H.W. #8

(1.)

Tangent line at \( x = a \)

(We must determine \( a \))

\((a, 4a-a^2)\)

\(y = 4x - x^2 \rightarrow y' = 4 - 2x\)

Slope of tangent line at \( x = a \) is

\[ 4 - 2a = \frac{5 - (4a - a^2)}{2 - a} \]

Derivative

\[ 8 - 4a - 4a + 2a^2 = 5 - 4a + a^2 \rightarrow a^2 - 4a + 3 = 0 \rightarrow \]

\[(a-1)(a-3) = 0 \rightarrow \]

\[ a = 1 \rightarrow \text{point } (1, 3) \]

\[ a = 3 \rightarrow \text{point } (3, 3) \]
2. The following chart represents the weight $W$ (lbs.) of a newborn baby as a function of time $t$ (months).

\[ W \text{ (lbs.)} \]

\[ 16 \]
\[ 12 \]
\[ 8 \]
\[ 4 \]

\[ t \text{ (months)} \]

a. What is the baby’s weight at birth? after 3 months? after 1 year?

b. What is an estimate of the baby’s growth rate (lbs./month) at birth? after 3 months? after 1 year?

c. When is the baby growing at the fastest rate during its first year of life and what is an estimate for this rate?
1. Use the given graph of function $f$ to sketch a graph of its derivative, $f'$. 

\begin{itemize}
  \item [f)] \hspace{2cm} f'
  \item [g)] \hspace{2cm} f'
  \item [h)] \hspace{2cm} f'
  \item [i)] \hspace{2cm} f'
  \item [j)] \hspace{2cm} f'
  \item [k)] \hspace{2cm} f'
\end{itemize}
2.)

a.) \( t = 0 \rightarrow W = 7 \text{ lbs.} \), \( t = 3 \text{ mo.} \rightarrow W = 5 \text{ lbs.} \), \( t = 1 \text{ yr.} \rightarrow W = 14 \text{ lbs.} \).

b.) growth rate: slope of tangent line
\( t = 0 \rightarrow \text{slope} = -\frac{3}{2.5} = -1.2 \text{ lbs./mo.} \),
\( t = 3 \text{ mo.} \rightarrow \text{slope} = 0 \text{ lbs./mo.} \),
\( t = 1 \text{ yr.} \rightarrow \text{slope} = \frac{8}{6.5} = 1.23 \text{ lbs./mo.} \).

c.) The baby is growing at the fastest rate when \( t = 4\frac{1}{2} \text{ months} \). The growth rate is \( \frac{8}{2.5} = 3.2 \text{ lbs./mo.} \).

\[ \text{SA4:} \quad \text{Cylinder:} \quad C = 2\pi r \rightarrow 25 = 2\pi r \rightarrow \]
\[ r = \frac{25}{2\pi} \text{ m.} \quad \text{so volume} \]
\[ V = \pi r^2 h = \pi \left(\frac{25}{2\pi}\right)^2(15) = 746.04 \text{ m}^3 \quad \text{(ice)} \]
(conversion: \( 1 \text{ ft}^3 = 0.028 \text{ m}^3 \))
\[ V = 26,641.243 \text{ ft}^3 \quad \text{(ice)} \]
(density ice = 0.917, density water = 0.917)
\[ \text{volume water} = 0.917 \text{ volume ice} \]
\[ V = 24,432.771 \text{ ft}^3 \quad \text{(water)} \]
1 acre = 43,560 ft. so depth of water is 
\[
\frac{24,432.771 \text{ ft.}^3}{43,560 \text{ ft}} = 0.56 \text{ ft.} = 6.7 \text{ inches}
\]

\[\text{SA7:}\]

\[
\begin{array}{c}
\text{A} \\
\text{B} \\
\text{C}
\end{array}
\]

\[
\begin{align*}
\text{a.} \quad \cos 30^\circ &= \frac{2}{A} \\
A &= \frac{2}{\cos 30^\circ} = \frac{2}{\frac{\sqrt{3}}{2}} = \frac{4}{\sqrt{3}} \text{ mi.}
\end{align*}
\]

\[
\begin{align*}
\text{b.} \quad \sin 30^\circ &= \frac{B}{A} \\
B &= A \sin 30^\circ = \frac{4}{\sqrt{3}} \cdot \frac{1}{2} = \frac{2}{\sqrt{3}} \text{ mi.}
\end{align*}
\]

\[\text{SA9:}\]

\[
\begin{align*}
\text{Area large } \Delta &= \text{ Areas of 4 small } \Delta \text{'s} \\
&+ \text{ Area } \square
\end{align*}
\]

\[
\frac{1}{2} (1)(2) = \frac{1}{2} (1-r) + \frac{1}{2} (1-r) + \frac{1}{2} (2-r) + \frac{1}{2} (2-r) + r^2
\]

\[
\rightarrow 1 = \frac{1}{2} (r-r^2) + \frac{1}{2} (r-r^2) + \frac{1}{2} (2r-r^2) + \frac{1}{2} (2r-r^2) + r^2
\]

\[
\rightarrow 1 = r-r^2+2r-r^2+r^2 ightarrow r^2-3r+1=0
\]

\[
r = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3-\sqrt{5}}{2} \approx 0.38
\]
\[ \frac{\sqrt{5}}{1} = \frac{2-r}{r} \rightarrow \sqrt{5} r = 2-r \rightarrow r + \sqrt{5} r = 2 \rightarrow (1+\sqrt{5}) r = 2 \rightarrow r = \frac{2}{1+\sqrt{5}} \]

**SA11**: Old K.E. = \( \frac{1}{2} mv^2 \)

a.) New K.E. = \( \frac{1}{2} (2m)(3v)^2 = 18 \left( \frac{1}{2} mv^2 \right) = 18 \) (Old K.E.)

b.) New K.E. = \( \frac{1}{2} (1.4m)(0.75v)^2 = 0.7875 \left( \frac{1}{2} mv^2 \right) = 0.7875 \) (Old K.E.)

So K.E. decreases by 0.2125 = 21.25%.

c.) New K.E. = \( \frac{1}{2} (0.25m)(2v)^2 = \frac{1}{2} mv^2 \) = Old K.E.

So K.E. remains unchanged.