PREREQUISITE REVIEW 8.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, find the area of the triangle.

1. Base: 10 cm; height: 7 cm

2. Base: 4 in.; height: 6 in.

In Exercises 3–6, let a and b represent the lengths of the legs, and let c represent the length of the hypotenuse, of a right triangle. Solve for the missing side length.

3. $a = 5, b = 12$

4. $a = 3, c = 5$

5. $a = 8, c = 17$

6. $b = 8, c = 10$

In Exercises 7–10, let a, b, and c represent the side lengths of a triangle. Use the information below to determine whether the figure is a right triangle, an isosceles triangle, or an equilateral triangle.

7. $a = 4, b = 4, c = 4$

8. $a = 1, b = 1, c = 2$

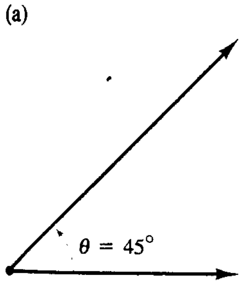
9. $a = 12, b = 16, c = 20$

10. $a = 1, b = 1, c = \sqrt{2}$

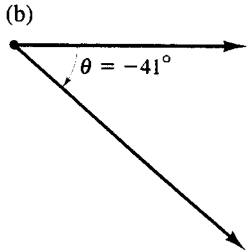
EXERCISES 8.1

In Exercises 1–4, determine two coterminal angles (one positive and one negative) for the given angle. Give the answers in degrees.

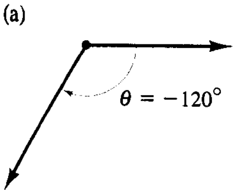
1. (a)



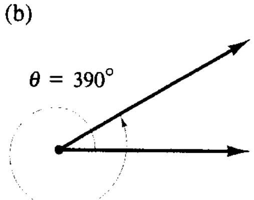
(b)



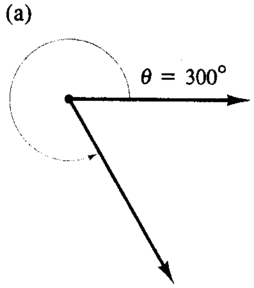
2. (a)



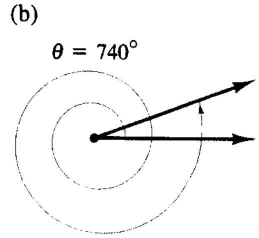
(b)



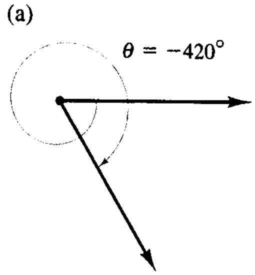
3. (a)



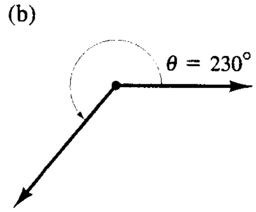
(b)



4. (a)

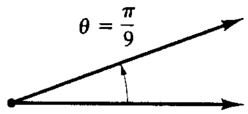


(b)

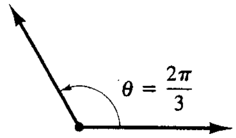


In Exercises 5–8, determine two coterminal angles (one positive and one negative) for each angle. Give the answers in radians.

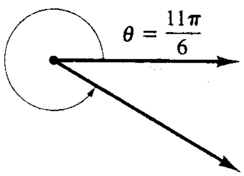
5. (a)



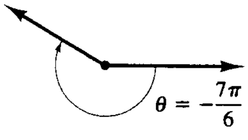
(b)



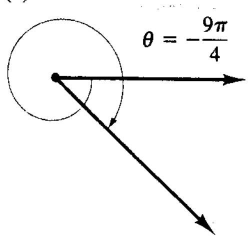
6. (a)



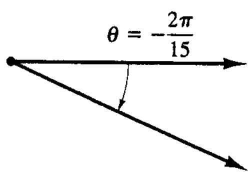
(b)



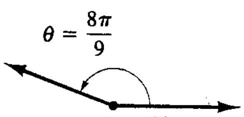
7. (a)



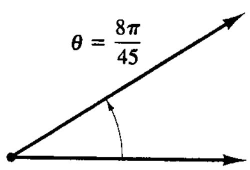
(b)



8. (a)



(b)



In Exercises 9–20, express the angle in radian measure as a multiple of π. Use a calculator to verify your result.

9. 30° **10.** 150°

11. 225° **12.** 210°

13. 315° **14.** 120°

15. −30° **16.** −240°

17. −270° **18.** −330°

19. 390° **20.** 405°

In Exercises 21–30, express the angle in degree measure. Use a calculator to verify your result.

21. $\frac{3\pi}{2}$ **22.** $\frac{7\pi}{6}$

23. $\frac{11\pi}{6}$ **24.** $\frac{7\pi}{4}$

25. $-\frac{5\pi}{3}$ **26.** $-\frac{3\pi}{4}$

27. $\frac{9\pi}{4}$ **28.** $\frac{5\pi}{2}$

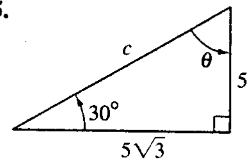
29. $\frac{19\pi}{6}$ **30.** $\frac{8\pi}{3}$

In Exercises 31–34, find the indicated measure of the angle. Express radian measure as a multiple of π.

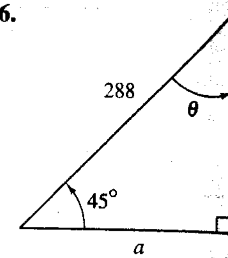
	Degree Measure	*Radian Measure*
31.	−270°	
32.		$\frac{\pi}{9}$
33.	144°	
34.		$-\frac{7\pi}{12}$

In Exercises 35–42, solve the triangle for the indicated side and/or angle.

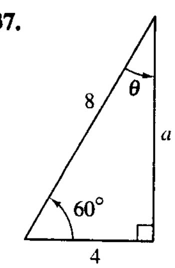
35.



36.



37.



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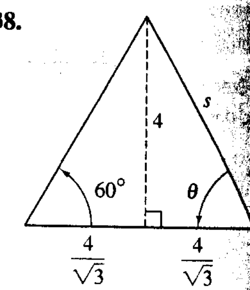

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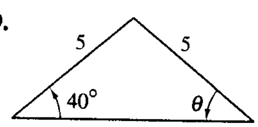
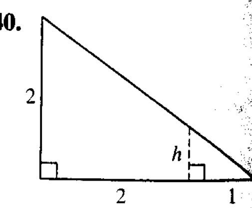


40.



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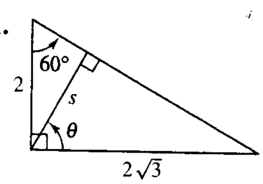
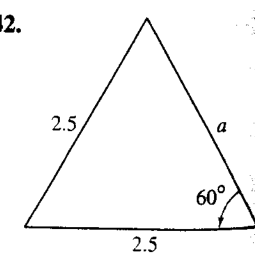


42.



In Exercises 43–46, find the a…
sides of length s.

43. $s = 6$ in.

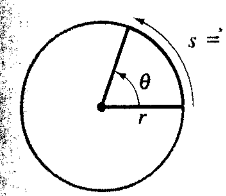
45. $s = 5$ ft

47. ***Height*** A person 6 …
streetlight casts a shado…
the height of the streetli…

Figure for 47

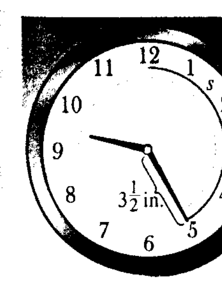
48. ***Length*** A guy wire …
tower at a point 200 fee…
feet from the base (see …

49. Let r represent the rad…
(measured in radians), a…
by the angle (see figur…
complete the table.



r	8 ft	15 in.
s	12 ft	
θ		1.6

50. ***Arc Length*** The m…
long (see figure). Throu…
minute hand move in 2…



In Exercises 43–46, find the area of the equilateral triangle with sides of length s.

43. $s = 6$ in.

44. $s = 8$ m

45. $s = 5$ ft

46. $s = 12$ cm

47. ***Height*** A person 6 feet tall standing 16 feet from a streetlight casts a shadow 8 feet long (see figure). What is the height of the streetlight?

Figure for 47

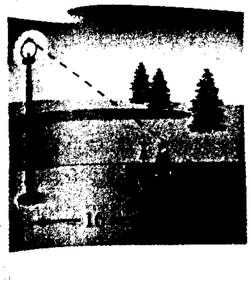
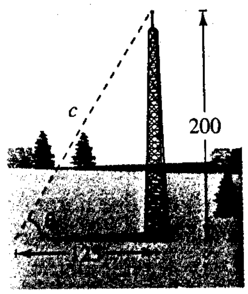
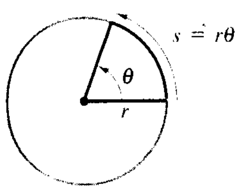


Figure for 48



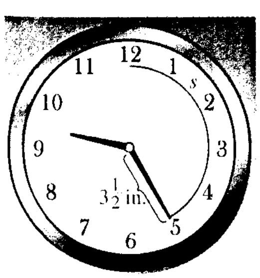
48. ***Length*** A guy wire is stretched from a broadcasting tower at a point 200 feet above the ground to an anchor 125 feet from the base (see figure). How long is the wire?

49. Let r represent the radius of a circle, θ the central angle (measured in radians), and s the length of the arc interceped by the angle (see figure). Use the relationship $\theta = s/r$ to complete the table.



r	8 ft	15 in.	85 cm		
s	12 ft			96 in.	8642 mi
θ		1.6	$\frac{3\pi}{4}$	4	$\frac{2\pi}{3}$

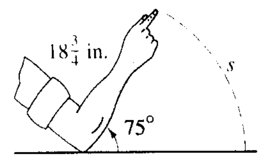
50. ***Arc Length*** The minute hand on a clock is $3\frac{1}{2}$ inches long (see figure). Through what distance does the tip of the minute hand move in 25 minutes?



51. ***Distance*** A man bends his elbow through 75°. The distance from his elbow to the tip of his index finger is $18\frac{3}{4}$ inches (see figure).

(a) Find the radian measure of this angle.

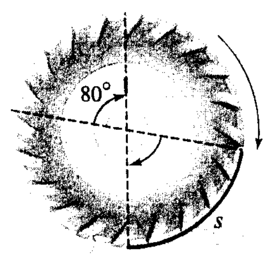
(b) Find the distance the tip of the index finger moves.



52. ***Distance*** A tractor tire that is 5 feet in diameter d is partially filled with a liquid ballast for additional traction. To check the air pressure, the tractor operator rotates the tire until the valve stem is at the top so that the liquid will not enter the gauge. On a given occasion, the operator notes that the tire must be rotated 80° to have the stem in the proper position (see figure).

(a) Find the radian measure of this rotation.

(b) How far must the tractor be moved to get the valve stem in the proper position?



53. ***Speed of Revolution*** A compact disc can have an angular speed up to 3142 radians per minute.

(a) At this angular speed, how many revolutions per minute would the CD make?

(b) How long would it take the CD to make 10,000 revolutions?

54. ***Speed of Revolution*** The radius of the magnetic disk in a 3.5-inch diskette is 1.68 inches. Find the linear speed of a point on the circumference of the disk if it rotates at a speed of 360 revolutions per minute.

True or False? In Exercises 55–58, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

55. An angle whose measure is 75° is obtuse.

56. $\theta = -35°$ is coterminal to 325°.

57. A right triangle can have one angle whose measure is 89°.

58. An angle whose measure is π radians is a straight angle.

PREREQUISITE REVIEW 8.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–8, convert the angle to radian measure.

1. $135°$
2. $315°$
3. $-210°$
4. $-300°$
5. $-120°$
6. $-225°$
7. $540°$
8. $390°$

In Exercises 9–16, solve for x.

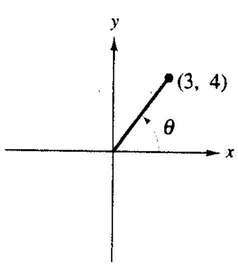
9. $x^2 - x = 0$
10. $2x^2 + x = 0$
11. $2x^2 - x = 1$
12. $x^2 - 2x = 3$
13. $x^2 - 2x = -1$
14. $2x^2 + x = 1$
15. $x^2 - 5x = -6$
16. $x^2 + x = 2$

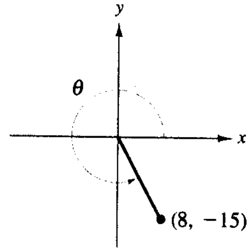
In Exercises 17–20, solve for t.

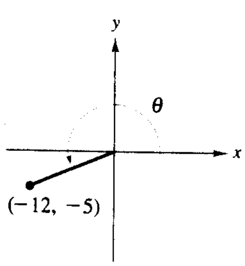
17. $\frac{2\pi}{24}(t - 4) = \frac{\pi}{2}$
18. $\frac{2\pi}{12}(t - 2) = \frac{\pi}{4}$
19. $\frac{2\pi}{365}(t - 10) = \frac{\pi}{4}$
20. $\frac{2\pi}{12}(t - 4) = \frac{\pi}{2}$

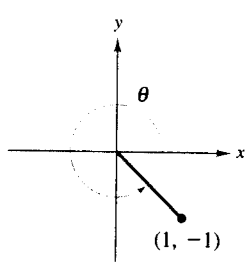
EXERCISES 8.2

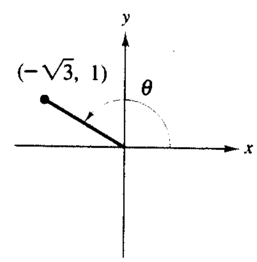
In Exercises 1–6, determine all six trigonometric functions for the angle θ.

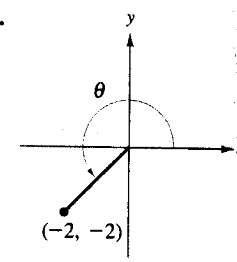
1.


2.


3.


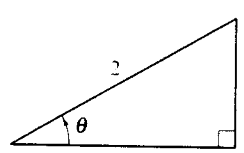
4.


5.


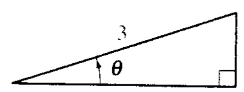
6.


In Exercises 7–12, find the indicated trigonometric function from the given function.

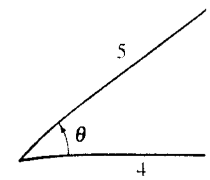
7. Given $\sin\theta = \frac{1}{2}$, find $\csc\theta$.



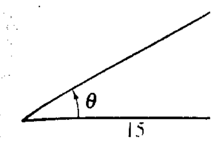
8. Given $\sin\theta = \frac{1}{3}$, find $\tan\theta$.



9. Given $\cos\theta = \frac{4}{5}$, find $\cot\theta$.



11. Given $\cot\theta = \frac{15}{8}$, find $\sec\theta$.



In Exercises 13–18, sketch
trigonometric function o
trigonometric functions o

13. $\sin\theta = \frac{1}{3}$
15. $\sec\theta = \frac{3}{2}$
17. $\tan\theta = 3.5$

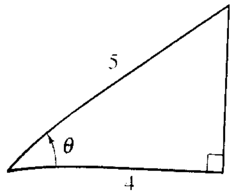
In Exercises 19–24, determ

19. $\sin\theta < 0$, $\cos\theta >$ (
20. $\sin\theta > 0$, $\cos\theta <$ (
21. $\sin\theta > 0$, $\sec\theta >$ (
22. $\cot\theta < 0$, $\cos\theta >$ (
23. $\csc\theta > 0$, $\tan\theta <$ (
24. $\cos\theta > 0$, $\tan\theta <$ (

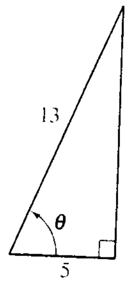
In Exercises 25–32, evalu
the angles *without* using

25. (a) $60°$
26. (a) $\frac{\pi}{4}$
27. (a) $-\frac{\pi}{6}$
28. (a) $-\frac{\pi}{2}$
29. (a) $225°$
30. (a) $300°$
31. (a) $750°$

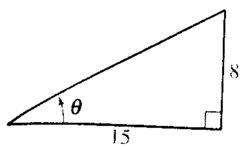
9. Given $\cos\theta = \frac{4}{5}$, find $\cot\theta$.



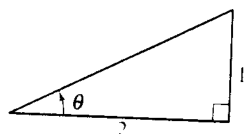
10. Given $\sec\theta = \frac{13}{5}$, find $\cot\theta$.



11. Given $\cot\theta = \frac{15}{8}$, find $\sec\theta$.



12. Given $\tan\theta = \frac{1}{2}$, find $\sin\theta$.



In Exercises 13–18, sketch a right triangle corresponding to the trigonometric function of the angle θ and find the other five trigonometric functions of θ.

13. $\sin\theta = \frac{1}{3}$
14. $\cot\theta = 5$
15. $\sec\theta = \frac{3}{2}$
16. $\cos\theta = \frac{5}{7}$
17. $\tan\theta = 3.5$
18. $\csc\theta = 4.25$

In Exercises 19–24, determine the quadrant in which θ lies.

19. $\sin\theta < 0$, $\cos\theta > 0$
20. $\sin\theta > 0$, $\cos\theta < 0$
21. $\sin\theta > 0$, $\sec\theta > 0$
22. $\cot\theta < 0$, $\cos\theta > 0$
23. $\csc\theta > 0$, $\tan\theta < 0$
24. $\cos\theta > 0$, $\tan\theta < 0$

In Exercises 25–32, evaluate the sines, cosines, and tangents of the angles *without* using a calculator.

25. (a) $60°$ (b) $-\frac{2\pi}{3}$
26. (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{4}$
27. (a) $-\frac{\pi}{6}$ (b) $150°$
28. (a) $-\frac{\pi}{2}$ (b) $\frac{\pi}{2}$
29. (a) $225°$ (b) $-225°$
30. (a) $300°$ (b) $330°$
31. (a) $750°$ (b) $510°$
32. (a) $\frac{10\pi}{3}$ (b) $\frac{17\pi}{3}$

In Exercises 33–40, use a calculator to evaluate the trigonometric functions to four decimal places.

33. (a) $\sin 12°$ (b) $\csc 12°$
34. (a) $\sec 225°$ (b) $\sec 135°$
35. (a) $\tan\frac{\pi}{9}$ (b) $\tan\frac{10\pi}{9}$
36. (a) $\cot 1.35$ (b) $\tan 1.35$
37. (a) $\cos(-110°)$ (b) $\cos 250°$
38. (a) $\tan 240°$ (b) $\cot 210°$
39. (a) $\csc 2.62$ (b) $\csc 150°$
40. (a) $\sin(-0.65)$ (b) $\sin 5.63$

In Exercises 41–46, find two values of θ corresponding to each function. List the measure of θ in radians ($0 \le \theta \le 2\pi$). Do not use a calculator.

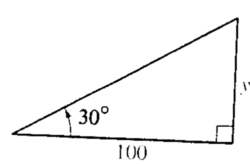
41. (a) $\sin\theta = \frac{1}{2}$ (b) $\sin\theta = -\frac{1}{2}$
42. (a) $\cos\theta = \frac{\sqrt{2}}{2}$ (b) $\cos\theta = -\frac{\sqrt{2}}{2}$
43. (a) $\csc\theta = \frac{2\sqrt{3}}{3}$ (b) $\cot\theta = -1$
44. (a) $\sec\theta = 2$ (b) $\sec\theta = -2$
45. (a) $\tan\theta = -1$ (b) $\cot\theta = -\sqrt{3}$
46. (a) $\sin\theta = \frac{\sqrt{3}}{2}$ (b) $\sin\theta = -\frac{\sqrt{3}}{2}$

In Exercises 47–56, solve the equation for θ ($0 \le \theta \le 2\pi$). For some of the equations you should use the trigonometric identities listed in this section. Use the *trace* feature of a graphing utility to verify your results.

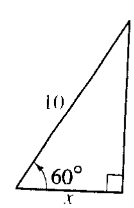
47. $2\sin^2\theta = 1$
48. $\tan^2\theta = 3$
49. $\tan^2\theta - \tan\theta = 0$
50. $2\cos^2\theta - \cos\theta = 1$
51. $\sin 2\theta - \cos\theta = 0$
52. $\cos 2\theta + 3\cos\theta + 2 = 0$
53. $\sin\theta = \cos\theta$
54. $\sec\theta\csc\theta = 2\csc\theta$
55. $\cos^2\theta + \sin\theta = 1$
56. $\cos\frac{\theta}{2} - \cos\theta = 1$

In Exercises 57–62, solve for x, y, or r as indicated.

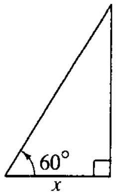
57. Solve for y.



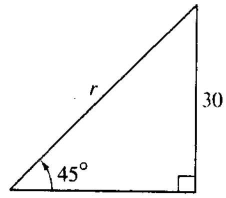
58. Solve for x.



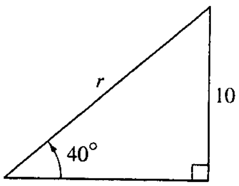
59. Solve for x.



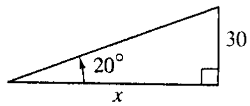
60. Solve for r.



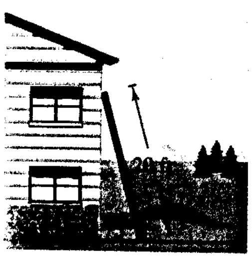
61. Solve for r.



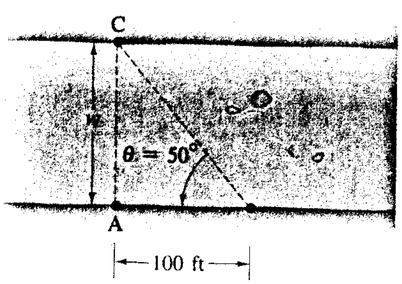
62. Solve for x.



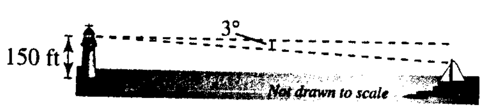
63. ***Length*** A 20-foot ladder leaning against the side of a house makes a 75° angle with the ground (see figure). How far up the side of the house does the ladder reach?



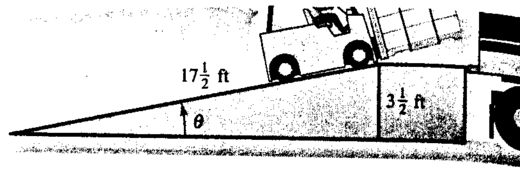
64. ***Width*** A biologist wants to know the width w of a river in order to set instruments to study the pollutants in the water. From point A the biologist walks downstream 100 feet and sights to point C. From this sighting it is determined that $\theta = 50°$ (see figure). How wide is the river?



65. ***Distance*** From a 150-foot observation tower on the coast, a Coast Guard officer sights a boat in difficulty. The angle of depression of the boat is 3° (see figure). How far is the boat from the shoreline?



66. ***Angle Measure*** A ramp $17\frac{1}{2}$ feet in length rises to a loading platform that is $3\frac{1}{2}$ feet off the ground (see figure). Find the angle that the ramp makes with the ground.



67. ***Medicine*** The temperature T in degrees Fahrenheit of a patient t hours after arriving at the emergency room of a hospital at 10:00 P.M. is given by

$$T(t) = 98.6 + 4\cos\frac{\pi t}{36}, \quad 0 \le t \le 18.$$

Find the patient's temperature at each time.

(a) 10:00 P.M.

(b) 4:00 A.M.

(c) 10:00 A.M.

At what time do you expect the patient's temperature to return to normal? Explain your reasoning.

68. ***Sales*** A company that produces a window and door insulating kit forecasts monthly sales over the next 2 years to be

$$S = 23.1 + 0.442t + 4.3\sin\frac{\pi t}{6}$$

where S is measured in thousands of units and t is the time in months, with $t = 1$ corresponding to January 2006. Use a graphing utility to estimate sales for each month.

(a) February 2006 (b) February 2007

(c) September 2006 (d) September 2007

In Exercises 69 and 70, use a graphing utility to complete the table. Then graph the function.

x	0	2	4	6	8	10
$f(x)$						

69. $f(x) = \frac{2}{5}x + 2\sin\frac{\pi x}{5}$

70. $f(x) = \frac{1}{2}(5 - x) + 3\cos\frac{\pi x}{5}$

8.3

Graphs of Trigon

When you are sketching
use x (rather than θ) as
sketch the graph of a fu

$f(x) = \sin x$

by constructing a table
them with a smooth cur
shown in the table belo

x	0	$\frac{\pi}{6}$	
$\sin x$	0.00	0.50	C

In Figure 8.21, note that
is −1. The **amplitude**
be half of the differenc
amplitude of $f(x) = \sin$
The periodic nature
that as x increases beyo
oscillating about the x-
x-axis) between success

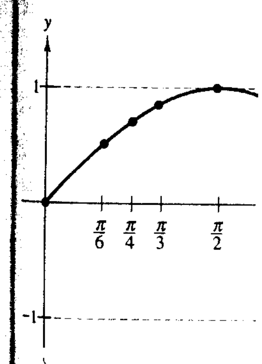


FIGURE 8.21

PREREQUISITE REVIEW 8.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, find the limit.

1. $\lim_{x \to 2} (x^2 + 4x + 2)$
2. $\lim_{x \to 3} (x^3 - 2x^2 + 1)$

In Exercises 3–10, evaluate the trigonometric function without using a calculator.

3. $\cos \frac{\pi}{2}$
4. $\sin \pi$
5. $\tan \frac{5\pi}{4}$
6. $\cot \frac{2\pi}{3}$
7. $\sin \frac{11\pi}{6}$
8. $\cos \frac{5\pi}{6}$
9. $\cos \frac{5\pi}{3}$
10. $\sin \frac{4\pi}{3}$

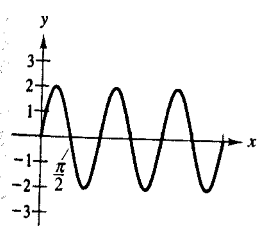
In Exercises 11–18, use a calculator to evaluate the trigonometric function to four decimal places.

11. $\cos 15°$
12. $\sin 220°$
13. $\sin 275°$
14. $\cos 310°$
15. $\sin 103°$
16. $\cos 72°$
17. $\tan 327°$
18. $\tan 140°$

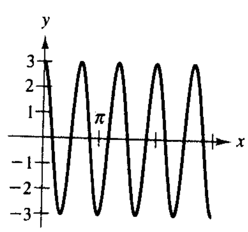
EXERCISES 8.3

In Exercises 1–14, determine the period and amplitude of the function.

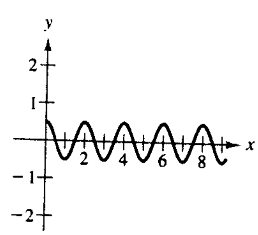
1. $y = 2 \sin 2x$



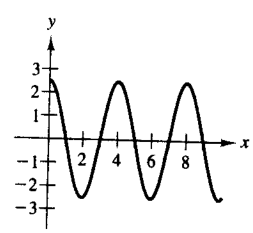
2. $y = 3 \cos 3x$



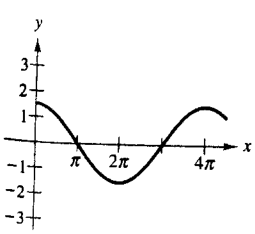
5. $y = \frac{1}{2} \cos \pi x$



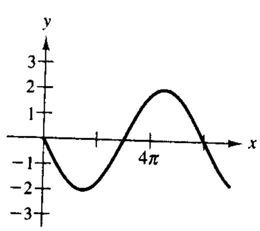
6. $y = \frac{5}{2} \cos \frac{\pi x}{2}$



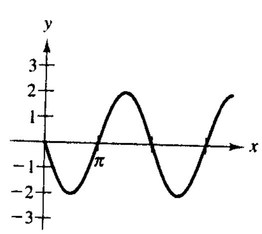
3. $y = \frac{3}{2} \cos \frac{x}{2}$



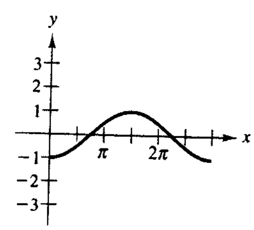
4. $y = -2 \sin \frac{x}{3}$



7. $y = -2 \sin x$



8. $y = -\cos \frac{2x}{3}$



9. $y = -2 \sin 10x$

10. $y = \frac{1}{3} \sin 8x$

11. $y = \dfrac{1}{2} \sin \dfrac{2x}{3}$

12. $y = 5 \cos \dfrac{x}{4}$

13. $y = 3 \sin 4\pi x$

14. $y = \dfrac{2}{3} \cos \dfrac{\pi x}{10}$

In Exercises 15–20, find the period of the function.

15. $y = 5 \tan 2x$

16. $y = 7 \tan 2\pi x$

17. $y = 3 \sec 5x$

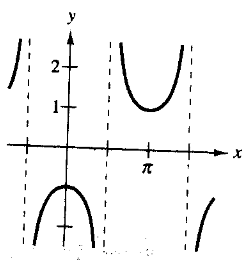
18. $y = \csc 4x$

19. $y = \cot \dfrac{\pi x}{6}$

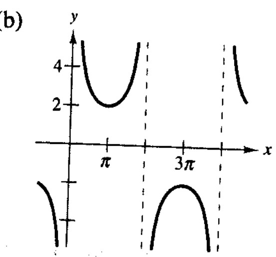
20. $y = 5 \tan \dfrac{2\pi x}{3}$

In Exercises 21–26, match the trigonometric function with the correct graph and give the period of the function. [The graphs are labeled (a)–(f).]

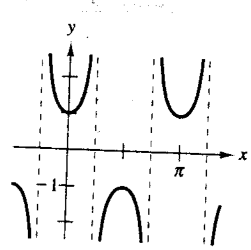
(a)



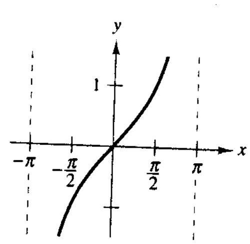
(b)



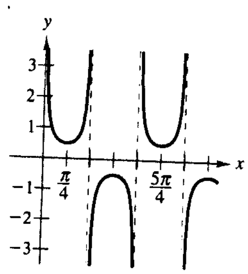
(c)



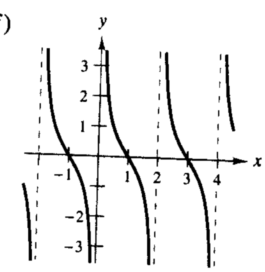
(d)



(e)



(f)



21. $y = \sec 2x$

22. $y = \frac{1}{2} \csc 2x$

23. $y = \cot \dfrac{\pi x}{2}$

24. $y = -\sec x$

25. $y = 2 \csc \dfrac{x}{2}$

26. $y = \tan \dfrac{x}{2}$

In Exercises 27–36, sketch the graph of the function by hand. Use a graphing utility to verify your sketch.

27. $y = \sin \dfrac{x}{2}$

28. $y = 4 \sin \dfrac{x}{3}$

29. $y = 2 \cos \dfrac{2x}{3}$

30. $y = \dfrac{3}{2} \cos \dfrac{2x}{3}$

31. $y = -2 \sin 6x$

32. $y = -3 \cos 4x$

33. $y = \cos 2\pi x$

34. $y = \dfrac{3}{2} \sin \dfrac{\pi x}{4}$

35. $y = 2 \tan x$

36. $y = 2 \cot x$

In Exercises 37–46, sketch the graph of the function.

37. $y = -\sin \dfrac{2\pi x}{3}$

38. $y = 10 \cos \dfrac{\pi x}{6}$

39. $y = \cot 2x$

40. $y = 3 \tan \pi x$

41. $y = \csc \dfrac{2x}{3}$

42. $y = \csc \dfrac{x}{3}$

43. $y = 2 \sec 2x$

44. $y = \sec \pi x$

45. $y = \csc 2\pi x$

46. $y = -\tan x$

In Exercises 47–56, complete the table (using a graphing utility set in *radian* mode) to estimate $\lim_{x \to 0} f(x)$.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

47. $f(x) = \dfrac{1 - \cos 2x}{2x}$

48. $f(x) = \dfrac{\sin 2x}{\sin 3x}$

49. $f(x) = \dfrac{\sin x}{5x}$

50. $f(x) = \dfrac{1 - \cos 2x}{x}$

51. $f(x) = \dfrac{3(1 - \cos x)}{x}$

52. $f(x) = \dfrac{2 \sin(x/4)}{x}$

53. $f(x) = \dfrac{\tan 2x}{x}$

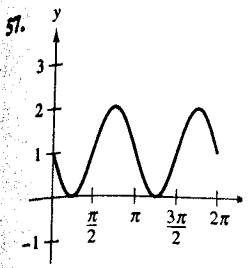
54. $f(x) = \dfrac{\tan 4x}{3x}$

55. $f(x) = \dfrac{1 - \cos^2 4x}{x}$

56. $f(x) = \dfrac{1 - \cos^2 x}{2x}$

In Exercises 57 and 58, deter
that the graph of $y = a \sin($

57.



59. ***Health*** For a person
second) of air flow int
piratory cycle is given

$$v = 0.9 \sin \frac{\pi t}{3}$$

where t is the time in
$v > 0$, and exhalation c

(a) Find the time for o

(b) Find the number of

(c) Use a graphing util

60. ***Health*** After exercis
a respiratory cycle for
approximated by

$$y = 1.75 \sin \frac{\pi t}{2}.$$

Use this model to repe

61. ***Music*** When tuning
fork for the A above mi
can be approximated b

$$y = 0.001 \sin 880\pi$$

where t is the time in s

(a) What is the period

(b) What is the freque

(c) Use a graphing util

62. ***Health*** The function

$$P = 100 - 20 \cos$$

approximates the blo
mercury) at time t in se

(a) Find the period of

(b) Find the number o

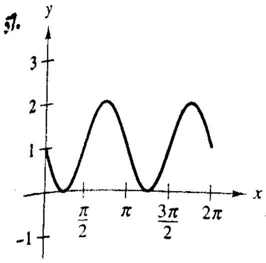
(c) Use a graphing util

63. ***Biology: Predator-P***
predator at time t (in m

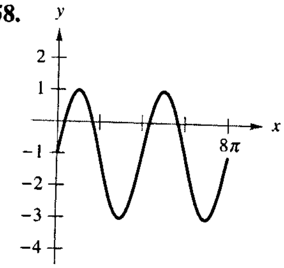
$$P = 8000 + 2500$$

In Exercises 57 and 58, determine the values of a, b, c, and d such that the graph of $y = a\sin(bx + c) + d$ is shown.

57.



58.



59. ***Health*** For a person at rest, the velocity v (in liters per second) of air flow into and out of the lungs during a respiratory cycle is given by

$$v = 0.9\sin\frac{\pi t}{3}$$

where t is the time in seconds. Inhalation occurs when $v > 0$, and exhalation occurs when $v < 0$.

(a) Find the time for one full respiratory cycle.

(b) Find the number of cycles per minute.

(c) Use a graphing utility to graph the velocity function.

60. ***Health*** After exercising for a few minutes, a person has a respiratory cycle for which the velocity of air flow is approximated by

$$y = 1.75\sin\frac{\pi t}{2}.$$

Use this model to repeat Exercise 59.

61. ***Music*** When tuning a piano, a technician strikes a tuning fork for the A above middle C and sets up wave motion that can be approximated by

$$y = 0.001\sin 880\pi t$$

where t is the time in seconds.

(a) What is the period p of this function?

(b) What is the frequency f of this note $(f = 1/p)$?

(c) Use a graphing utility to graph this function.

62. ***Health*** The function

$$P = 100 - 20\cos\frac{5\pi t}{3}$$

approximates the blood pressure P (in millimeters of mercury) at time t in seconds for a person at rest.

(a) Find the period of the function.

(b) Find the number of heartbeats per minute.

(c) Use a graphing utility to graph the pressure function.

63. ***Biology: Predator-Prey Cycle*** The population P of a predator at time t (in months) is modeled by

$$P = 8000 + 2500\sin\frac{2\pi t}{24}$$

and the population p of its prey is modeled by

$$p = 12{,}000 + 4000\cos\frac{2\pi t}{24}.$$

(a) Use a graphing utility to graph both models in the same viewing window.

(b) Explain the oscillations in the size of each population.

64. ***Biology: Predator-Prey Cycle*** The population P of a predator at time t (in months) is modeled by

$$P = 5700 + 1200\sin\frac{2\pi t}{24}$$

and the population p of its prey is modeled by

$$p = 9800 + 2750\cos\frac{2\pi t}{24}.$$

(a) Use a graphing utility to graph both models in the same viewing window.

(b) Explain the oscillations in the size of each population.

Sales In Exercises 65 and 66, sketch the graph of the sales function over 1 year where S is sales in thousands of units and t is the time in months, with $t = 1$ corresponding to January.

65. $S = 22.3 - 3.4\cos\dfrac{\pi t}{6}$ **66.** $S = 74.50 + 43.75\sin\dfrac{\pi t}{6}$

67. ***Biorhythms*** For the person born on July 20, 1984, use the biorhythm cycles given in Example 6 to calculate this person's three energy levels on December 31, 2008. Assume this is the 8930th day of the person's life.

68. ***Biorhythms*** Use your birthday and the concept of biorhythms as given in Example 6 to calculate your three energy levels on December 31, 2008. Use the internet or some other reference source to calculate the number of days between your birthday and December 31, 2008.

In Exercises 69 and 70, use a graphing utility to graph the functions in the same viewing window where $0 \le x \le 2$.

69. (a) $y = \dfrac{4}{\pi}\sin \pi x$

(b) $y = \dfrac{4}{\pi}\left(\sin \pi x + \dfrac{1}{3}\sin 3\pi x\right)$

70. (a) $y = \dfrac{1}{2} - \dfrac{4}{\pi^2}\cos \pi x$

(b) $y = \dfrac{1}{2} - \dfrac{4}{\pi^2}\left(\cos \pi x + \dfrac{1}{9}\cos 3\pi x\right)$

In Exercises 71–74, use a graphing utility to graph the function f and find $\lim\limits_{x\to 0} f(x)$.

71. $f(x) = \dfrac{\sin x}{x}$ **72.** $f(x) = \dfrac{\sin 5x}{2x}$

73. $f(x) = \dfrac{\sin 5x}{\sin 2x}$ **74.** $f(x) = \dfrac{\tan 2x}{3x}$

75. ***Sales*** The sales S (in thousands of units) of snowmobiles are modeled by

$$S = 58.3 + 32.5 \cos \frac{\pi t}{6}$$

where t is the time in months, with $t = 1$ corresponding to January and $t = 12$ corresponding to December.

(a) Use a graphing utility to graph S.

(b) Determine the months when sales exceed 75,000 units.

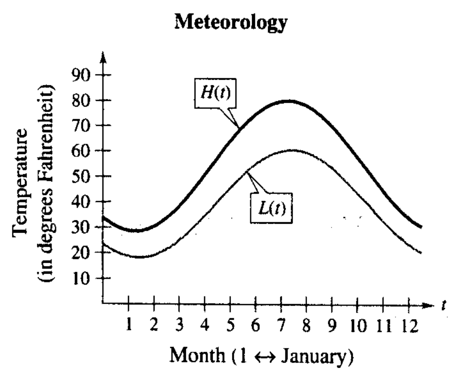
76. ***Meteorology*** The normal monthly high temperatures for Erie, Pennsylvania are approximated by

$$H(t) = 54.33 - 20.38 \cos \frac{\pi t}{6} - 15.69 \sin \frac{\pi t}{6}$$

and the normal monthly low temperatures are approximated by

$$L(t) = 39.36 - 15.70 \cos \frac{\pi t}{6} - 14.16 \sin \frac{\pi t}{6}$$

where t is the time in months, with $t = 1$ corresponding to January. *(Source: NOAA)* Use the figure to answer the questions below.

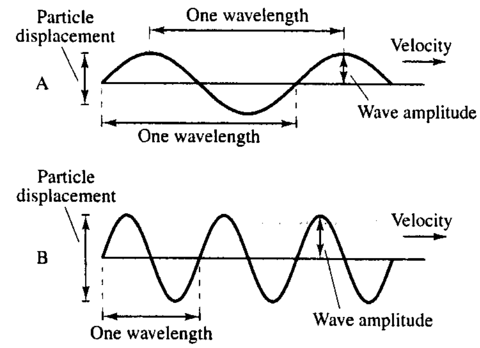


(a) During what part of the year is the difference between the normal high and low temperatures greatest? When is it smallest?

(b) The sun is the farthest north in the sky around June 21, but the graph shows the highest temperatures at a later date. Approximate the lag time of the temperatures relative to the position of the sun.

77. ***Finance: Cyclical Stocks*** The term "cyclical stock" describes the stock of a company whose profits are greatly influenced by changes in the economic business cycle. The market prices of cyclical stocks mirror the general state of the economy and reflect the various phases of the business cycle. Give a description and sketch of the graph of a given corporation's stock prices during recurrent periods of prosperity and recession. *(Source: Adapted from Garman/Forgue,* Personal Finance, *Fifth Edition)*

78. ***Physics*** Use the graphs below to answer each question.



(a) Which graph (A or B) has a longer wavelength, or period?

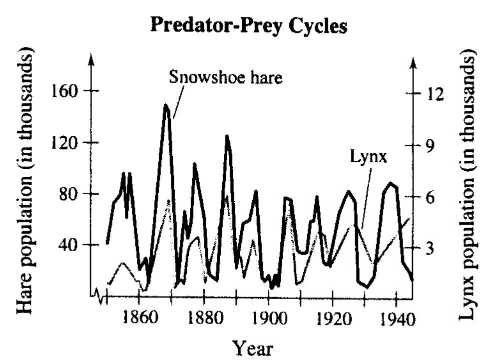
(b) Which graph (A or B) has a greater amplitude?

(c) The frequency of a graph is the number of oscillations or cycles that occur during a given period of time. Which graph (A or B) has a greater frequency?

(d) Based on the definition of frequency, how are frequency and period related?

(Source: Adapted from Shipman/Wilson/Todd, An Introduction to Physical Science, *Tenth Edition)*

79. ***Biology: Predator-Prey Cycles*** The graph below demonstrates snowshoe hare and lynx population fluctuations. The cycles of each population follow a periodic pattern. Describe several factors that could be contributing to the cyclical patterns. *(Source: Adapted from Levine/Miller,* Biology: Discovering Life, *Second Edition)*



True or False? In Exercises 80–83, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

80. The amplitude of $f(x) = -3 \cos 2x$ is -3.

81. The period of $f(x) = 5 \cot\left(-\frac{4x}{3}\right)$ is $\frac{3\pi}{2}$.

82. $\lim\limits_{x \to 0} \frac{\sin 5x}{3x} = \frac{5}{3}$

83. One solution of $\tan \frac{x}{2} = 1$ is $\frac{5\pi}{4}$.

PREREQUISITE REVIEW 8.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, find the derivative of the function.

1. $f(x) = 3x^3 - 2x^2 + 4x - 7$
2. $g(x) = (x^3 + 4)^4$
3. $f(x) = (x - 1)(x^2 + 2x + 3)$
4. $g(x) = \dfrac{2x}{x^2 + 5}$

In Exercises 5 and 6, find the relative extrema of the function.

5. $f(x) = x^2 + 4x + 1$
6. $f(x) = \frac{1}{3}x^3 - 4x + 2$

In Exercises 7–10, solve the trigonometric equation for x where $0 \le x \le 2\pi$.

7. $\sin x = \dfrac{\sqrt{3}}{2}$
8. $\cos x = -\dfrac{1}{2}$
9. $\cos \dfrac{x}{2} = 0$
10. $\sin \dfrac{x}{2} = -\dfrac{\sqrt{2}}{2}$

EXERCISES 8.4

In Exercises 1–26, find the derivative of the function.

1. $y = \frac{1}{2} - 3 \sin x$
2. $y = 5 + \sin x$
3. $y = x^2 - \cos x$
4. $g(t) = \pi \cos t - \dfrac{1}{t^2}$
5. $f(x) = 4\sqrt{x} + 3 \cos x$
6. $f(x) = \sin x + \cos x$
7. $f(t) = t^2 \cos t$
8. $f(x) = (x + 1) \cos x$
9. $g(t) = \dfrac{\cos t}{t}$
10. $f(x) = \dfrac{\sin x}{x}$
11. $y = \tan x + x^2$
12. $y = x + \cot x$
13. $y = e^{x^2} \sec x$
14. $y = e^{-x} \sin x$
15. $y = \cos 3x + \sin^2 x$
16. $y = \csc^2 x - \cos 2x$
17. $y = \sin \pi x$
18. $y = \frac{1}{2} \csc 2x$
19. $y = x \sin \dfrac{1}{x}$
20. $y = x^2 \sin \dfrac{1}{x}$
21. $y = 3 \tan 4x$
22. $y = \tan e^x$
23. $y = 2 \tan^2 4x$
24. $y = -\sin^4 2x$
25. $y = e^{2x} \sin 2x$
26. $y = e^{-x} \cos \dfrac{x}{2}$

In Exercises 27–38, find the derivative of the function and simplify your answer by using the trigonometric identities listed in Section 8.2.

27. $y = \cos^2 x$
28. $y = \frac{1}{4} \sin^2 2x$
29. $y = \cos^2 x - \sin^2 x$
30. $y = \dfrac{x}{2} + \dfrac{\sin 2x}{4}$
31. $y = \ln|\sin x|$
32. $y = -\ln|\cos x|$
33. $y = \ln|\csc x^2 - \cot x^2|$
34. $y = \ln|\sec x + \tan x|$
35. $y = \tan x - x$
36. $y = \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5}$
37. $y = \ln(\sin^2 x)$
38. $y = \frac{1}{2}(x \tan x - \sec x)$

In Exercises 39–46, find an equation of the tangent line to the graph of the function at the given point.

	Function	*Point*
39.	$y = \tan x$	$\left(-\frac{\pi}{4}, -1\right)$
40.	$y = \sec x$	$\left(\frac{\pi}{3}, 2\right)$
41.	$y = \sin 4x$	$(\pi, 0)$
42.	$y = \csc^2 x$	$\left(\frac{\pi}{2}, 1\right)$
43.	$y = \dfrac{\cos x}{\sin x}$	$\left(\frac{3\pi}{4}, -1\right)$
44.	$y = \sin x \cos x$	$\left(\frac{3\pi}{2}, 0\right)$
45.	$y = \ln\lvert\cot x\rvert$	$\left(\frac{\pi}{4}, 0\right)$
46.	$y = \sqrt{\sin x}$	$\left(\frac{\pi}{6}, \frac{\sqrt{2}}{2}\right)$

In Exercises 47 and 48, use implicit differentiation to find dy/dx and evaluate the derivative at the given point.

Function	*Point*
47. $\sin x + \cos 2y = 1$	$\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$
48. $\tan(x + y) = x$	$(0, 0)$

In Exercises 49–52, show that the function satisfies the differential equation.

49. $y = 2 \sin x + 3 \cos x$

$y'' + y = 0$

50. $y = \dfrac{10 - \cos x}{x}$

$xy' + y = \sin x$

51. $y = \cos 2x + \sin 2x$

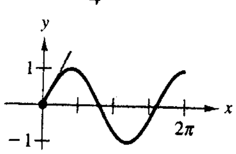
$y'' + 4y = 0$

52. $y = e^x(\cos \sqrt{2}x + \sin \sqrt{2}x)$

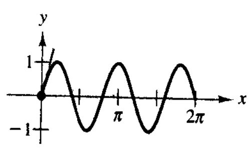
$y'' - 2y' + 3y = 0$

In Exercises 53–58, find the slope of the tangent line to the given sine function at the origin. Compare this value with the number of complete cycles in the interval $[0, 2\pi]$.

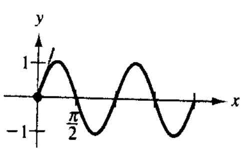
53. $y = \sin \dfrac{5x}{4}$



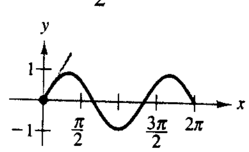
54. $y = \sin \dfrac{5x}{2}$



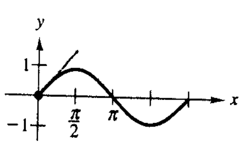
55. $y = \sin 2x$



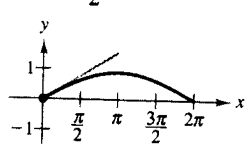
56. $y = \sin \dfrac{3x}{2}$



57. $y = \sin x$



58. $y = \sin \dfrac{x}{2}$



In Exercises 59–64, determine the relative extrema of the function on the interval $(0, 2\pi)$. Use a graphing utility to confirm your result.

59. $y = 2 \sin x + \sin 2x$

60. $y = 2 \sin x + \cos 2x$

61. $y = x - 2 \sin x$

62. $y = e^{-x} \sin x$

63. $y = e^{-x} \cos x$

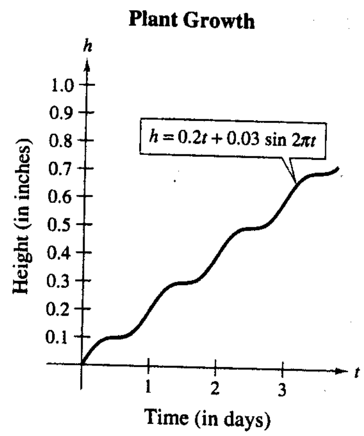
64. $y = \sec \dfrac{x}{2}$

65. ***Biology*** Plants do not grow at constant rates during a normal 24-hour period because their growth is affected by sunlight. Suppose that the growth of a certain plant species in a controlled environment is given by the model

$$h = 0.2t + 0.03 \sin 2\pi t$$

where h is the height of the plant in inches and t is the time in days, with $t = 0$ corresponding to midnight of day 1 (see figure). During what time of day is the rate of growth of this plant

(a) a maximum? (b) a minimum?



66. ***Meteorology*** The normal average daily temperature in degrees Fahrenheit for a city is given by

$$T = 55 - 21 \cos \frac{2\pi(t - 32)}{365}$$

where t is the time in days, with $t = 1$ corresponding to January 1. Find the expected date of

(a) the warmest day. (b) the coldest day.

67. ***Physics*** An amusement park ride is constructed such that its height h in feet above ground in terms of the horizontal distance x in feet from the starting point can be modeled by

$$h = 50 + 45 \sin \frac{\pi x}{150}, \quad 0 \le x \le 300.$$

(a) Use a graphing utility to graph the model. Be sure to choose an appropriate viewing window.

(b) Determine dh/dx and evaluate for $x = 50, 150, 200,$ and 250. Interpret these values of dh/dx.

(c) Find the maximum height and the minimum height of the ride.

(d) Find the distance from the starting point at which the ride's rate of change is the greatest.

68. ***Meteorology*** The nun
Orleans can be modeled

$D = 12.2 - 1.9 \cos$

where t represents the m
January. Find the month
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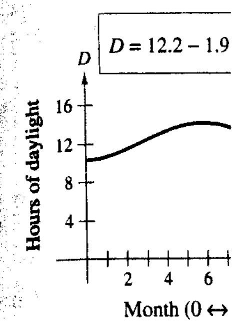
number of daylight hour



69. For $f(x) = \sec^2 x$ and $g($

70. For $f(x) = \sin^2 x$ and $g(x$

71. ***Physics*** A 15-centim
the equation

$\theta = 0.2 \cos 8t$

where θ is the angular
radians and t is the tim

(a) Determine the max

(b) Find the rate of cha

72. ***Tides*** Throughout th
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particular day can be m

$D = 3.5 + 1.5 \cos$

where $t = 0$ represents

(a) Determine dD/dt.

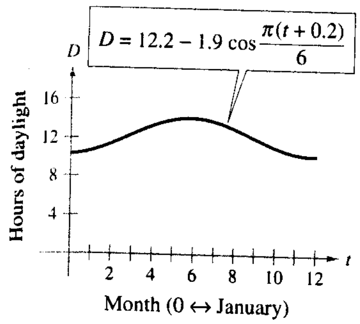
(b) Evaluate dD/dt fo
results.

(c) Find the time(s) w
and the time(s) wh

68. ***Meteorology*** The number of hours of daylight D in New Orleans can be modeled by

$$D = 12.2 - 1.9 \cos \frac{\pi(t + 0.2)}{6}, \quad 0 \le t \le 12$$

where t represents the month, with $t = 0$ corresponding to January. Find the month t when New Orleans has the maximum number of daylight hours. What is this maximum number of daylight hours?



69. For $f(x) = \sec^2 x$ and $g(x) = \tan^2 x$, show that $f'(x) = g'(x)$.

70. For $f(x) = \sin^2 x$ and $g(x) = \cos^2 x$, show that $f'(x) = -g'(x)$.

71. ***Physics*** A 15-centimeter pendulum moves according to the equation

$$\theta = 0.2 \cos 8t$$

where θ is the angular displacement from the vertical in radians and t is the time in seconds.

(a) Determine the maximum angular displacement.

(b) Find the rate of change of θ when $t = 3$ seconds.

72. ***Tides*** Throughout the day, the depth of water D in meters at the end of a dock varies with the tides. The depth for one particular day can be modeled by

$$D = 3.5 + 1.5 \cos \frac{\pi t}{6}, \quad 0 \le t \le 24$$

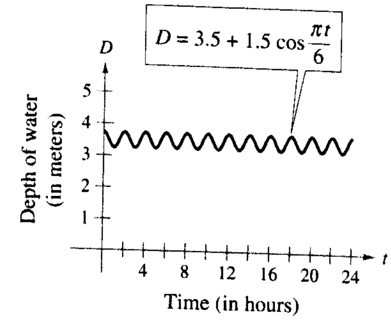
where $t = 0$ represents midnight.

(a) Determine dD/dt.

(b) Evaluate dD/dt for $t = 4$ and $t = 20$ and interpret your results.

(c) Find the time(s) when the water depth is the greatest and the time(s) when the water depth is the least.

(d) What is the greatest depth? What is the least depth? Did you have to use calculus to determine these depths? Explain your reasoning.



In Exercises 73–78, use a graphing utility (a) to graph f and f' on the same coordinate axes over the specified interval, (b) to find the critical numbers of f, and (c) to find the interval(s) on which f' is positive and the interval(s) on which it is negative. Note the behavior of f in relation to the sign of f'.

Function	*Interval*
73. $f(t) = t^2 \sin t$	$(0, 2\pi)$
74. $f(x) = \frac{x}{2} + \cos \frac{x}{2}$	$(0, 4\pi)$
75. $f(x) = \sin x - \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x$	$(0, \pi)$
76. $f(x) = x \sin x$	$(0, \pi)$
77. $f(x) = \sqrt{2x} \sin x$	$(0, 2\pi)$
78. $f(x) = 4e^{-0.5x} \sin \pi x$	$(0, 4)$

In Exercises 79–84, use a graphing utility to find the relative extrema of the trigonometric function. Let $0 < x < 2\pi$.

79. $f(x) = \dfrac{x}{\sin x}$

80. $f(x) = \dfrac{x^2 - 2}{\sin x} - 5x$

81. $f(x) = \ln x \cos x$

82. $f(x) = \ln x \sin x$

83. $f(x) = \sin(0.1x^2)$

84. $f(x) = \sin \sqrt{x}$

True or False? In Exercises 85–88, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

85. If $y = (1 - x)^{1/2}$, then $y' = \frac{1}{2}(1 - x)^{-1/2}$.

86. If $f(x) = \sin^2(2x)$, then $f'(x) = 2(\sin 2x)(\cos 2x)$.

87. If $y = x \sin^3 x$, then $y' = 3x \sin^2 x$.

88. The maximum value of $y = 3 \sin x + 2 \cos x$ is 5.