Section 1.6

3.) Since $Y=1$ (line) and $Y=X^2-4$ (parabola) are continuous for all X-values, it follows that $f(x)=\frac{1}{x^2-4}$ is continuous for all (quotient) X-values except where $x^2-4=(x-2)(x+2)=0$, that is, except at $X=2$ and $X=-2$.

5.) Since $Y=1$ (line) and $Y=4+x^2$ (parabola) are continuous for all X-values, it follows that $f(x)=\frac{1}{x^2+4}$ is continuous for all (quotient) X-values (since x^2+4 is never zero).

11.) Since $Y=X^2-1$ (parabola) and $Y=X$ (line) are continuous for all X-values, it follows that $f(x)=\frac{x^2-1}{x}$ is continuous for all X-values except $X=0$. (quotient)

14.) $y=x^3-8$ (polynomial) and $y=X-2$ (line) are continuous for all X-values, so it follows that $f(x)=\frac{x^3-8}{x-2}$ is continuous for all values of X except $X=2$.

21.) Since $Y = X-5$ (line) and $Y = X^2-9X+20$ (parabola) are continuous for all values of X, it follows that $f(X) = \dfrac{X-5}{X^2-9X+20}$ is (quotient) → continuous for all values of X except where $X^2-9X+20 = (X-4)(X-5) = 0$, that is, except at $X=4$ and $X=5$.

25.) $f(x) = \begin{cases} -2X+3, & X<1 \\ x^2, & X \geq 1 \end{cases}$ We need only check for continuity at $X=1$ since $Y = -2X+3$ (line) and $Y = X^2$ (parabola) are continuous graphs:

i.) $f(1) = (1)^2 = 1$,

ii) $\lim\limits_{X\to 1^+} f(x) = \lim\limits_{X\to 1^+} (X^2) = (1)^2 = 1$ and

$\lim\limits_{X\to 1^-} f(x) = \lim\limits_{X\to 1^-} (-2X+3) = -2+3 = 1$ so $\lim\limits_{X\to 1} f(x) = 1$

and iii) i.) and ii) are equal.

Thus f is continuous at $X=1$ and all other X-values.

27.) $f(X) = \begin{cases} \frac{1}{2}X+1, & X \leq 2 \\ 3-X, & X > 2 \end{cases}$ We need only check for continuity at $X=2$ since $Y = \frac{1}{2}X+1$ (line) and $Y = 3-X$ (line) are continuous graphs:

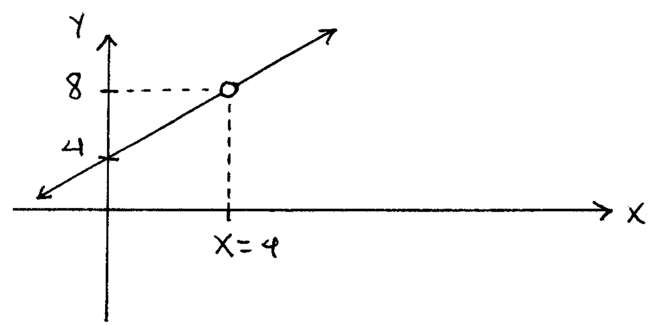
i.) $f(2) = \frac{1}{2}(2)+1 = 2$,

ii.) $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3-x) = 1$ and

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (\frac{1}{2}x+1) = 2$, so $\lim_{x \to 2} f(x)$ does not exist and f is NOT continuous at $x=2$. Thus, f is continuous for all x-values except $x=2$.

30.) $y = |4-x|$ is continuous for all values of x (well known: $\vee$) and $y = 4-x$ is continuous (line) for all values of x, so it follows that $f(x) = \frac{|4-x|}{4-x}$ (quotient) is continuous for all values of x except $x=4$.

39.) $f(x) = \frac{x^2-16}{x-4} = \frac{(x-4)(x+4)}{x-4}$ for $x \neq 4$; that is, $f(x) = x+4$ for $x \neq 4$. Function f has a removable discontinuity at $x=4$:



i.) $f(4)$ is NOT defined, but

ii.) $\lim_{x \to 4} f(x) = \lim_{x \to 4} (x+4) = 8$.

44.) $f(x) = \begin{cases} x^2-4 & \text{if } x \le 0 \\ 2x+4 & \text{if } x > 0 \end{cases}$;

$y = x^2-4$ (parabola) is continuous for $x<0$ and $y = 2x+4$ (line) is continuous for $x>0$; check at $x=0$:

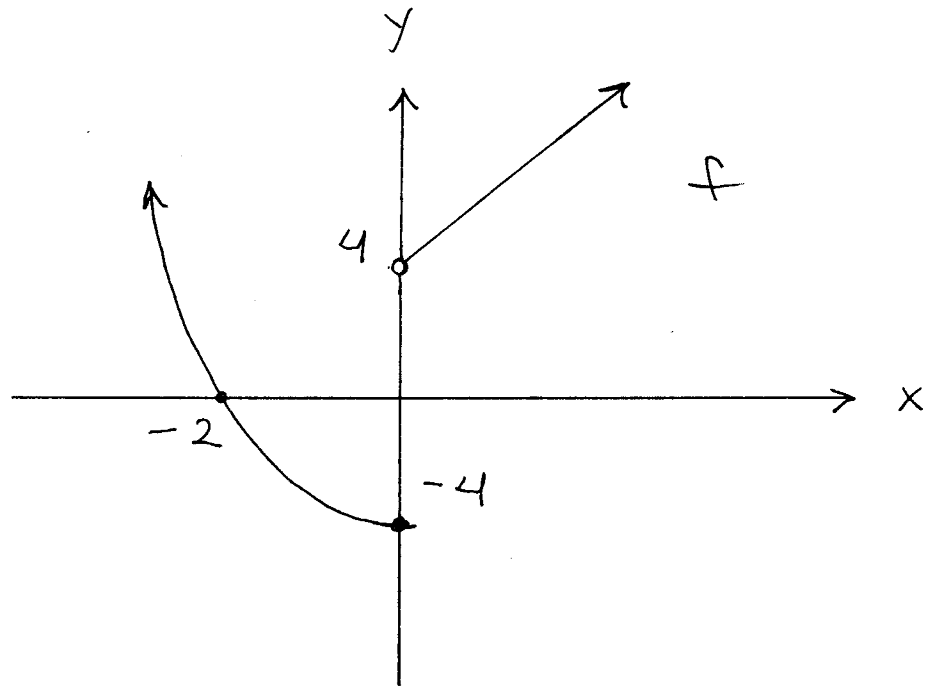
i.) $f(0) = 0^2-4 = -4$,

ii.) $\displaystyle\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (2x+4) = 4$ and

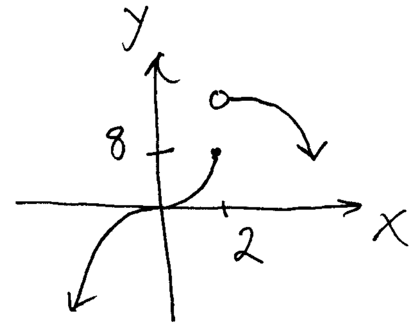
$\displaystyle\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x^2-4) = -4$, so

$\displaystyle\lim_{x\to 0} f(x)$ D.N.E. and f is NOT continuous at $x=0$;

thus, f is continuous for all values of x except $x=0$.

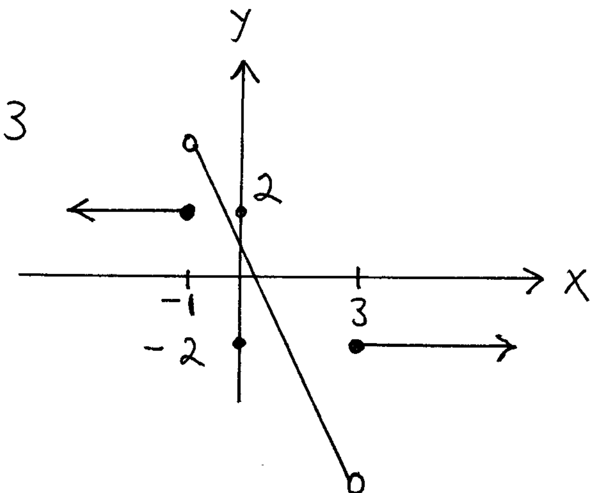


45.) $f(x) = \begin{cases} x^3 & \text{if } x \le 2 \\ ax^2 & \text{if } x > 2 \end{cases}$



$$\left.\begin{aligned} \lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} (ax^2) = 4a \\ \lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} (x^3) = 8 \end{aligned}\right\} \quad 4a = 8 \text{ so } a = 2.$$

46.) $f(x) = \begin{cases} 2 & \text{if } x \le -1 \\ ax+b & \text{if } -1 < x < 3 \\ -2 & \text{if } x \ge 3 \end{cases}$



$$\left.\begin{aligned} \lim_{x\to -1^+} f(x) &= \lim_{x\to -1^+} (ax+b) = b-a \\ \lim_{x\to -1^-} f(x) &= \lim_{x\to -1^-} (2) = 2 \end{aligned}\right\} \quad b-a = 2$$

$$\left.\begin{aligned} \lim_{x\to 3^+} f(x) &= \lim_{x\to 3^+} (-2) = -2 \\ \lim_{x\to 3^-} f(x) &= \lim_{x\to 3^-} (ax+b) = 3a+b \end{aligned}\right\} \quad 3a+b = -2 \qquad \text{so}$$

$$\left.\begin{aligned} b-a &= 2 \\ 3a+b &= -2 \end{aligned}\right\} \to b = a+2 \to 3a + (a+2) = -2 \to 4a = -4 \to$$

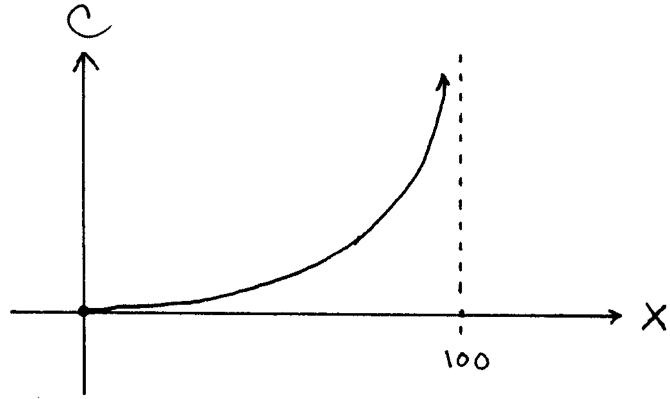
$a = -1$ and $b = 1$.

60.) $C = \dfrac{2X}{100-X}$, X percent

a.) Since X represents a measure of percent, negative values of X are impossible, and X is at least 0 and at most 100, but function C is not defined at $X=100$; thus the domain of C is

$$0 \le X < 100 .$$

b.)



Function C is continuous on $0 \le X < 100$ since it is the ratio of continuous functions on $0 \le X < 100$.

c.) If $X = 75$, then

$C = \dfrac{2(75)}{100-75} = 6$, so that the cost of removing 75% of the pollutants is \$6 million.

Worksheet 2

1.) a.) $\lim_{x\to 6} f(x) = \lim_{x\to 6} \frac{x^2-7x+6}{x-6} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 6} \frac{\cancel{(x-6)}(x-1)}{\cancel{x-6}} = 5$

so choose $a=5$.

b.) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (a^2x-a) = a^2-a$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2) = 2$

$\Big\}\ a^2-a=2 \rightarrow$

$a^2-a-2=0 \rightarrow (a-2)(a+1)=0 \rightarrow a=2$ or $a=-1$.

c.) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (ax^3+3) = 3$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \left(\frac{a+x}{a+1}\right) = \frac{a}{a+1}$

$\Big\}\ 3 = \frac{a}{a+1} \rightarrow$

$3a+3=a \rightarrow 2a=-3 \rightarrow a=-3/2$.

d.) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (ax^2+b) = a+b$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (3) = 3$

$\Big\}\ a+b=3$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (5) = 5$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (ax^2+b) = 4a+b$

$\Big\}\ 4a+b=5$

$a+b=3 \rightarrow b=3-a$

$4a+b=5 \quad \rightarrow 4a+(3-a)=5 \rightarrow$

$3a = 2 \rightarrow$ $a = 2/3$, $b = 7/3$.

e.) $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (2x + 3a + b) = -2 + 3a + b$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (ax - b) = -a - b$

$-2 + 3a + b = -a - b \rightarrow$

$4a + 2b = 2 \rightarrow$ $\boxed{2a + b = 1}$;

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (4) = 4$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (2x + 3a + b) = 2 + 3a + b$

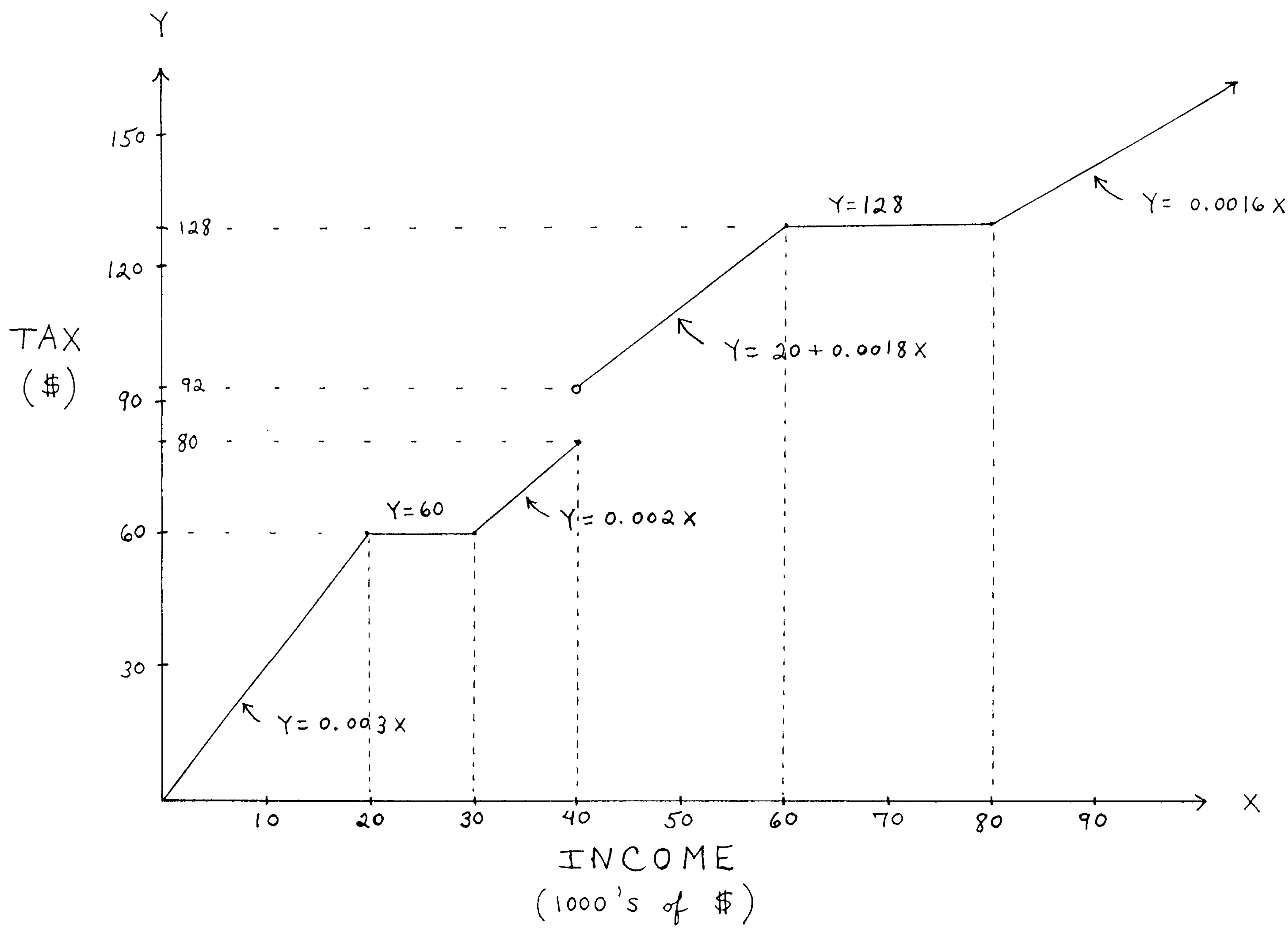
$2 + 3a + b = 4 \rightarrow$ $\boxed{3a + b = 2}$;

$b = 1 - 2a$ so $3a + (1 - 2a) = 2 \rightarrow$ $a = 1$ and $b = -1$.

2.) a.) See next page.

b.) This tax scheme is unfair for 2 reasons. The flat portions of the scheme are unfair to those with incomes at the beginning of the flat portion. For example, a person making \$20,000 pays the same tax, \$60, as one making \$30,000. The point of discontinuity is also unfair. A person making \$40,000 pays \$80 in taxes and a person making \$40,001 pays \$92 in taxes!

An Example of Continuity as Fairness



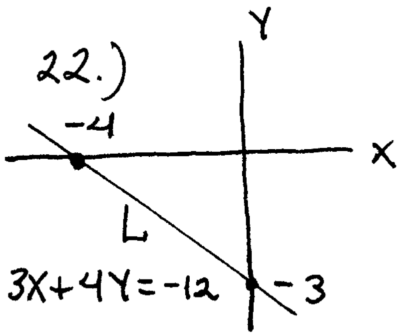
Worksheet 1

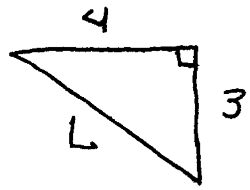
21.) Find equation of line passing through pts. (5.2, 27.8) and (5.3, 29.2) :

slope $m = \dfrac{29.2 - 27.8}{5.3 - 5.2} = 14$ so line is

$Y - 27.8 = 14\,(X - 5.2)$ OR $Y = 14X - 45$

22.)





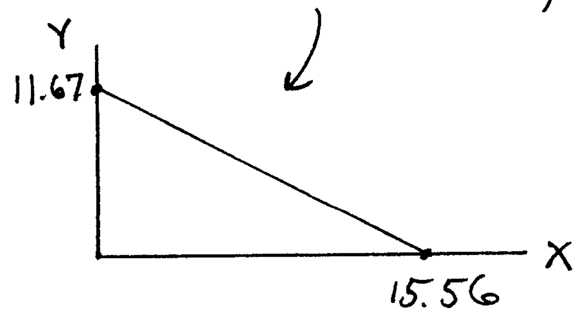
$3^2 + 4^2 = L^2 \rightarrow$

$L = 5$

24.) a.) Let X : # of Mexican peppers

Y : # of Indian peppers

$900X + 1200Y = 14{,}000$

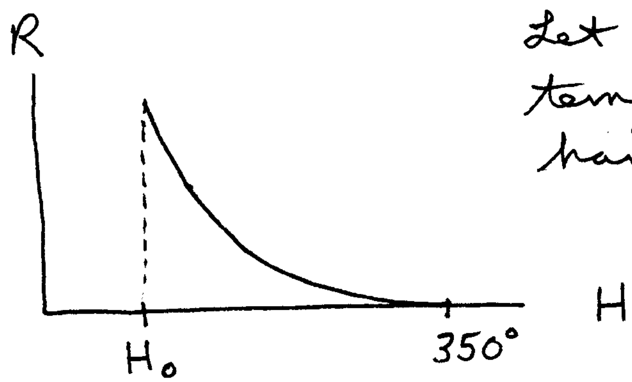


b.) Solve for Y :

$Y = {}^{-}0.75X + 11.67$

25.) a.) $R = k\,(350 - H)$

b.)



Let H_0 : initial temperature of ham

27.) a.) $C = 100 + 2q = f(q) \rightarrow$

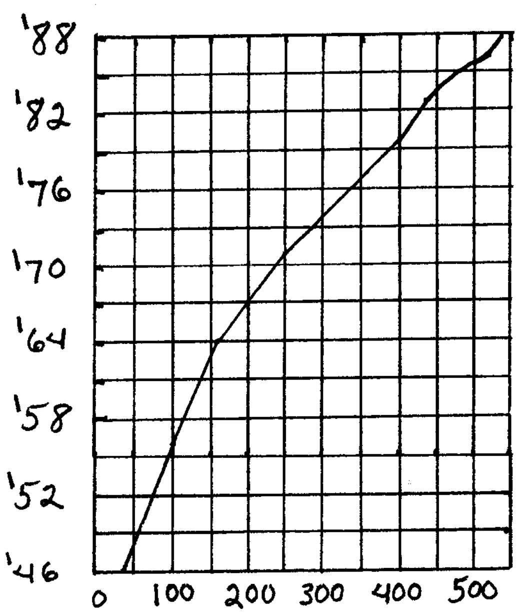
$$q = \frac{1}{2}(C - 100) = f^{-1}(C)$$

b.) The inverse function inputs total cost and outputs the number of articles produced.

28.) a.) Function f is invertible since it satisfies the horizontal line test.

b.) $f^{-1}(400) = 1980$

c.) Graph of f^{-1} $\longrightarrow$



29.) a.) supply curve

b.) demand curve

<u>Manufacturer</u>: As the selling price p of an item increases, the manufacturer will eagerly produce more items q (a.)

<u>Consumer</u>: As the selling price p of an item increases, the consumer will respond by buying fewer items q (b.)