Thm (11.2)
Let $V \subseteq \mathbb{R}^2$ be open, $(x_0, y_0) \in \mathbb{R}^2$, and $f : V \to \mathbb{R}$. If $f \in C^2(V)$ and if one of the mixed second partial derivatives of $f$ exists on $V$ and is continuous at the point $(x_0, y_0)$, then the other partial derivative exists at $(x_0, y_0)$ and

$$f_{xy}(x_0, y_0) = f_{yx}(y_0, x_0).$$

- Note: The conditions are met if $f \in C^2(V)$.

Thm (11.4)
Let $H = [a, b] \times [c, d]$ and $f : H \to \mathbb{R}$ be continuous. If

$$F(y) := \int_a^b f(x, y) \, dx,$$

then $F$ is continuous on $[c, d]$. More specifically, we have

$$\lim_{y \to y_0} \int_a^b f(x, y) \, dx = \int_a^b \lim_{y \to y_0} f(x, y) \, dx \quad \forall y_0 \in [c, d].$$

Thm (11.5)
Let $H = [a, b] \times [c, d]$ and $f : H \to \mathbb{R}$. Let $f(\cdot, y) \in C^{1}(I, [a, b])$, $\forall y \in [c, d]$ and $f_y(x, \cdot)$ exists on $[c, d]$, $\forall x \in [a, b]$. If $f_y(x, y) \in C(H)$,

$$\frac{d}{dy} \int_a^b f(x, y) \, dx = \int_a^b \frac{df}{dy}(x, y) \, dx \quad \forall y \in [c, d].$$

- Note: The conditions are satisfied if $f \in C^1(H)$.

- Note: This list is restricted to real-valued functions of two variables and can easily be extended to vector-valued functions of more than two variables (Optional HW).