

Section 8.4

$$13.) y = e^{x^2} \sec x \xrightarrow{D} y' = e^{x^2} \sec x \tan x + 2xe^{x^2} \sec x$$

$$14.) y = e^{-x} \sin x \xrightarrow{D} y' = e^{-x} \cos x - e^{-x} \sin x$$

$$15.) y = \cos 3x + \sin^2 x \xrightarrow{D} y' = -\sin 3x \cdot (3) + 2 \sin x \cdot \cos x$$

$$16.) y = \csc^2 x - \cos 2x \xrightarrow{D}$$

$$y' = 2 \csc x \cdot -\csc x \cot x - -\sin 2x \cdot (2) \rightarrow$$

$$17.) y = \sin \pi x \xrightarrow{D} y' = \cos \pi x \cdot (\pi)$$

$$18.) y = \frac{1}{2} \csc 2x \xrightarrow{D} y' = \frac{1}{2} \cdot -\csc 2x \cot 2x \cdot (2)$$

$$19.) y = x \sin\left(\frac{1}{x}\right) \xrightarrow{D} y' = x \cdot \cos\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) + (1) \cdot \sin\left(\frac{1}{x}\right)$$

$$20.) y = x^2 \sin\left(\frac{1}{x}\right) \xrightarrow{D} y' = x^2 \cdot \cos\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) + (2x) \cdot \sin\left(\frac{1}{x}\right)$$

$$21.) y = 3 \tan(4x) \xrightarrow{D} y' = 3 \cdot \sec^2(4x) \cdot (4)$$

$$22.) y = \tan(e^x) \xrightarrow{D} y' = \sec^2(e^x) \cdot e^x$$

$$23.) y = 2 \tan^2(4x) \xrightarrow{D} y' = 2 \cdot 2 \tan(4x) \cdot \sec^2(4x) \cdot (4)$$

$$24.) y = -\sin^4(2x) \xrightarrow{D} y' = -4 \sin^3(2x) \cdot \cos(2x) \cdot (2)$$

$$25.) y = e^{2x} \sin(2x) \xrightarrow{D} y' = e^{2x} \cos(2x) \cdot (2) + 2e^{2x} \cdot \sin(2x)$$

$$26.) y = e^{-x} \cos\left(\frac{x}{2}\right) \xrightarrow{D} y' = e^{-x} \cdot \sin\left(\frac{x}{2}\right) \cdot \left(\frac{1}{2}\right) - e^{-x} \cdot \cos\left(\frac{x}{2}\right)$$

$$28.) y = \frac{1}{4} \sin^2(2x) \xrightarrow{D} y' = \frac{1}{4} \cdot 2 \sin(2x) \cdot \cos(2x) \cdot (2)$$

$$31.) y = \ln|\sin x| \xrightarrow{D} y' = \frac{1}{\sin x} \cdot \cos x$$

$$36.) y = \frac{1}{7} \sec^7 x - \frac{1}{5} \sec^5 x \xrightarrow{D}$$

$$y' = \frac{1}{7} \cdot 7 \sec^6 x \cdot \sec x \tan x - \frac{1}{5} \cdot 5 \sec^4 x \cdot \sec x \tan x$$

$$41.) y = \sin(4x) \xrightarrow{D} y' = \cos(4x) \cdot 4 \text{ and}$$

$$x = \pi, y = 0 \text{ so slope } m = 4 \cos(4\pi) = 4(1) = 4$$

$$\text{and line is } y - y_1 = m(x - x_1) \rightarrow$$

$$y - 0 = 4(x - \pi) \rightarrow \underline{y = 4x - 4\pi}$$

$$46.) y = (\sin x)^{1/2} \xrightarrow{D} y' = \frac{1}{2} (\sin x)^{-1/2} \cdot \cos x$$

$$\text{and } x = \frac{\pi}{6}, y = \frac{\sqrt{2}}{2} \text{ so slope}$$

$$m = \frac{1}{2} \cdot \frac{\cos(\pi/6)}{\sqrt{\sin(\pi/6)}} = \frac{1}{2} \cdot \frac{\sqrt{3}/2}{\sqrt{1/2}} = \frac{\sqrt{3}/2}{\sqrt{1/2}} = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4}$$

$$\text{and line is } y - y_1 = m(x - x_1) \rightarrow$$

$$\underline{y - \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} (x - \frac{\pi}{6})}$$

Section 2.5

$$17.) y = (2x-7)^3 \xrightarrow{D} y' = 3(2x-7)^2 \cdot (2)$$

$$22.) f(x) = (4x-x^2)^3 \xrightarrow{D} f'(x) = 3(4x-x^2)^2 \cdot (4-2x)$$

$$23.) f(x) = (x^2-9)^{2/3} \xrightarrow{D} f'(x) = \frac{2}{3}(x^2-9)^{-1/3} \cdot (2x)$$

$$30.) y = 2(4-x^2)^{1/2} \xrightarrow{D} y' = 2 \cdot \frac{1}{2}(4-x^2)^{-1/2} \cdot (-2x)$$

$$38.) f(x) = x \cdot \sqrt{x^2+5} \xrightarrow{D}$$

$$f'(x) = x \cdot \frac{1}{2}(x^2+5)^{-1/2} (2x) + (1) \cdot \sqrt{x^2+5} \rightarrow$$

$$f'(x) = \frac{x^2}{\sqrt{x^2+5}} + \sqrt{x^2+5} \quad \text{and } x=2, f(2)=6$$

so slope $m = \frac{4}{\sqrt{9}} + \sqrt{9} = \frac{4}{3} + 3 = \frac{13}{3}$ and
line is $y - y_1 = m(x - x_1) \rightarrow \underline{y - 6 = \frac{13}{3}(x - 2)}$.

$$39.) f(x) = (x^2-2x+1)^{1/2} \xrightarrow{D} f'(x) = \frac{1}{2}(x^2-2x+1)^{-1/2} \cdot (2x-2)$$

$$\rightarrow f'(x) = \frac{x-1}{\sqrt{x^2-2x+1}} \quad \text{and } x=2, f(2)=1 \text{ so}$$

slope $m = \frac{1}{\sqrt{1}} = 1$ and line is $y - y_1 = m(x - x_1)$
 $\rightarrow y - 1 = 1 \cdot (x - 2) \rightarrow \underline{y = x - 1}$.

$$49.) f(x) = (x^2-3x)^{-2} \xrightarrow{D} f'(x) = -2(x^2-3x)^{-3} (2x-3)$$

$$59.) f(x) = \left(\frac{3-2x}{4x}\right)^{1/2} \xrightarrow{D}$$

$$f'(x) = \frac{1}{2} \left(\frac{3-2x}{4x}\right)^{-1/2} \cdot \frac{4x \cdot (-2) - (3-2x) \cdot (4)}{(4x)^2}$$

$$64.) y = \left(\frac{4x^2}{3-x} \right)^3 \xrightarrow{D}$$

$$y' = 3 \left(\frac{4x^2}{3-x} \right)^2 \cdot \frac{(3-x) \cdot (8x) - (4x^2) \cdot (-1)}{(3-x)^2}$$

$$67.) f(t) = (t^2-9)\sqrt{t+2} \xrightarrow{D}$$

$$f'(t) = (t^2-9) \cdot \frac{1}{2}(t+2)^{-1/2} + (2t) \cdot \sqrt{t+2} \text{ and}$$

$$t=-1, f(-1) = -8 \text{ so slope}$$

$$m = (-8) \left(\frac{1}{2} \right) (1)^{-1/2} - 2\sqrt{1} = -4 - 2 = -6$$

$$\text{and line is } y - (-8) = -6(x - (-1)) \rightarrow$$

$$y + 8 = -6(x + 1) = -6x - 6 \rightarrow \underline{y = -6x - 14}$$

$$70.) y = \frac{x}{\sqrt{25+x^2}} \xrightarrow{D} y' = \frac{\sqrt{25+x^2} \cdot (1) - x \cdot \frac{1}{2}(25+x^2)^{-1/2}(2x)}{25+x^2}$$

$$\text{and } x=0, y=0 \text{ so slope } m = \frac{5-0}{25} = \frac{1}{5}$$

$$\text{and line is } y - 0 = \frac{1}{5}(x - 0) \rightarrow \underline{y = \frac{1}{5}x}$$

$$73.) N = 400 - 1200(t^2+2)^{-2} \rightarrow$$

$$\frac{dN}{dt} = 2400(t^2+2)^{-3} \cdot 2t = \frac{4800t}{(t^2+2)^3}$$

t	0	1	2	3	4	days
$\frac{dN}{dt}$	0	177.8	44.4	10.8	3.3	<u>bacteria</u> day

The growth rate for the number of bacteria is decreasing.

74.) FACT: "A is inversely proportional to B" means $A = \frac{k}{B}$ for some constant k. Then ...

a.) $V = \frac{k}{\sqrt{t+1}}$ and $t=0, V = \$10,000$ so

$$10,000 = \frac{k}{\sqrt{0+1}} \rightarrow k = 10,000 \rightarrow \boxed{V = 10,000 (t+1)^{-1/2}};$$

b.) c.) rate of depreciation is

$$\frac{dV}{dt} = -5000 (t+1)^{-3/2} \quad \text{so}$$

$$t=1 \rightarrow \frac{dV}{dt} = -\$1767.77 / \text{yr.} \quad \text{and}$$

$$t=3 \rightarrow \frac{dV}{dt} = -\$625 / \text{yr.}$$

Worksheet 5

1.) a.) $Y' = \cos(x^3) \cdot 3x^2$

b.) $Y' = 3 \sin^2 x \cdot \cos x$

c.) $Y' = 3 \sin^2(x^3) \cdot \cos(x^3) \cdot 3x^2$

d.) $f'(x) = \cos(3+\sqrt{x}) \cdot 2 \sin(5x) \cdot \cos(5x) \cdot 5$
 $+ -\sin(3+\sqrt{x}) \cdot \frac{1}{2} x^{-1/2} \cdot \sin^2(5x)$

e.) $g'(x) = -\sin(\sin^4(7x^3)) \cdot 4 \sin^3(7x^3) \cdot \cos(7x^3) \cdot 21x^2$

f.) $Y' = \frac{\cos^2(3x) \cdot 3 \sec^2(5x) \cdot -\sec(5x) \cot(5x) \cdot 5 - \sec^3(5x) \cdot 2 \cos(3x) \cdot \sin(3x) \cdot 3}{\cos^4(3x)}$

2.) $Y = \frac{\sin 2x}{\cos 3x + \tan x}$

$$\rightarrow Y' = \frac{(\cos 3x + \tan x) \cdot 2 \cos 2x - \sin 2x \cdot (-3 \sin 3x + \sec^2 x)}{(\cos 3x + \tan x)^2},$$

at $x = \frac{\pi}{6}$, $Y = \frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{2} + \tan \frac{\pi}{6}} = \frac{\frac{\sqrt{3}}{2}}{0 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{1} = \frac{3}{2}$,

and slope $Y' = \frac{(\cos \frac{\pi}{2} + \tan \frac{\pi}{6}) \cdot 2 \cos \frac{\pi}{3} - \sin \frac{\pi}{3} (-3 \sin \frac{\pi}{2} + \sec^2 \frac{\pi}{6})}{(\cos \frac{\pi}{2} + \tan \frac{\pi}{6})^2}$
 $= \frac{(0 + \frac{1}{\sqrt{3}}) \cdot 2 \cdot \frac{1}{2} - \frac{\sqrt{3}}{2} (-3 \cdot 1 + (\frac{2}{\sqrt{3}})^2)}{(0 + \frac{1}{\sqrt{3}})^2} = \left(\frac{1}{\sqrt{3}} - \frac{-5}{2\sqrt{3}} \right) \cdot 3$

$$= \sqrt{3} + \frac{5}{2} \sqrt{3} = \frac{7}{2} \sqrt{3} \text{ so tangent line is}$$

$$Y - \frac{3}{2} = \frac{7}{2} \sqrt{3} \left(x - \frac{\pi}{6} \right).$$

$$3.) Y = \sin\left(\frac{\pi}{3} \tan 2x\right) \rightarrow$$

$$Y' = \cos\left(\frac{\pi}{3} \tan 2x\right) \cdot \frac{\pi}{3} \sec^2 2x \cdot 2, \text{ at } x = \frac{\pi}{8}$$

$$Y' = \cos\left(\frac{\pi}{3} \tan \frac{\pi}{4}\right) \cdot \frac{\pi}{3} \sec^2 \frac{\pi}{4} \cdot 2$$

$$= \frac{2\pi}{3} \cos\left(\frac{\pi}{3} \cdot 1\right) (\sqrt{2})^2 = \frac{4\pi}{3} \cdot \frac{1}{2} = \frac{2\pi}{3} \text{ so}$$

slope of line $\perp$ is $m = -\frac{3}{2\pi}$.

$$4.) a.) f(x) = \frac{1}{2} \sin 2x + \sin x \rightarrow$$

$$f'(x) = \frac{1}{2} \cdot 2 \cos 2x + \cos x = (2 \cos^2 x - 1) + \cos x = 0 \rightarrow$$

$$(2 \cos x - 1)(\cos x + 1) = 0$$

$$\downarrow$$

$$\cos x = \frac{1}{2}$$

$$\downarrow$$

$$\cos x = -1 \rightarrow x = \pi$$

$$\hookrightarrow \text{Unit Circle Diagram} \quad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$b.) f(x) = \cos^2 x + \sin x \rightarrow$$

$$f'(x) = -2 \cos x \cdot \sin x + \cos x = 0 \rightarrow$$

$$\cos x (1 - 2 \sin x) = 0$$

$$\downarrow$$

$$\cos x = 0$$

$$\downarrow$$

$$\sin x = \frac{1}{2}$$

$$\text{Unit Circle Diagram}$$

$$\rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\cos x = 0 \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

Supplemental Trig

$$ST6.) Y = \sin^3(\tan^2(3x))$$

$$Y' = 3 \sin^2(\tan^2(3x)) \cdot \cos(\tan^2(3x)) \cdot 2 \tan(3x) \cdot \sec^2(3x) \cdot 3$$