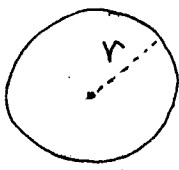


Section 2.8

$$2.) \quad Y = x^2 - 3x \rightarrow \frac{dY}{dt} = 2x \cdot \frac{dx}{dt} - 3 \cdot \frac{dx}{dt} = (2x - 3) \cdot \frac{dx}{dt}$$

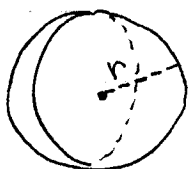
$$a.) \quad \frac{dY}{dt} = (2 \cdot 3 - 3) \cdot (2) = 6$$

$$b.) \quad 5 = (2 \cdot 1 - 3) \cdot \frac{dx}{dt} \rightarrow \frac{dx}{dt} = -5$$

5.)  $A = \pi r^2 \rightarrow \frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$
and $\frac{dr}{dt} = 2 \text{ in./min.}$

$$a.) \quad \frac{dA}{dt} = \pi \cdot 2(6)(2) = 24\pi \text{ in}^2/\text{min.}$$

$$b.) \quad \frac{dA}{dt} = \pi \cdot 2(24)(2) = 96\pi \text{ in}^2/\text{min.}$$

6.)  $V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$
and $\frac{dr}{dt} = 2 \text{ in./min.}$

$$a.) \quad \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3(6)^2 \cdot (2) = 288\pi \text{ in}^3/\text{min.}$$

$$b.) \quad \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3(24)^2 \cdot (2) = 4608\pi \text{ in}^3/\text{min.}$$

9.) $V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$ and
 $\frac{dV}{dt} = 20 \text{ ft}^3/\text{min.}$

$$a.) \quad 20 = \frac{4}{3}\pi \cdot 3(1)^2 \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{5}{\pi} \text{ ft./min.}$$

$$b.) \quad 20 = \frac{4}{3}\pi \cdot 3(2)^2 \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{5}{4\pi} \text{ ft./min.}$$

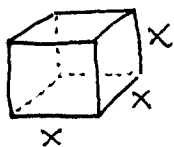
10.] $V = \frac{1}{3}\pi r^2 h$ and $h = 3r$ so $V = \pi r^3 \rightarrow$

$\frac{dV}{dt} = \pi \cdot 3r^2 \cdot \frac{dr}{dt}$ and $\frac{dr}{dt} = 2$ in./min.

a.) $\frac{dV}{dt} = \pi \cdot 3(6)^2 \cdot (2) = 216\pi$ in.³/min.

b.) $\frac{dV}{dt} = \pi \cdot 3(24)^2 \cdot (2) = 3456\pi$ in.³/min.

13.]



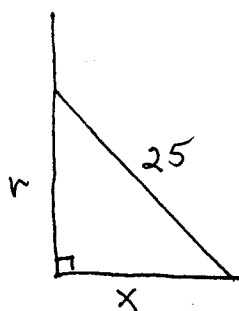
$V = x^3 \rightarrow \frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$

and $\frac{dx}{dt} = 3$ cm./sec.

a.) $\frac{dV}{dt} = 3(1)^2(3) = 9$ cm.³/sec.

b.) $\frac{dV}{dt} = 3(10)^2(3) = 900$ cm.³/sec.

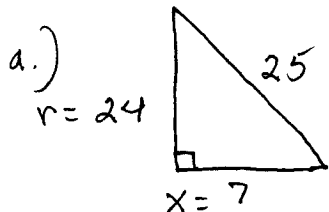
17.]



$x^2 + r^2 = 25^2$ and $\frac{dx}{dt} = 2$ ft./sec

find $\frac{dr}{dt}$:

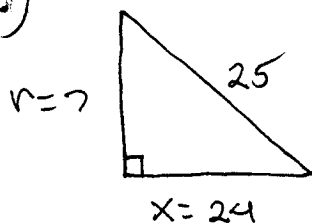
$2x \cdot \frac{dx}{dt} + 2r \cdot \frac{dr}{dt} = 0 \rightarrow$



$2(7)(2) + 2(24) \frac{dr}{dt} = 0 \rightarrow$

$\frac{dr}{dt} = \frac{-28}{48} = \frac{-7}{12}$ ft./sec.

c.)



$2(24)(2) + 2(7) \frac{dr}{dt} = 0 \rightarrow$

$\frac{dr}{dt} = \frac{-96}{14} = \frac{-48}{7}$ ft./sec.

Worksheet 6

1.) a.) $XY^3 + X^2Y = X - Y \rightarrow$

$$X \cdot 3Y^2 \cdot Y' + Y^3 + X^2Y' + 2XY = 1 - Y' \rightarrow$$

$$3XY^2Y' + X^2Y' + Y' = 1 - 2XY - Y^3 \rightarrow$$

$$(3XY^2 + X^2 + 1)Y' = 1 - 2XY - Y^3 \rightarrow$$

$$Y' = \frac{1 - 2XY - Y^3}{3XY^2 + X^2 + 1}$$

b.) $Y^2(X+Y)^3 = X^2 \rightarrow$

$$Y^2 \cdot 3(X+Y)^2(1+Y') + 2YY'(X+Y)^3 = 2X \rightarrow$$

$$3Y^2(X+Y)^2 + 3Y^2(X+Y)^2Y' + 2Y(X+Y)^3Y' = 2X \rightarrow$$

$$[3Y^2(X+Y)^2 + 2Y(X+Y)^3]Y' = 2X - 3Y^2(X+Y)^2 \rightarrow$$

$$Y' = \frac{2X - 3Y^2(X+Y)^2}{3Y^2(X+Y)^2 + 2Y(X+Y)^3}$$

c.) $\sin(3X+2Y) = \tan(X^3) \rightarrow$

$$\cos(3X+2Y) \cdot (3+2Y') = \sec^2(X^3) \cdot 3X^2 \rightarrow$$

$$3\cos(3X+2Y) + 2\cos(3X+2Y)Y' = 3X^2\sec^2(X^3) \rightarrow$$

$$2\cos(3X+2Y)Y' = 3X^2\sec^2(X^3) - 3\cos(3X+2Y) \rightarrow$$

$$Y' = \frac{3X^2\sec^2(X^3) - 3\cos(3X+2Y)}{2\cos(3X+2Y)}$$

d.) $Y \cdot \sec(Y^2+1) = \cos(X+Y) \rightarrow$

$$Y \cdot \sec(Y^2+1) \tan(Y^2+1) \cdot 2YY' + Y' \sec(Y^2+1) = -\sin(X+Y) \cdot (1+Y') \rightarrow$$

$$2Y^2 \sec(Y^2+1) \tan(Y^2+1) \cdot Y' + \sec(Y^2+1) \cdot Y' \\ = -\sin(x+Y) - \sin(x+Y) \cdot Y' \rightarrow$$

$$2Y^2 \sec(Y^2+1) \tan(Y^2+1) Y' + \sec(Y^2+1) \cdot Y' \\ + \sin(x+Y) \cdot Y' = -\sin(x+Y) \rightarrow$$

$$[2Y^2 \sec(Y^2+1) \tan(Y^2+1) + \sec(Y^2+1) + \sin(x+Y)] \cdot Y' \\ = -\sin(x+Y) \rightarrow$$

$$Y' = \frac{-\sin(x+Y)}{2Y^2 \sec(Y^2+1) \tan(Y^2+1) + \sec(Y^2+1) + \sin(x+Y)}$$

$$e.) (x - \csc(3Y))^5 = 7+Y \rightarrow$$

$$5(x - \csc(3Y))^4 \cdot [1 + \csc(3Y) \cot(3Y) \cdot 3Y'] = Y' \rightarrow$$

$$5(x - \csc(3Y))^4 + 15(x - \csc(3Y))^4 \csc(3Y) \cot(3Y) \cdot Y' = Y' \rightarrow$$

$$15(x - \csc(3Y))^4 \csc(3Y) \cot(3Y) \cdot Y' - Y' = -5(x - \csc(3Y))^4 \rightarrow$$

$$[15(x - \csc(3Y))^4 \csc(3Y) \cot(3Y) - 1] Y' = -5(x - \csc(3Y))^4 \rightarrow$$

$$Y' = \frac{-5(x - \csc(3Y))^4}{15(x - \csc(3Y))^4 \csc(3Y) \cot(3Y) - 1}$$

$$2.) \quad XY + X^2 + Y^3 = 8 \xrightarrow{D} XY' + Y + 2X + 3Y^2Y' = 0 \rightarrow \\ (X + 3Y^2)Y' = -2X - Y \rightarrow Y' = \frac{-2X - Y}{X + 3Y^2},$$

at $x=0$, $Y^3=8$ so $Y=2$ and slope of tangent line is

$$Y' = \frac{0 - 2}{0 + 12} = -\frac{1}{6}.$$

$$3.) \quad X^2Y + Y^3 = X + 1 \xrightarrow{D} X^2Y' + 2XY + 3Y^2Y' = 1 \rightarrow \\ (X^2 + 3Y^2)Y' = 1 - 2XY \quad \left| \begin{array}{l} \text{differentiate again} \end{array} \right. \xrightarrow{D}$$

$$\rightarrow Y' = \frac{1 - 2XY}{X^2 + 3Y^2}$$

$$\text{at } x=0, Y^3=1$$

$\rightarrow Y=1$ so that

$$Y' = \frac{1}{3}$$

$$X^2Y'' + 2XY' + 2XY' + 2Y \\ + 3Y^2Y'' + 6YY' \cdot Y' = 0,$$

$$\text{let } x=0, Y=1, \text{ and } Y' = \frac{1}{3}$$

so that

$$(0)Y'' + (0)\left(\frac{1}{3}\right) + (0)\left(\frac{1}{3}\right) + 2(1) + 3(1)Y'' + 6(1)\left(\frac{1}{3}\right)^2 = 0$$

$$\rightarrow 3Y'' = -2 - \frac{2}{3} = -\frac{8}{3} \rightarrow Y'' = -\frac{8}{9} < 0$$

so graph is concave down at $x=0$

Supplemental Trig

$$\text{ST4.) } x^3 + \sin Y = Y^2 + 7x \xrightarrow{D}$$

$$3x^2 + \cos Y \cdot Y' = 2Y \cdot Y' + 7 \rightarrow$$

$$\cos Y \cdot Y' - 2Y \cdot Y' = 7 - 3x^2 \rightarrow$$

$$(\cos Y - 2Y) \cdot Y' = 7 - 3x^2 \rightarrow$$

$$Y' = \frac{7 - 3x^2}{\cos Y - 2Y} \quad ; \quad \text{if } x=0, Y=0 \text{ then}$$

$$\text{slope } m = Y' = \frac{7 - 0}{\cos 0 - 0} = \frac{7}{1} = 7 \text{ and}$$

tangent line is $\boxed{Y = 7X}$.

$$\text{ST5.) } (x + \tan Y)^2 = 8 + \sin(xY) \xrightarrow{D}$$

$$2(x + \tan Y) \cdot (1 + \sec^2 Y \cdot Y') = \cos(xY) \cdot (x \cdot Y' + (1) \cdot Y) \rightarrow$$

$$2(x + x \sec^2 Y \cdot Y' + \tan Y + \tan Y \sec^2 Y \cdot Y') = x \cos(xY) \cdot Y' + Y \cos(xY) \rightarrow$$

$$2x + 2x \sec^2 Y \cdot Y' + 2 \tan Y + 2 \tan Y \sec^2 Y \cdot Y' = x \cos(xY) \cdot Y' + Y \cos(xY) \rightarrow$$

$$2x \sec^2 Y \cdot Y' + 2 \tan Y \sec^2 Y \cdot Y' - x \cos(xY) \cdot Y' = Y \cos(xY) - 2x - 2 \tan Y \rightarrow$$

$$(2x \sec^2 Y + 2 \tan Y \sec^2 Y - x \cos(xY)) \cdot Y' = Y \cos(xY) - 2x - 2 \tan Y \rightarrow$$

$$Y' = \frac{Y \cos(xY) - 2x - 2 \tan Y}{2x \sec^2 Y + 2 \tan Y \sec^2 Y - x \cos(xY)} \quad ; \quad \text{if } x=2, Y=\frac{\pi}{4}$$

slope of tangent line is

$$m = Y' = \frac{\frac{\pi}{4} \cos \frac{\pi}{2} - 4 - 2 \tan \frac{\pi}{4}}{4 \sec^2 \frac{\pi}{4} + 2 \tan \frac{\pi}{4} \sec^2 \frac{\pi}{4} - 2 \cos \frac{\pi}{2}}$$

$$= \frac{\frac{\pi}{4}(0) - 4 - 2(1)}{4(\sqrt{2})^2 + 2(1)(\sqrt{2})^2 - 2(0)} = \frac{-6}{12} = -\frac{1}{2}, \text{ and}$$

slope of $\perp$ line is $\boxed{m=2}$