrac{3x^2}{x^2 + 2}$

10. $f(x) = \dfrac{2x}{\sqrt{x^2 + 2}}$

11. $f(x) = \dfrac{x}{x^2 + 2}$

12. $f(x) = 2 + \dfrac{x^2}{x^4 + 1}$

13. $f(x) = 5 - \dfrac{1}{x^2 + 1}$

14. $f(x) = \dfrac{2x^2 - 3x + 5}{x^2 + 1}$

In Exercises 15–22, find the limit.

15. $\displaystyle\lim_{x \to -2^-} \frac{1}{(x + 2)^2}$

16. $\displaystyle\lim_{x \to -2^-} \frac{1}{x + 2}$

17. $\displaystyle\lim_{x \to 3^+} \frac{x - 4}{x - 3}$

18. $\displaystyle\lim_{x \to 1^+} \frac{2 + x}{1 - x}$

19. $\displaystyle\lim_{x \to 4^-} \frac{x^2}{x^2 - 16}$

20. $\displaystyle\lim_{x \to 4} \frac{x^2}{x^2 + 16}$

21. $\displaystyle\lim_{x \to 0^-} \left(1 + \frac{1}{x}\right)$

22. $\displaystyle\lim_{x \to 0^-} \left(x^2 - \frac{1}{x}\right)$

In Exercises 23–32, find the limit.

23. $\displaystyle\lim_{x \to \infty} \frac{2x - 1}{3x + 2}$

24. $\displaystyle\lim_{x \to \infty} \frac{5x^3 + 1}{10x^3 - 3x^2 + 7}$

25. $\displaystyle\lim_{x \to \infty} \frac{3x}{4x^2 - 1}$

26. $\displaystyle\lim_{x \to \infty} \frac{2x^{10} - 1}{10x^{11} - 3}$

27. $\displaystyle\lim_{x \to -\infty} \frac{5x^2}{x + 3}$

28. $\displaystyle\lim_{x \to \infty} \frac{x^3 - 2x^2 + 3x + 1}{x^2 - 3x + 2}$

29. $\displaystyle\lim_{x \to \infty} (2x - x^{-2})$

30. $\displaystyle\lim_{x \to \infty} (2 - x^{-3})$

31. $\displaystyle\lim_{x \to -\infty} \left(\frac{2x}{x - 1} + \frac{3x}{x + 1}\right)$

32. $\displaystyle\lim_{x \to \infty} \left(\frac{2x^2}{x - 1} + \frac{3x}{x + 1}\right)$

In Exercises 33 and 34, complete the table. Then use the result to estimate the limit of $f(x)$ as x approaches infinity.

33. $f(x) = \dfrac{x + 1}{x\sqrt{x}}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

34. $f(x) = x^2 - x\sqrt{x(x - 1)}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

In Exercises 35 and 36, use a spreadsheet software program to complete the table and use the result to estimate the limit of $f(x)$ as x approaches infinity.

35. $f(x) = \dfrac{x^2 - 1}{0.02x^2}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

36. $f(x) = \dfrac{3x^2}{0.1x^2 + 1}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

In Exercises 37 and 38, use a graphing utility to complete the table and use the result to estimate the limit of $f(x)$ as x approaches infinity and as x approaches negative infinity.

37. $f(x) = \dfrac{2x}{\sqrt{x^2 + 4}}$

x	-10^6	-10^4	-10^2	10^0	10^2	10^4	10^6
$f(x)$							

38. $f(x) = x - \sqrt{x(x-1)}$

x	-10^6	-10^4	-10^2	10^0	10^2	10^4	10^6
$f(x)$							

In Exercises 39–56, sketch the graph of the equation. Use intercepts, extrema, and asymptotes as sketching aids.

39. $y = \dfrac{2+x}{1-x}$

40. $y = \dfrac{x-3}{x-2}$

41. $f(x) = \dfrac{x^2}{x^2+9}$

42. $f(x) = \dfrac{x}{x^2+4}$

43. $g(x) = \dfrac{x^2}{x^2-16}$

44. $g(x) = \dfrac{x}{x^2-4}$

45. $xy^2 = 4$

46. $x^2y = 4$

47. $y = \dfrac{2x}{1-x}$

48. $y = \dfrac{2x}{1-x^2}$

49. $y = 3(1 - x^{-2})$

50. $y = 1 + x^{-1}$

51. $f(x) = \dfrac{1}{x^2-x-2}$

52. $f(x) = \dfrac{x-2}{x^2-4x+3}$

53. $g(x) = \dfrac{x^2-x-2}{x-2}$

54. $g(x) = \dfrac{x^2-9}{x+3}$

55. $y = \dfrac{2x^2-6}{(x-1)^2}$

56. $y = \dfrac{x}{(x+1)^2}$

57. ***Cost*** The cost C (in dollars) of producing x units of a product is $C = 1.35x + 4570$.

(a) Find the average cost function $\overline{C}$.

(b) Find $\overline{C}$ when $x = 100$ and when $x = 1000$.

(c) What is the limit of $\overline{C}$ as x approaches infinity?

58. ***Average Cost*** A business has a cost (in dollars) of $C = 0.5x + 500$ for producing x units.

(a) Find the average cost function $\overline{C}$.

(b) Find $\overline{C}$ when $x = 250$ and when $x = 1250$.

(c) What is the limit of $\overline{C}$ as x approaches infinity?

59. ***Cost*** The cost C (in millions of dollars) for the federal government to seize $p\%$ of a type of illegal drug as it enters the country is modeled by

$$C = 528p/(100 - p), \qquad 0 \le p < 100.$$

(a) Find the cost of seizing 25%, 50%, and 75%.

(b) Find the limit of C as $p \to 100^-$.

(c) Use a graphing utility to verify the result of part (b).

60. ***Cost*** The cost C (in dollars) of removing $p\%$ of the air pollutants in the stack emission of a utility company that burns coal is modeled by

$$C = 80{,}000p/(100 - p), \qquad 0 \le p < 100.$$

(a) Find the cost of removing 15%, 50%, and 90%.

(b) Find the limit of C as $p \to 100^-$.

(c) Use a graphing utility to verify the result of part (b).

61. ***Learning Curve*** Psychologists have developed mathematical models to predict performance P (the percent of correct responses) as a function of n, the number of times a task is performed. One such model is

$$P = \frac{b + \theta a(n-1)}{1 + \theta(n-1)}$$

where a, b, and θ are constants that depend on the actual learning situation. Find the limit of P as n approaches infinity.

62. ***Learning Curve*** Consider the learning curve given by

$$P = \frac{0.5 + 0.9(n-1)}{1 + 0.9(n-1)}, \qquad 0 < n.$$

(a) Complete the table for the model.

n	1	2	3	4	5	6	7	8	9	10
P										

(b) Find the limit as n approaches infinity.

(c) Use a graphing utility to graph this learning curve, and interpret the graph in the context of the problem.

63. ***Biology: Wildlife Management*** The state game commission introduces 30 elk into a new state park. The population N of the herd is modeled by

$$N = [10(3 + 4t)]/(1 + 0.1t)$$

where t is the time in years.

(a) Find the size of the herd after 5, 10, and 25 years.

(b) According to this model, what is the limiting size of the herd as time progresses?

64. ***Average Profit*** The cost and revenue functions for a product are $C = 34.5x + 15{,}000$ and $R = 69.9x$.

(a) Find the average profit function

$$\overline{P} = (R - C)/x.$$

(b) Find the average profit when x is 1000, 10,000, and 100,000.

(c) What is the limit of the average profit function as x approaches infinity? Explain your reasoning.

65. ***Average Profit*** The cost and revenue functions for a product are $C = 25.5x + 1000$ and $R = 75.5x$.

(a) Find the average profit function $\overline{P} = \dfrac{R - C}{x}$.

(b) Find the average profit when x is 100, 500, and 1000.

(c) What is the limit of the average profit function as x approaches infinity? Explain your reasoning.

3.7

T

Summary of Curve

It would be difficult to ov
Descartes's introduction
rapid advances in calculu
So far, you have stu
graph of a function.

- x-intercepts and y-inter
- Domain and range
- Continuity
- Differentiability
- Relative extrema
- Concavity
- Points of inflection
- Vertical asymptotes
- Horizontal asymptotes

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Guidelines for Analy

1. Determine the dor
models a real-life
2. Determine the inte
3. Locate the x-value
the results to dete

In these guidelines, note
ing the equations $f(x) =$

PREREQUISITE REVIEW 3.7

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, find the vertical and horizontal asymptotes of the graph.

1. $f(x) = \dfrac{1}{x^2}$

2. $f(x) = \dfrac{8}{(x-2)^2}$

3. $f(x) = \dfrac{40x}{x+3}$

4. $f(x) = \dfrac{x^2-3}{x^2-4x+3}$

In Exercises 5–10, determine the open intervals on which the function is increasing or decreasing.

5. $f(x) = x^2 + 4x + 2$

6. $f(x) = -x^2 - 8x + 1$

7. $f(x) = x^3 - 3x + 1$

8. $f(x) = \dfrac{-x^3 + x^2 - 1}{x^2}$

9. $f(x) = \dfrac{x-2}{x-1}$

10. $f(x) = -x^3 - 4x^2 + 3x + 2$

EXERCISES 3.7

In Exercises 1–20, sketch the graph of the function. Choose a scale that allows all relative extrema and points of inflection to be identified on the graph.

1. $y = -x^2 - 2x + 3$

2. $y = 2x^2 - 4x + 1$

3. $y = x^3 - 4x^2 + 6$

4. $y = -\frac{1}{3}(x^3 - 3x + 2)$

5. $y = 2 - x - x^3$

6. $y = x^3 + 3x^2 + 3x + 2$

7. $y = 3x^3 - 9x + 1$

8. $y = -4x^3 + 6x^2$

9. $y = 3x^4 + 4x^3$

10. $y = 3x^4 - 6x^2$

11. $y = x^3 - 6x^2 + 3x + 10$

12. $y = -x^3 + 3x^2 + 9x - 2$

13. $y = x^4 - 8x^3 + 18x^2 - 16x + 5$

14. $y = x^4 - 4x^3 + 16x - 16$

15. $y = x^4 - 4x^3 + 16x$

16. $y = x^5 + 1$

17. $y = x^5 - 5x$

18. $y = (x-1)^5$

19. $y = \begin{cases} x^2 + 1, & x \le 0 \\ 1 - 2x, & x > 0 \end{cases}$

20. $y = \begin{cases} x^2 + 4, & x < 0 \\ 4 - x, & x \ge 0 \end{cases}$

In Exercises 21–32, use a graphing utility to graph the function. Choose a window that allows all relative extrema and points of inflection to be identified on the graph.

21. $y = \dfrac{x^2+2}{x^2+1}$

22. $y = \dfrac{x}{x^2+1}$

23. $y = 3x^{2/3} - 2x$

24. $y = 3x^{2/3} - x^2$

25. $y = 1 - x^{2/3}$

26. $y = (1-x)^{2/3}$

27. $y = x^{1/3} + 1$

28. $y = x^{-1/3}$

29. $y = x^{5/3} - 5x^{2/3}$

30. $y = x^{4/3}$

31. $y = x\sqrt{x^2-9}$

32. $y = \dfrac{x}{\sqrt{x^2-4}}$

In Exercises 33–42, sketch the graph of the function. Label the intercepts, relative extrema, points of inflection, and asymptotes. Then state the domain of the function.

33. $y = \dfrac{5-3x}{x-2}$

34. $y = \dfrac{x^2+1}{x^2-2}$

35. $y = \dfrac{2x}{x^2-1}$

36. $y = \dfrac{x^2-6x+12}{x-4}$

37. $y = x\sqrt{4-x}$

38. $y = x\sqrt{4-x^2}$

39. $y = \dfrac{x-3}{x}$

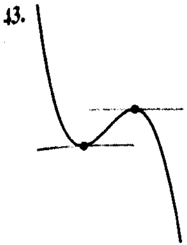
40. $y = x + \dfrac{32}{x^2}$

41. $y = \dfrac{x^3}{x^3-1}$

42. $y = \dfrac{x^4}{x^4-1}$

In Exercises 43–46, find
graph of $f(x) = ax^3 + b$
graph. Then use a graphir
many correct answers.)

43.



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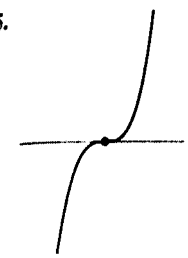
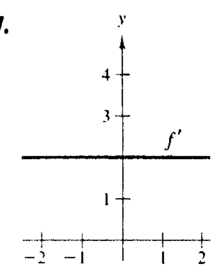


In Exercises 47–50, use th
of f. (There are many corr

47.



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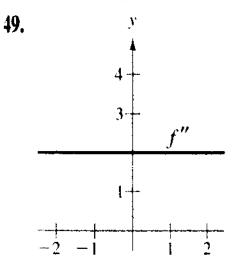


In Exercises 51 and 52, sk
given characteristics. (The

51. $f(-2) = 0$

$f(0) = 0$

$f'(x) > 0, \quad -\infty <$

$f'(-1) = 0$

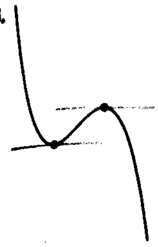
$f'(x) < 0, \quad -1 <$

$f'(0) = 0$

$f'(x) > 0, \quad 0 < x$

In Exercises 43–46, find values of a, b, c, and d such that the graph of $f(x) = ax^3 + bx^2 + cx + d$ will resemble the given graph. Then use a graphing utility to verify your result. (There are many correct answers.)

43.



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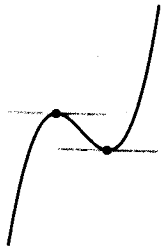

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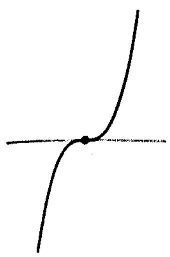

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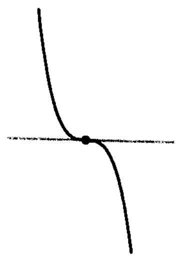
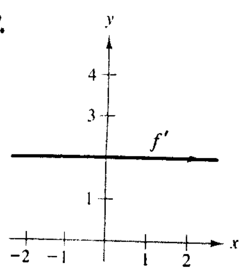


In Exercises 47–50, use the graph of f' or f'' to sketch the graph of f. (There are many correct answers.)

47.



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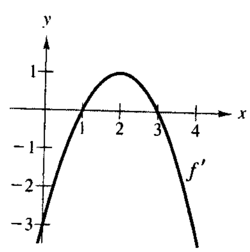

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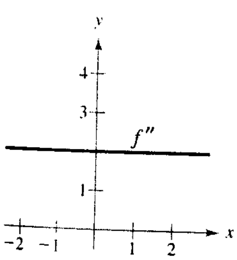

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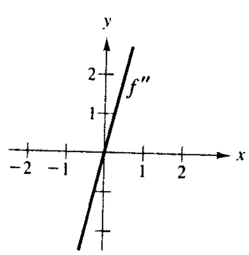


In Exercises 51 and 52, sketch a graph of a function f having the given characteristics. (There are many correct answers.)

51. $f(-2) = 0$

$f(0) = 0$

$f'(x) > 0, \quad -\infty < x < -1$

$f'(-1) = 0$

$f'(x) < 0, \quad -1 < x < 0$

$f'(0) = 0$

$f'(x) > 0, \quad 0 < x < \infty$

52. $f(-1) = 0$

$f(3) = 0$

$f'(1)$ is undefined.

$f'(x) < 0, \quad -\infty < x < 1$

$f'(x) > 0, \quad 1 < x < \infty$

$f''(x) < 0, \quad x \neq 1$

$\lim_{x\to\infty} f(x) = 4$

53. ***Cost*** An employee of a delivery company earns \$9 per hour driving a delivery van in an area where gasoline costs \$1.80 per gallon. When the van is driven at a constant speed s (in miles per hour, with $40 \le s \le 65$), the van gets $500/s$ miles per gallon.

(a) Find the cost C as a function of s for a 100-mile trip on an interstate highway.

(b) Use a graphing utility to graph the function found in part (a) and determine the most economical speed.

54. ***Profit*** The management of a company is considering three possible models for predicting the company's profits from 2001 through 2006. Model I gives the expected annual profits if the current trends continue. Models II and III give the expected annual profits for various combinations of increased labor and energy costs. In each model, p is the profit (in billions of dollars) and $t = 0$ corresponds to 2001.

Model I: $p = 0.03t^2 - 0.01t + 3.39$

Model II: $p = 0.08t + 3.36$

Model III: $p = -0.07t^2 + 0.05t + 3.38$

(a) Use a graphing utility to graph all three models in the same viewing window.

(b) For which models are profits increasing during the interval from 2001 through 2006?

(c) Which model is the most optimistic? Which is the most pessimistic?

55. ***Meteorology*** The monthly normal temperature T (in degrees Fahrenheit) for Pittsburgh, Pennsylvania can be modeled by

$$T = \frac{23.011 - 1.0t + 0.048t^2}{1 - 0.204t + 0.014t^2}, \quad 1 \le t \le 12$$

where t is the month, with $t = 1$ corresponding to January. Use a graphing utility to graph the model and find all absolute extrema. Explain the meaning of those values.

Writing In Exercises 56 and 57, use a graphing utility to graph the function. Explain why there is no vertical asymptote when a superficial examination of the function may indicate that there should be one.

56. $h(x) = \dfrac{6 - 2x}{3 - x}$

57. $g(x) = \dfrac{x^2 + x - 2}{x - 1}$

PREREQUISITE REVIEW 3.8

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–12, find the derivative.

1. $C = 44 + 0.09x^2$
2. $C = 250 + 0.15x$
3. $R = x(1.25 + 0.02\sqrt{x})$
4. $R = x(15.5 - 1.55x)$
5. $P = -0.03x^{1/3} + 1.4x - 2250$
6. $P = -0.02x^2 + 25x - 1000$
7. $A = \frac{1}{4}\sqrt{3}x^2$
8. $A = 6x^2$
9. $C = 2\pi r$
10. $P = 4w$
11. $S = 4\pi r^2$
12. $P = 2x + \sqrt{2}x$

In Exercises 13–16, write a formula for the quantity.

13. Area A of a circle of radius r
14. Area A of a square of side x
15. Volume V of a cube of edge x
16. Volume V of a sphere of radius r

EXERCISES 3.8

In Exercises 1–6, find the differential dy.

1. $y = 3x^2 - 4$
2. $y = 2x^{3/2}$
3. $y = (4x - 1)^3$
4. $y = (1 - 2x^2)^4$
5. $y = \sqrt{x^2 + 1}$
6. $y = \sqrt[3]{6x^2}$

In Exercises 7–10, let $x = 1$ and $\Delta x = 0.01$. Find Δy.

7. $f(x) = 5x^2 - 1$
8. $f(x) = \sqrt{3x}$
9. $f(x) = \dfrac{4}{\sqrt[3]{x}}$
10. $f(x) = \dfrac{x}{x^2 + 1}$

In Exercises 11–14, compare the values of dy and Δy.

11. $y = x^3$ $\quad x = 1 \quad \Delta x = dx = 0.1$
12. $y = 1 - 2x^2$ $\quad x = 0 \quad \Delta x = dx = -0.1$
13. $y = x^4 + 1$ $\quad x = -1 \quad \Delta x = dx = 0.01$
14. $y = 2x^3 + 1$ $\quad x = 2 \quad \Delta x = dx = 0.01$

In Exercises 15–20, let $x = 2$ and complete the table for the function.

$dx = \Delta x$	dy	Δy	$\Delta y - dy$	$dy/\Delta y$
1.000				
0.500				
0.100				
0.010				
0.001				

15. $y = x^2$
16. $y = x^5$
17. $y = \dfrac{1}{x^2}$
18. $y = \dfrac{1}{x}$
19. $y = \sqrt[4]{x}$
20. $y = \sqrt{x}$

In Exercises 21–24, find an equation of the tangent line to the function at the given point. Then find the function value and the tangent line value at $f(x + \Delta x)$ and $y(x + \Delta x)$ for $\Delta x = -0.01$ and 0.01.

	Function	*Point*
21.	$f(x) = 2x^3 - x^2 + 1$	$(-2, -19)$
22.	$f(x) = 3x^2 - 1$	$(2, 11)$
23.	$f(x) = \dfrac{x}{x^2 + 1}$	$(0, 0)$
24.	$f(x) = \sqrt{25 - x^2}$	$(3, 4)$

25. ***Demand*** The demanc by

$p = 75 - 0.25x.$

(a) If x changes from change in p? Compa

(b) Repeat part (a) whe

26. ***Biology: Wildlife M*** mission introduces 50 d lands. The population N

$$N = \frac{10(5 + 3t)}{1 + 0.04t}$$

where t is the time in mate the change in the

Marginal Analysis In E approximate the change in c to an increase in sales of or approximate the change in c units. Then use a graphing u the *trace* feature to verify yo

Function

27. $C = 0.05x^2 + 4x + 10$
28. $C = 0.025x^2 + 8x + 5$
29. $R = 30x - 0.15x^2$
30. $R = 50x - 1.5x^2$
31. $P = -0.5x^3 + 2500x$
32. $P = -x^2 + 60x - 10C$

33. ***Marginal Analysis*** monthly sales x of a w \$50, but decreases to Assume that the dema Find the revenue R as change in revenue for $x = 141$. Make a sketc

34. ***Marginal Analysis*** demand x for a prod square of the price p. 2500. Find the revenu mate the change in rev when $x = 3000$. Make

35. ***Marginal Analysis*** units per week when th the price is \$20. The i cost per unit is \$17. function of the price. and approximate the increase in sales when dP and ΔP.

25. ***Demand*** The demand function for a product is modeled by

$$p = 75 - 0.25x.$$

(a) If x changes from 7 to 8, what is the corresponding change in p? Compare the values of Δp and dp.

(b) Repeat part (a) when x changes from 70 to 71 units.

26. ***Biology: Wildlife Management*** A state game commission introduces 50 deer into newly acquired state game lands. The population N of the herd can be modeled by

$$N = \frac{10(5 + 3t)}{1 + 0.04t}$$

where t is the time in years. Use differentials to approximate the change in the herd size from $t = 5$ to $t = 6$.

Marginal Analysis In Exercises 27–32, use differentials to approximate the change in cost, revenue, or profit corresponding to an increase in sales of one unit. For instance, in Exercise 27, approximate the change in cost as x increases from 12 units to 13 units. Then use a graphing utility to graph the function, and use the *trace* feature to verify your result.

Function	*x-Value*
27. $C = 0.05x^2 + 4x + 10$	$x = 12$
28. $C = 0.025x^2 + 8x + 5$	$x = 10$
29. $R = 30x - 0.15x^2$	$x = 75$
30. $R = 50x - 1.5x^2$	$x = 15$
31. $P = -0.5x^3 + 2500x - 6000$	$x = 50$
32. $P = -x^2 + 60x - 100$	$x = 25$

33. ***Marginal Analysis*** A retailer has determined that the monthly sales x of a watch is 150 units when the price is \$50, but decreases to 120 units when the price is \$60. Assume that the demand is a linear function of the price. Find the revenue R as a function of x and approximate the change in revenue for a one-unit increase in sales when $x = 141$. Make a sketch showing dR and ΔR.

34. ***Marginal Analysis*** A manufacturer determines that the demand x for a product is inversely proportional to the square of the price p. When the price is \$10, the demand is 2500. Find the revenue R as a function of x and approximate the change in revenue for a one-unit increase in sales when $x = 3000$. Make a sketch showing dR and ΔR.

35. ***Marginal Analysis*** The demand x for a radio is 30,000 units per week when the price is \$25 and 40,000 units when the price is \$20. The initial investment is \$275,000 and the cost per unit is \$17. Assume that the demand is a linear function of the price. Find the profit P as a function of x and approximate the change in profit for a one-unit increase in sales when $x = 28{,}000$. Make a sketch showing dP and ΔP.

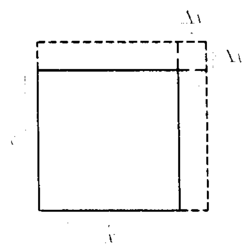
36. ***Marginal Analysis*** The variable cost for the production of a calculator is \$14.25 and the initial investment is \$110,000. Find the total cost C as a function of x, the number of units produced. Then use differentials to approximate the change in the cost for a one-unit increase in production when $x = 50{,}000$. Make a sketch showing dC and ΔC. Explain why $dC = \Delta C$ in this problem.

37. ***Area*** The area A of a square of side x is $A = x^2$.

(a) Compute dA and ΔA in terms of x and Δx.

(b) In the figure, identify the region whose area is dA.

(c) Identify the region whose area is $\Delta A - dA$.



38. ***Area*** The side of a square is measured to be 12 inches, with a possible error of $\frac{1}{64}$ inch. Use differentials to approximate the possible error and the relative error in computing the area of the square.

39. ***Area*** The radius of a circle is measured to be 10 inches, with a possible error of $\frac{1}{8}$ inch. Use differentials to approximate the possible error and the relative error in computing the area of the circle.

40. ***Volume and Surface Area*** The edge of a cube is measured to be 12 inches, with a possible error of 0.03 inch. Use differentials to approximate the possible error and the relative error in computing (a) the volume of the cube and (b) the surface area of the cube.

41. ***Volume*** The radius of a sphere is measured to be 6 inches, with a possible error of 0.02 inch. Use differentials to approximate the possible error and the relative error in computing the volume of the sphere.

42. ***Medical Science*** The concentration C (in milligrams per milliliter) of a drug in a patient's bloodstream t hours after injection into muscle tissue is modeled by

$$C = \frac{3t}{27 + t^3}.$$

Use differentials to approximate the change in the concentration when t changes from $t = 1$ to $t = 1.5$.

True or False? In Exercises 43 and 44, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

43. If $y = x + c$, then $dy = dx$.

44. If $y = ax + b$, then $\Delta y/\Delta x = dy/dx$.