Section 2.6

1.) $f(x) = 5 - 4x \xrightarrow{D} f'(x) = -4 \xrightarrow{D} f''(x) = 0$

4.) $f(x) = 3x^2 + 4x \xrightarrow{D} f'(x) = 6x + 4 \xrightarrow{D} f''(x) = 6$

7.) $f(t) = \frac{3}{4}t^{-2} \xrightarrow{D} f'(t) = \frac{3}{4}(-2)t^{-3} = -\frac{3}{2}t^{-3} \xrightarrow{D}$
$f''(t) = \frac{9}{2}t^{-4}$

11.) $f(x) = \frac{x+1}{x-1} \xrightarrow{D} f'(x) = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2}$
$\rightarrow f'(x) = \frac{\cancel{x} - 1 - \cancel{x} - 1}{(x-1)^2} = \frac{-2}{(x-1)^2} = -2(x-1)^{-2}$
$\xrightarrow{D} f''(x) = 4(x-1)^{-3}$

13.) $y = x^4 + 4x^3 + 8x^2 \xrightarrow{D} y' = 4x^3 + 12x^2 + 16x \xrightarrow{D}$
$y'' = 12x^2 + 24x + 16$

16.) $f(x) = x^4 - 2x^3 \xrightarrow{D} f'(x) = 4x^3 - 6x^2 \xrightarrow{D}$
$f''(x) = 12x^2 - 12x \xrightarrow{D} f'''(x) = 24x - 12$

19.) $f(x) = \frac{3}{16}x^{-2} \xrightarrow{D} f'(x) = -\frac{6}{16}x^{-3} = -\frac{3}{8}x^{-3} \xrightarrow{D}$
$f''(x) = \frac{9}{8}x^{-4} \xrightarrow{D} f'''(x) = -\frac{36}{8}x^{-5} = -\frac{9}{2}x^{-5}$

20.) $f(x) = x^{-1} \xrightarrow{D} f'(x) = -x^{-2} \xrightarrow{D} f''(x) = 2x^{-3}$
$\xrightarrow{D} f'''(x) = -6x^{-4}$

21.) $g(t) = 5t^4 + 10t^2 + 3 \xrightarrow{D} g'(t) = 20t^3 + 20t \xrightarrow{D}$
$g''(t) = 60t^2 + 20$ so $g''(2) = 60(4) + 20 = 260$

24.) $f(t)=\sqrt{2t+3} \xrightarrow{D} f'(t)=\frac{1}{2}(2t+3)^{-1/2}\cdot(2)$

$=(2t+3)^{-1/2} \xrightarrow{D} f''(t)=-\frac{1}{2}(2t+3)^{-3/2}\cdot(2)$

$=-(2t+3)^{-3/2} \xrightarrow{D} f'''(t)=\frac{3}{2}(2t+3)^{-5/2}\cdot(2)$

$\rightarrow f'''(t)=3(2t+3)^{-5/2}$ so

$f'''(\frac{1}{2})=3(4)^{-5/2}=3\cdot\frac{1}{(4^{1/2})^5}=3\cdot\frac{1}{2^5}=\frac{3}{32}$

34.) $f(x)=3x^3-9x+1 \xrightarrow{D} f'(x)=9x^2-9=9(x-1)(x+1)=0$

$\longrightarrow$ $x=1$, $x=-1$; then

$f''(x)=18x=0 \longrightarrow$ $x=0$.

35.) $f(x)=(x+3)(x-4)(x+5) \xrightarrow{D}$ (TPR)

$f'(x)=(1)(x-4)(x+5)+(x+3)(1)(x+5)+(x+3)(x-4)(1)$

$=x^2+x-20+x^2+8x+15+x^2-x-12$

$=3x^2+8x-17=0 \longrightarrow$

$$x=\frac{-8\pm\sqrt{64-4(3)(-17)}}{2(3)}=\frac{-8\pm\sqrt{268}}{6}$$

$$\rightarrow x=\frac{-8\pm2\sqrt{67}}{6}=\frac{-4\pm\sqrt{67}}{3}$$; then

$f''(x)=6x+8=0 \longrightarrow x=-8/6 \longrightarrow$ $x=-4/3$.

38.) $f(x)=x\sqrt{4-x^2} \xrightarrow{D}$

$f'(x)=x\cdot\frac{1}{2}(4-x^2)^{-1/2}\cdot(-2x)+(1)\cdot(4-x^2)^{1/2}$

$$=\frac{-x^2}{(4-x^2)^{1/2}}+\frac{(4-x^2)^{1/2}}{1}=\frac{-x^2+(4-x^2)}{(4-x^2)^{1/2}}$$

$$=\frac{4-2x^2}{(4-x^2)^{1/2}}=0 \rightarrow 4-2x^2=2(2-x^2)=0 \rightarrow$$

$\boxed{x=\sqrt{2},\ x=-\sqrt{2}}$; $\xrightarrow{D}$

$$f''(x) = \frac{(4-x^2)^{1/2}(-4x) - (4-2x^2)\cdot\frac{1}{2}(4-x^2)^{-1/2}\cdot(-2x)}{(4-x^2)}$$

$$= \frac{\dfrac{-4x(4-x^2)^{1/2}}{1} + \dfrac{x(4-2x^2)}{(4-x^2)^{1/2}}}{\dfrac{(4-x^2)}{1}} = \frac{-4x(4-x^2)+x(4-2x^2)}{(4-x^2)^{1/2}(4-x^2)}$$

$$= \frac{2x\left[-2(4-x^2)+(2-x^2)\right]}{(4-x^2)^{3/2}} = \frac{2x\left[x^2-6\right]}{(4-x^2)^{3/2}} = 0 \longrightarrow$$

$$2x\left[x^2-6\right]=0 \longrightarrow \boxed{x=0,\ x=\sqrt{6},\ x=-\sqrt{6}}$$

40.) $f(x) = \dfrac{x}{x^2+1} \rightarrow f'(x) = \dfrac{(x^2+1)(1)-x(2x)}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2} = 0$

$$\rightarrow 1-x^2=0 \rightarrow \boxed{x=1,\ x=-1}\ ;$$

$$f''(x) = \frac{(x^2+1)^2(-2x)-(1-x^2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$$

$$= \frac{-2x\,\cancel{(x^2+1)}\cdot\left[(x^2+1)+2(1-x^2)\right]}{(x^2+1)^{\cancel{4}\,3}} = \frac{-2x\left[3-x^2\right]}{(x^2+1)^3} = 0 \rightarrow$$

$$-2x\left[3-x^2\right]=0 \rightarrow x=0,\ x=\pm\sqrt{3}$$

FACT: Let $s(t)$ be height (feet) of object above ground at time t (seconds). Then it can be shown that

$$\boxed{s(t) = -16t^2 + v_0 t + s_0},$$

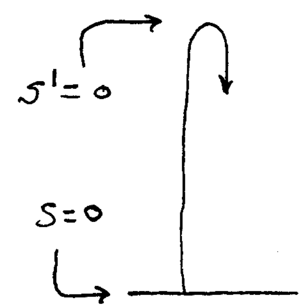
where v_0 is initial velocity and s_0 is initial height.

41.) a.) $v_0 = 144$ ft./sec. and $s_0 = 0$ ft. $\rightarrow$

$s(t) = -16t^2 + 144t$ is height at time t

b.) velocity: $s'(t) = -32t + 144$ and

acceleration: $s''(t) = -32$



c.) highest point: $s'(t) = 0 \rightarrow$

$-32t + 144 = 0 \rightarrow$ $t = 4.5$ sec.

and $s(4.5) = -16(4.5)^2 + 144(4.5)$

$= 324$ ft.

d.) hit ground: $s(t) = 0 \rightarrow 0 = -16t^2 + 144t \rightarrow$

$0 = 16t(-t+9) \rightarrow t = 0$ or $t = 9$ sec.,

$s'(9) = -144$ ft./sec. and velocity is 144 ft./sec., the same as initial speed.

42.) a.) $v_0 = 0$ ft./sec. and $s_0 = 1250$ ft. $\rightarrow$

$s(t) = -16t^2 + 0 \cdot t + 1250 = 1250 - 16t^2$

is height at time t

b.) velocity: $s'(t) = -32t$

acceleration: $s''(t) = -32$

c.) hit ground: $s(t) = 0 \rightarrow 1250 - 16t^2 = 0$

$\rightarrow t^2 = \frac{1250}{16} = 78.12 \rightarrow$ $t \approx 8.84$ sec.

d.) $s'(8.84) \approx -32(8.84) \approx -282.8$ ft./sec.

43.) velocity $s'(t) = \dfrac{90t}{t+10}$ ft./sec. and

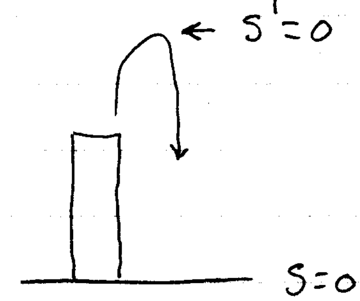
acceleration $s''(t) = \dfrac{(t+10)(90) - 90t(1)}{(t+10)^2} = \dfrac{900}{(t+10)^2}$ ft./sec.^{2}

t	:	0	10	20	30	40	50	60	sec.
s'	:	0	45	60	67.5	72	75	77.14	ft./sec.
s''	:	9	2.25	1	0.56	0.36	0.25	0.18	ft./sec.^{2}

Velocity is increasing;
acceleration is decreasing;
it appears car is approaching
a constant velocity (0 acceleration)

50.) a.) $V_0 = 48$ ft./sec. and $S_0 = 64$ ft. $\rightarrow$

$s(t) = -16t^2 + 48t + 64$ is height at time t



b.) velocity : $s'(t) = -32t + 48$

acceleration : $s''(t) = -32$

c.) hit ground : $s(t) = 0 \rightarrow$

$-16t^2 + 48t + 64 = -16(t^2 - 3t - 4)$

$= -16(t-4)(t+1) = 0 \rightarrow t = -1$ (no) or

$t = 4$ sec.

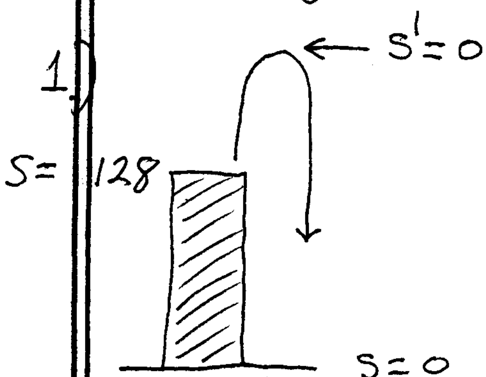
d.) $s'(t) = 0 \rightarrow -32t + 48 = 0 \rightarrow$

$t = \frac{48}{32} = \frac{3}{2}$ sec.

e.) $s\left(\frac{3}{2}\right) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 64 = 100$ ft.

Gravity Problems

1.)



$s(t) = -16t^2 + V_0 t + s_0 \rightarrow$ $\boxed{s(t) = -16t^2 + 112t + 128}$

$\xrightarrow{D}$ $\boxed{s'(t) = -32t + 112}$

a.) highest point: $s'(t) = 0$

$\rightarrow -32t + 112 = 0 \rightarrow t = \frac{112}{32} = \underline{3.5 \text{ sec.}} \rightarrow$

$s(3.5) = -16(3.5)^2 + 112(3.5) + 128 = 324 \text{ ft.}$

b.) hit ground: $s(t) = 0 \rightarrow$

$-16t^2 + 112t + 128 = 0 \rightarrow -16(t^2 - 7t - 8) = 0 \rightarrow$

$-16(t-8)(t+1) = 0 \rightarrow t = -1$ (NO) or

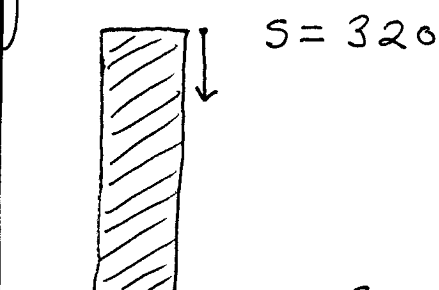
$t = 8$ sec.

c.) i.) $s'(3) = -32(3) + 112 = 16$ ft./sec

ii.) $s'(4) = -32(4) + 112 = -16$ ft./sec.

iii.) $s'(8) = -32(8) + 112 = -144$ ft./sec.

2.)



$s(t) = -16t^2 + V_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 - 16t + 320} \rightarrow$

$\boxed{s'(t) = -32t - 16}$

a.) hit ground: $s(t) = 0 \rightarrow$

$-16t^2 - 16t + 320 = 0 \rightarrow -16(t^2 + t - 20) = 0 \rightarrow$

$-16(t-4)(t+5) = 0 \rightarrow t = -5$ (NO) or

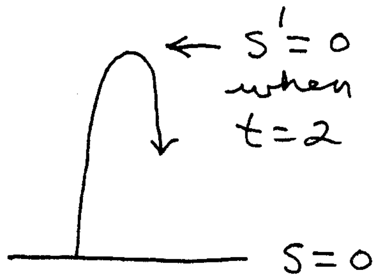
$\boxed{t = 4 \text{ sec.}}$

b.) i.) $s'(1) = -32(1) - 16 = -48$ ft./sec.

ii.) $s'(2) = -32(2) - 16 = -80$ ft./sec.

iii.) $s'(4) = -32(4) - 16 = -144$ ft./sec.

3.)



$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 + v_0 t} \rightarrow$

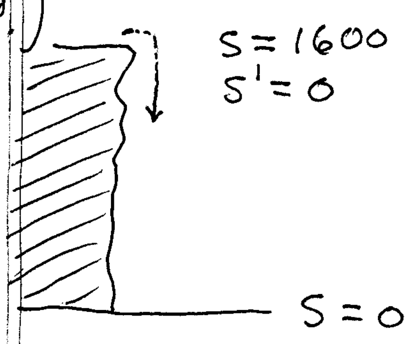
$\boxed{s'(t) = -32t + v_0}$

and $s'(2) = 0 \rightarrow -32(2) + v_0 = 0$

$\rightarrow$ b.) $v_0 = 64$ ft./sec.

a.) $s(2) = -16(2)^2 + 64(2) = 64$ ft.

4.)



$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 + 1600} \rightarrow$

$\boxed{s'(t) = -32t}$

a.) hit ground: $s(t) = 0 \rightarrow$

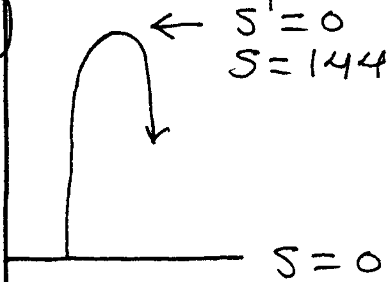
$-16t^2 + 1600 = 0 \rightarrow 16t^2 = 1600 \rightarrow t^2 = 100 \rightarrow$

$\boxed{t = 10 \text{ sec.}}$

b.) i.) $s'(5) = -32(5) = -160$ ft./sec.

ii.) $s'(10) = -32(10) = \boxed{-320 \text{ ft./sec}}$;

$$-\frac{320 \text{ ft.}}{\text{sec}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft.}} \cdot \frac{3600 \text{ sec.}}{1 \text{ hr.}} \approx \boxed{-218.2 \text{ mph}}$$

5.) 

$s(t) = -16t^2 + V_0 t + S_0 \rightarrow$

$\boxed{s(t) = -16t^2 + V_0 t} \rightarrow$

$\boxed{s'(t) = -32t} + V_0;$

Let $t = T$ be time required to reach highest point. Then

$$\left.\begin{array}{l} s'(T) = -32T + V_0 = 0 \\ s(T) = -16T^2 + V_0 T = 144 \end{array}\right\} \rightarrow$$

$\boxed{V_0 = 32T} \xrightarrow{(SUB)} -16T^2 + (32T)T = 144 \rightarrow$

$16T^2 = 144 \rightarrow T^2 = 9 \rightarrow T = 3$ sec.

a.) $T = 3$ sec

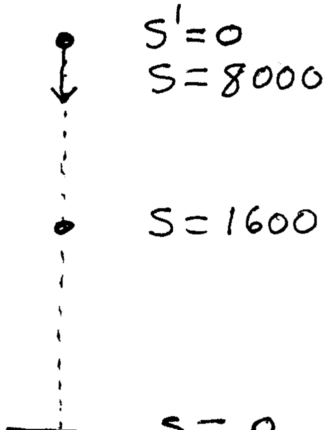
c.) $V_0 = 32T = 32(3) = 96$ ft./sec.

b.) hit ground : $s(t) = 0 \rightarrow$

$-16t^2 + 96t = 0 \rightarrow 16t(6 - t) = 0 \rightarrow$

$t = 0$ and $t = 6$ sec.

d.) You know.

6.) 

$s(t) = -16t^2 + V_0 t + S_0 \rightarrow$

$\boxed{s(t) = -16t^2 + 8000} \rightarrow$

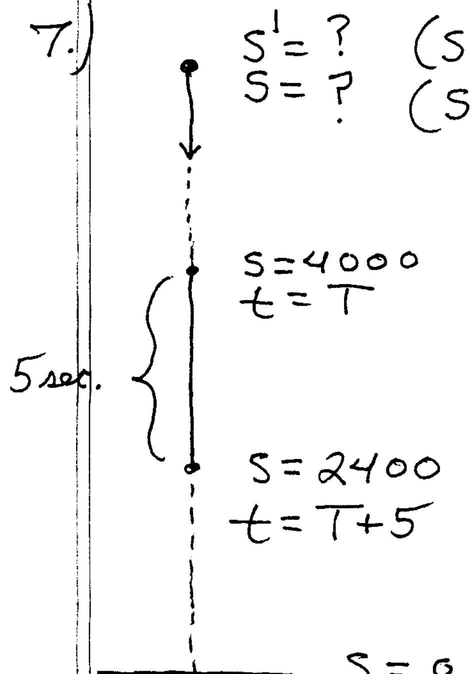
$\boxed{s'(t) = -32t}$

a.) $s(t) = 1600 \rightarrow -16t^2 + 8000 = 1600$

$\rightarrow 16t^2 = 6400 \rightarrow t^2 = 400 \rightarrow t = 20 \text{ sec.}$

b.) $s'(20) = -32(20) = -640$ ft./sec.

7.)



$(s' = v_0)$
$(s = s_0)$

$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$
$s'(t) = -32t + v_0$;

a.) $s'(10) = -400 \rightarrow$
$-32(10) + v_0 = -400 \rightarrow$
$v_0 = -80$ ft./sec.

b.) $\left.\begin{array}{l} s(T) = 4000 \\ s(T+5) = 2400 \end{array}\right\} \rightarrow$

$\left.\begin{array}{r} -16T^2 - 80T + s_0 = 4000 \\ -16(T+5)^2 - 80(T+5) + s_0 = 2400 \end{array}\right\} \rightarrow$

$\boxed{s_0 = 4000 + 80T + 16T^2} \rightarrow$ (SUB) $\rightarrow$

$-16(T^2 + 10T + 25) - 80T - 400$
$+ (4000 + 80T + 16T^2) = 2400 \rightarrow$

$\cancel{-16T^2} - 160T - 400 - \cancel{80T} - 400$
$+ 4000 + \cancel{80T} + \cancel{16T^2} = 2400 \rightarrow$

$160T = 800 \rightarrow T = 5$ sec. $\rightarrow$

$s_0 = 4000 + 80(5) + 16(5)^2 \rightarrow$

$s_0 = 4800$ ft.

c.) $s(t) = -16t^2 - 80t + 4800$,

strike ground: $s(t) = 0 \rightarrow$

$-16t^2 - 80t + 4800 = 0 \rightarrow$

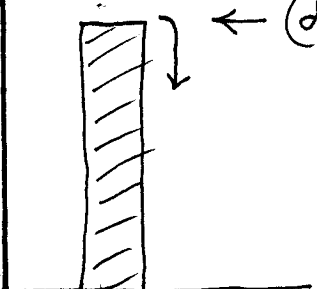
$-16(t^2 + 5t - 300) = 0 \rightarrow$

$-16(t-15)(t+20) = 0 \rightarrow$

$t = 15$ sec. $\quad$ $\cancel{t = -20}$

d.) Snapple Peach Ice Tea

8.)



$\leftarrow$ (dropped) $s' = 0$

$s = 0$

Let H be height of building. Then

$s(t) = -16t^2 + (0)t + H \rightarrow$

$\boxed{s(t) = -16t^2 + H} \xrightarrow{D}$

$\boxed{s'(t) = -32t}$;

a.) Given $s(5) = 0$ ft. $\rightarrow -16(5)^2 + H = 0$

$\rightarrow \boxed{H = 400 \text{ ft}}$

b.) $s'(1) = -32(1) =$ -32 ft./sec. ;

$s'(3) = -32(3) =$ -96 ft./sec.

c.) $s'(5) = -32(5) =$ -160 ft./sec.

$= \frac{-160 \text{ ft.}}{\text{sec.}} \times \frac{1 \text{ mi.}}{5280 \text{ ft.}} \times \frac{3600 \text{ sec.}}{1 \text{ hr.}} \approx$ 109.1 mph