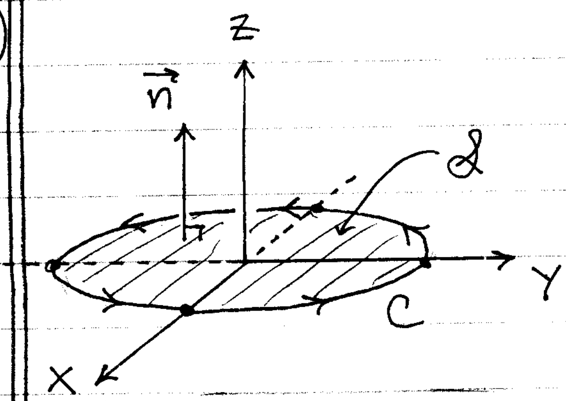
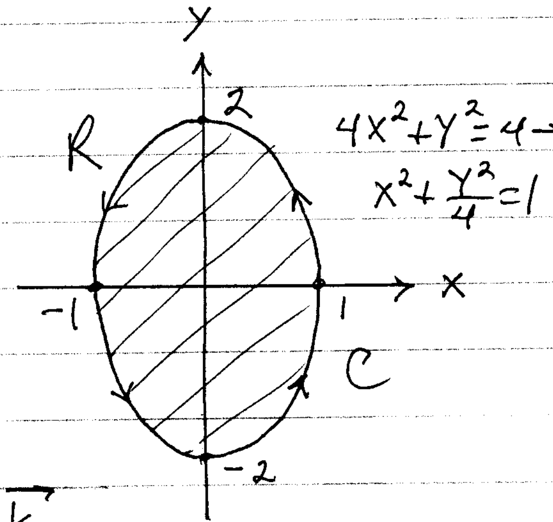
Section 16.7

1.)





$4x^2+y^2=4 \rightarrow$
$x^2+\frac{y^2}{4}=1$

$\vec{F}(x,y,z) = (x^2)\vec{i} + (2x)\vec{j} + (z^2)\vec{k}$,

$$\operatorname{curl} \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 2x & z^2 \end{vmatrix}$$

$= (0-0)\vec{i} - (0-0)\vec{j} + (2-0)\vec{k} = 2\vec{k}$;

$\vec{n} = \vec{k}$; so by Stoke's Theorem

$$\text{circ } \vec{F} = \oint_C \vec{F}\cdot\vec{T}\,ds = \iint_S \operatorname{curl}\vec{F}\cdot\vec{n}\,dS$$

$$= \iint_S 2\,dS = \iint_R 2\cdot\sec\gamma\,dA$$

$$= \iint_R 2\underbrace{\sec 0}_{1}\,dA = 2\iint_R 1\,dA$$

$= 2\cdot\text{area } R$

$= 2\cdot\frac{1}{2}\oint_C x\,dy - y\,dx$

$C: \begin{cases} x = \cos t \\ y = 2\sin t \end{cases}$
for $0 \le t \le 2\pi$;

$\frac{dx}{dt} = -\sin t,\ \frac{dy}{dt} = 2\cos t$

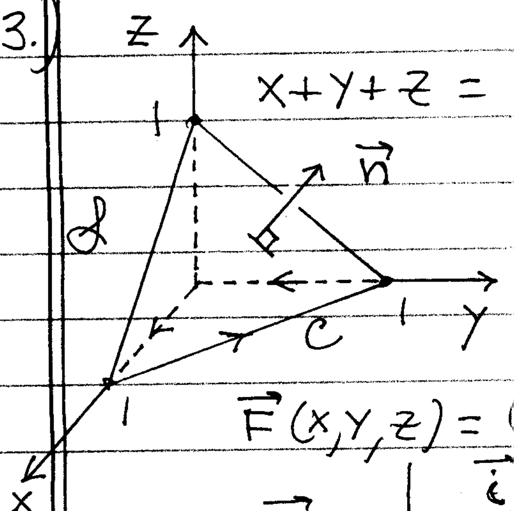
$$= \int_0^{2\pi} \left[\cos t \cdot \frac{dy}{dt} - 2\sin t \cdot \frac{dx}{dt}\right] dt$$

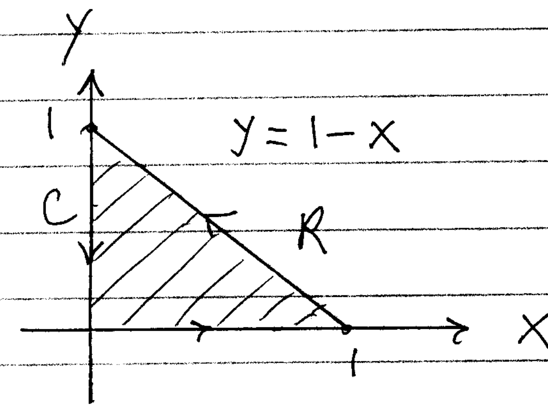
$$= \int_0^{2\pi} \left[\cos t \cdot 2\cos t - 2\sin t \cdot {-\sin t}\right] dt$$

$$= \int_0^{2\pi} 2(\underbrace{\cos^2 t + \sin^2 t}_{1})\, dt$$

$$= 2\theta \Big|_0^{2\pi} = 4\pi$$

3.)





$$\vec{F}(x,y,z) = (y)\vec{i} + (xz)\vec{j} + (x^2)\vec{k},$$

$$\text{curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & xz & x^2 \end{vmatrix}$$

$$= (0 - x)\vec{i} - (2x - 0) + (z - 1)\vec{k}$$

$$= (-x)\vec{i} + (-2x)\vec{j} + (z-1)\vec{k};$$

$$\vec{n} = \frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k};$$

$$\sec\gamma = \frac{|\vec{\nabla} f|}{|f_z|} = \frac{\sqrt{3}}{1} = \sqrt{3};$$

so by Stoke's Theorem

$$\text{circ } \vec{F} = \oint_C \vec{F} \cdot \vec{T}\, ds = \iint_S \text{curl } \vec{F} \cdot \vec{n}\, dS$$

$$= \iint_S \frac{1}{\sqrt{3}}(-x - 2x + z - 1)\, dS$$

$$= \iint_R \frac{1}{\sqrt{3}}(-3x + (1-x-y) - 1) \cdot \underbrace{\sec\gamma}_{\sqrt{3}}\, dA$$

$$= \int_0^1 \int_0^{1-x} (-4x - y)\, dy\, dx$$

$$= \int_0^1 \left(-4xy - \frac{1}{2}y^2\right) \Big|_{y=0}^{y=1-x} dx$$

$$= \int_0^1 \left(-4x(1-x) - \frac{1}{2}(1-x)^2\right) dx$$

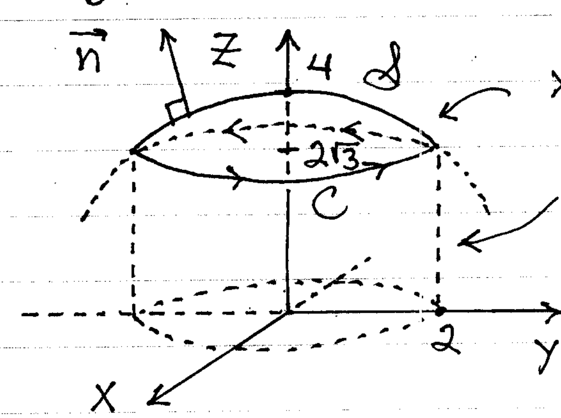
$$= \int_0^1 \left(-4x + 4x^2 - \frac{1}{2}(x^2 - 2x + 1)\right) dx$$

$$= \int_0^1 \left(-4x + 4x^2 - \frac{1}{2}x^2 + x - \frac{1}{2}\right) dx$$

$$= \int_0^1 \left(\frac{7}{2}x^2 - 3x - \frac{1}{2}\right) dx$$

$$= \left(\frac{7}{6}x^3 - \frac{3}{2}x^2 - \frac{1}{2}x\right) \Big|_0^1$$

$$= \frac{7}{6} - \frac{3}{2} - \frac{1}{2} = \frac{7}{6} - \frac{12}{6} = \frac{-5}{6}$$

6.) 

$x^2 + y^2 + z^2 = 16$

$x^2 + y^2 = 4 \longrightarrow$

$z^2 = 12 \rightarrow$

$z = 2\sqrt{3}$,

$$\vec{F}(x,y,z) = (x^2y^3)\vec{i} + (1)\vec{j} + (z)\vec{k},$$

$$\text{curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y^3 & 1 & z \end{vmatrix}$$

$$= (0-0)\vec{i} - (0-0)\vec{j} + (0 - 3x^2y^2)\vec{k}$$

$$= (-3x^2y^2)\vec{k} \ ;$$

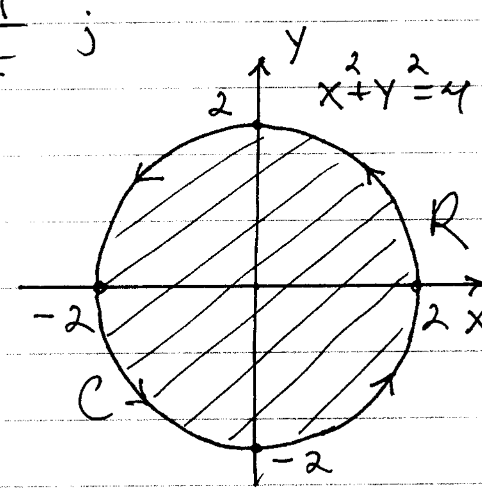
$$\vec{\nabla} f = (2x)\vec{i} + (2y)\vec{j} + (2z)\vec{k},$$

$$|\vec{\nabla} f| = \sqrt{(2x)^2 + (2y)^2 + (2z)^2}$$

$$= \sqrt{4(x^2+y^2+z^2)} = \sqrt{4(16)} = 8,$$

$$\vec{n} = \frac{\vec{\nabla} f}{|\vec{\nabla} f|} = \left(\frac{x}{4}\right)\vec{i} + \left(\frac{y}{4}\right)\vec{j} + \left(\frac{z}{4}\right)\vec{k},$$

$$\sec\gamma = \frac{|\vec{\nabla} f|}{|f_z|} = \frac{8}{2z} = \frac{4}{z} \ ;$$



so by Stoke's Theorem

$$\text{circ } \vec{F} = \oint_C \vec{F}\cdot\vec{T}\, ds$$

$$= \iint_S \text{curl } \vec{F}\cdot\vec{n}\, dS$$

$$= \iint_S (-3x^2y^2)\left(\frac{z}{4}\right) dS$$

$$= \iint_R (-3x^2y^2)\left(\frac{z}{4}\right)\cdot \sec\gamma\, dA$$

$$= \iint_R (-3x^2y^2)\left(\frac{\cancel{z}}{\cancel{4}}\right)\left(\frac{\cancel{4}}{\cancel{z}}\right) dA$$

$$= \int_0^{2\pi}\int_0^2 -3(r\cos\theta)^2(r\sin\theta)^2 \cdot r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^2 -3r^5\cos^2\theta\sin^2\theta\,dr\,d\theta$$

$$= \int_0^{2\pi} -\frac{1}{2}r^6(\sin\theta\cos\theta)^2\Big|_{r=0}^{r=2}\,d\theta$$

$$= \int_0^{2\pi} -32\left(\frac{1}{2}\sin 2\theta\right)^2 d\theta$$

$$= \int_0^{2\pi} -8\sin^2 2\theta\,d\theta$$

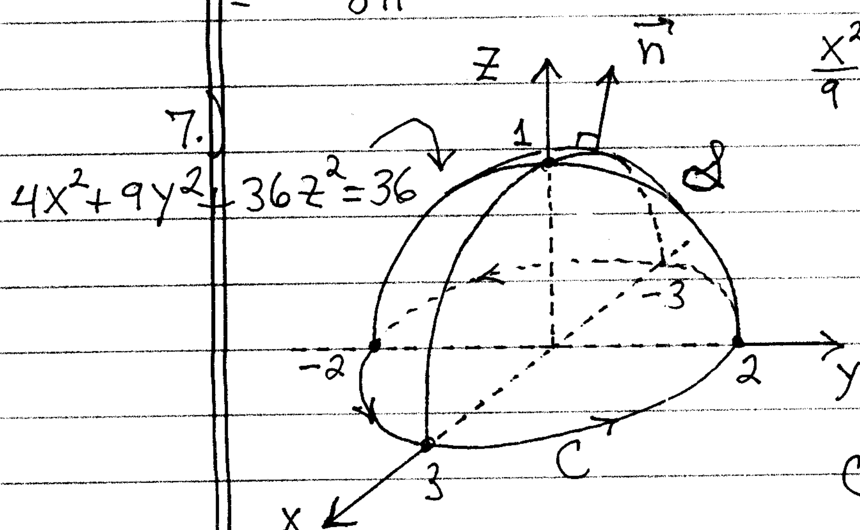
$$= \int_0^{2\pi} -8\cdot\frac{1}{2}(1-\cos 4\theta)\,d\theta$$

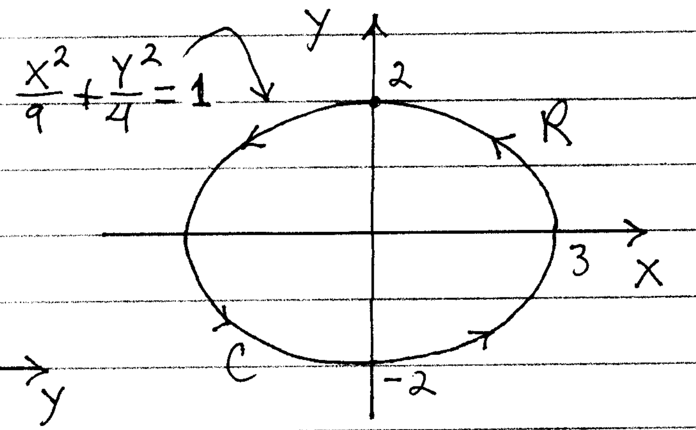
$$= -4\left(\theta - \frac{1}{4}\sin 4\theta\right)\Big|_0^{2\pi}$$

$$= -4\left(2\pi - \frac{1}{4}\cancelto{0}{\sin 8\pi}\right) - -4\left(0 - \frac{1}{4}\cancelto{0}{\sin 0}\right)$$

$$= -8\pi$$

7.





$$C: \begin{cases} x = 3\cos t \\ y = 2\sin t \end{cases} \quad \text{for } 0 \le t \le 2\pi$$

$$\frac{dx}{dt} = -3\sin t, \quad \frac{dy}{dt} = 2\cos t;$$

$$\vec{F}(x,y,z) = (y)\,\vec{i} + (x^2)\,\vec{j} + (x^2+y^4)^{3/2} \sin e^{\sqrt{xyz}}\,\vec{k}\,;$$

by Stoke's Theorem

$$\iint_{S} \vec{\nabla}\times\vec{F}\cdot n \, dS = \oint_{C} \vec{F}\cdot\vec{T}\, ds$$

$$= \oint_{C} M\,dx + N\,dy = \int_{C}\left[y\frac{dx}{dt} + x^2\frac{dy}{dt}\right] dt$$

$$= \int_0^{2\pi}\left[(2\sin t)(-3\sin t) + (9\cos^2 t)(2\cos t)\right] dt$$

$$= \int_0^{2\pi}\left[-6\cdot\tfrac{1}{2}(1-\cos 2t) + 18\cdot(1-\sin^2 t)\cos t\right] dt$$

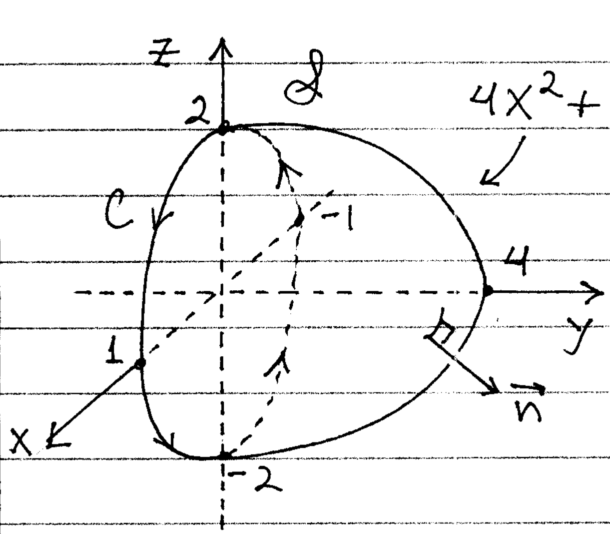
$$= \int_0^{2\pi}\left[-3(1-\cos 2t) + 18(\cos t - \sin^2 t\cos t)\right] dt$$

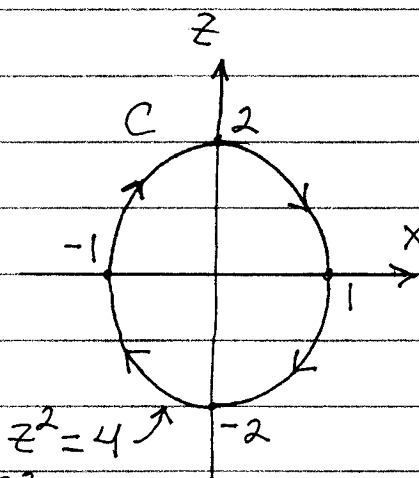
$$= \left[-3\left(\theta - \tfrac{1}{2}\sin 2t\right) + 18\left(\sin t - \tfrac{1}{3}\sin^3 t\right)\right]\Big|_0^{2\pi}$$

$$= \left[-3\left(2\pi - \tfrac{1}{2}\cancelto{0}{\sin 4\pi}\right) + 18\left(\cancelto{0}{\sin 2\pi} - \tfrac{1}{3}\cancelto{0}{\sin^3 2\pi}\right)\right]$$
$$-\left[-3\left(0 - \tfrac{1}{2}\cancelto{0}{\sin 0}\right) + 18\left(\cancelto{0}{\sin 0} - \tfrac{1}{3}\cancelto{0}{\sin^3 0}\right)\right]$$

$$= -6\pi$$

8.)





$$4x^2 + z^2 = 4$$
or $$x^2 + \frac{z^2}{4} = 1$$

C is CLOCKWISE for this problem:

$$C: \begin{cases} x = \sin t \\ z = 2\cos t \end{cases} \quad \text{for } 0 \le t \le 2\pi,$$

$$\frac{dx}{dt} = \cos t, \quad \frac{dz}{dt} = -2\sin t \ ;$$

$$\vec{F}(x,y,z) = \left(-z + \frac{1}{2+x}\right)\vec{i} + (\arctan y)\,\vec{j} + \left(x + \frac{1}{4+z}\right)\vec{k}$$

$$= M\vec{i} + N\vec{j} + P\vec{k} \quad ; \text{ then}$$

by Stoke's Theorem

$$\iint_{S} \vec{\nabla} \times \vec{F} \cdot \vec{n}\, dS = \oint_C \vec{F} \cdot \vec{T}\, ds$$

$$= \oint_C M\,dx + P\,dz$$

$$= \int_0^{2\pi} \left[\left(-z + \frac{1}{2+x}\right)\frac{dx}{dt} + \left(x + \frac{1}{z+4}\right)\frac{dz}{dt}\right] dt$$

$$= \int_0^{2\pi} \left[\left(-2\cos t + \frac{1}{2+\sin t}\right)\cdot \cos t + \left(\sin t + \frac{1}{4+2\cos t}\right)(-2\sin t)\right] dt$$

$$= \int_0^{2\pi} \left[-2(\cos^2 t + \sin^2 t) + \frac{\cos t}{2+\sin t} + \frac{-2\sin t}{4+2\cos t}\right] dt$$

$$= \left(-2t + \ln(2+\sin t) + \ln(4+2\cos t)\right)\Big|_0^{2\pi}$$

$$= \left(-4\pi + \ln(2+\overset{0}{\cancel{\sin 2\pi}}) + \ln(4+2\overset{1}{\cancel{\cos 2\pi}})\right) - \left(0 + \ln(2+\overset{0}{\cancel{\sin 0}}) + \ln(4+2\overset{1}{\cancel{\cos 0}})\right)$$

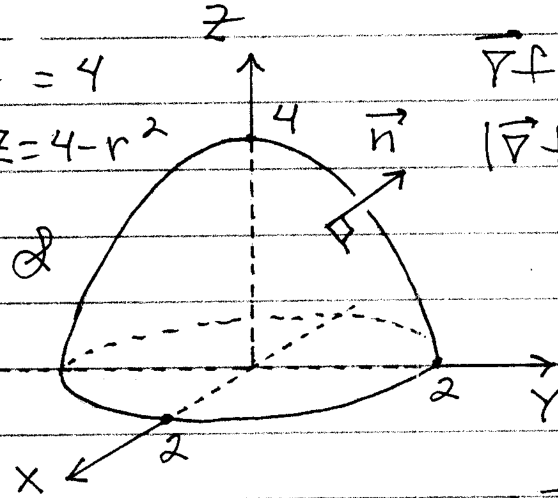
$$= -4\pi$$

13.) $\vec{F}(x,y,z) = (2z)\vec{i} + (3x)\vec{j} + (5y)\vec{k}$,

$$S: \begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 4 - r^2 \end{cases} \quad \text{for } 0 \le r \le 2, \quad 0 \le \theta \le 2\pi$$

$x^2 + y^2 + z = 4$

or $z = 4 - r^2$



$\vec{\nabla} f = (2x)\vec{i} + (2y)\vec{j} + (1)\vec{k}$,

$|\vec{\nabla} f| = \sqrt{4x^2 + 4y^2 + 1}$;

$$\text{curl}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2z & 3x & 5y \end{vmatrix}$$

$= (5-0)\vec{i} - (0-2)\vec{j} + (3-0)\vec{k}$

$= (5)\vec{i} + (2)\vec{j} + (3)\vec{k}$; then

$$\vec{n} = \frac{(2x)\vec{i} + (2y)\vec{j} + (1)\vec{k}}{\sqrt{4(x^2+y^2)+1}} = \frac{\vec{\nabla} f}{|\vec{\nabla} f|};$$

$$\vec{r}_\theta \times \vec{r}_r = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -r\sin\theta & r\cos\theta & 0 \\ \cos\theta & \sin\theta & -2r \end{vmatrix}$$

$= (-2r^2\cos\theta - 0)\vec{i} - (2r^2\sin\theta - 0)\vec{j}$
$\quad + (-r\sin^2\theta - r\cos^2\theta)\vec{k}$

$= (-2r^2\cos\theta)\vec{i} + (-2r^2\sin\theta)\vec{j}$
$\quad - r(\underbrace{\sin^2\theta + \cos^2\theta}_{1})\vec{k}$

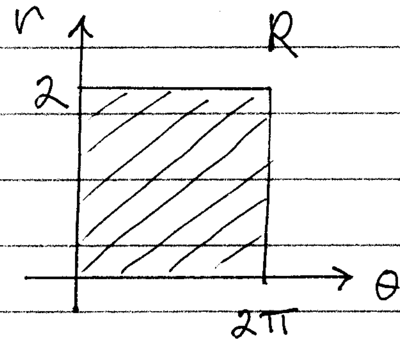
$= (-2r^2\cos\theta)\vec{i} + (-2r^2\sin\theta)\vec{j} + (-r)\vec{k}$;

$$|\vec{r}_\theta \times \vec{r}_r| = \sqrt{4r^4\cos^2\theta + 4r^4\sin^2\theta + r^2}$$

$$= \sqrt{4r^4(\underbrace{\cos^2\theta + \sin^2\theta}_{1}) + r^2}$$

$$= \sqrt{r^2(4r^2+1)} = r\sqrt{4r^2+1};$$

then



$$\iint_S \text{curl}\,\vec{F}\cdot\vec{n}\; dS$$

$$= \iint_R \text{curl}\,\vec{F}\cdot\vec{n}\cdot|\vec{r}_\theta \times \vec{r}_r|\; dA$$

$$= \int_0^{2\pi}\int_0^2 \frac{10x+4y+3}{\sqrt{4(x^2+y^2)+1}}\cdot r\sqrt{4r^2+1}\; dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^2 \frac{10r\cos\theta + 4r\sin\theta + 3}{\cancel{\sqrt{4r^2+1}}}\, r\cancel{\sqrt{4r^2+1}}\; dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^2 (10r^2\cos\theta + 4r^2\sin\theta + 3r)\; dr\, d\theta$$

$$= \int_0^{2\pi} \left(\frac{10}{3}r^3\cos\theta + \frac{4}{3}r^3\sin\theta + \frac{3}{2}r^2\right)\Big|_{r=0}^{r=2} d\theta$$

$$= \int_0^{2\pi} \left(\frac{80}{3}\cos\theta + \frac{32}{3}\sin\theta + 6\right) d\theta$$

$$= \left(\frac{80}{3}\sin\theta - \frac{32}{3}\cos\theta + 6\theta\right)\Big|_0^{2\pi}$$

$$= \left(\frac{80}{3}\overset{0}{\cancel{\sin 2\pi}} - \frac{32}{3}\overset{1}{\cancel{\cos 2\pi}} + 12\pi\right)$$

$$- \left(\frac{80}{3}\overset{0}{\cancel{\sin 0}} - \frac{32}{3}\overset{1}{\cancel{\cos 0}} + 0\right) = 12\pi$$

15.) $\vec{F}(x,y,z) = (x^2y)\vec{i} + (2y^3z)\vec{j} + (3z)\vec{k}$

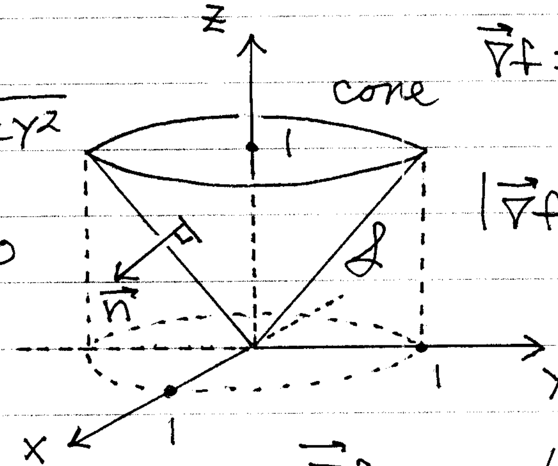
$$S: \begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = r \end{cases} \quad \text{for } \begin{array}{l} 0 \le r \le 1, \\ 0 \le \theta \le 2\pi \end{array},$$

$z = r$

or $z = \sqrt{x^2+y^2}$

or

$\sqrt{x^2+y^2} - z = 0$



$$\vec{\nabla} f = \frac{x}{\sqrt{x^2+y^2}}\vec{i} + \frac{y}{\sqrt{x^2+y^2}}\vec{j} + (-1)\vec{k},$$

$$|\vec{\nabla} f| = \sqrt{\frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2} + 1}$$

$$= \sqrt{\frac{x^2+y^2}{x^2+y^2} + 1} = \sqrt{2},$$

and $\vec{n} = \dfrac{\vec{\nabla} f}{|\vec{\nabla} f|} = \dfrac{1}{\sqrt{2}}\left(\dfrac{x}{\sqrt{x^2+y^2}}\vec{i} + \dfrac{y}{\sqrt{x^2+y^2}}\vec{j} + (-1)\vec{k}\right);$

$$\text{curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & 2y^3z & 3z \end{vmatrix}$$

$= (0 - 2y^3)\vec{i} - (0-0)\vec{j} + (0 - x^2)\vec{k}$

$= (-2y^3)\vec{i} + (-x^2)\vec{k}$;

$$\vec{r}_\theta \times \vec{r}_r = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -r\sin\theta & r\cos\theta & 0 \\ \cos\theta & \sin\theta & 1 \end{vmatrix},$$

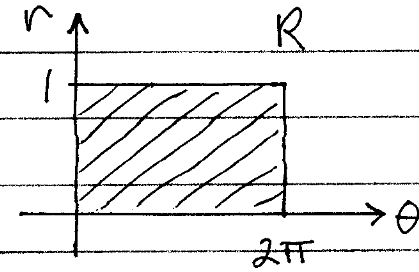
$= (r\cos\theta - 0)\vec{i} - (-r\sin\theta - 0)\vec{j}$

$\quad + (-r\sin^2\theta - r\cos^2\theta)\vec{k}$

$= (r\cos\theta)\vec{i} + (r\sin\theta)\vec{j} + (-r)\vec{k}$, then

$|\vec{r}_\theta \times \vec{r}_r| = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta + r^2}$

$$= \sqrt{r^2\underbrace{(\cos^2\theta+\sin^2\theta)}_{1}+r^2} = \sqrt{2r^2} = \sqrt{2}\cdot r \;;$$



then
$$\iint_{S} \operatorname{curl}\vec{F}\cdot\vec{n}\,dS$$

$$= \iint_{R} \operatorname{curl}\vec{F}\cdot\vec{n}\cdot\left|\vec{r}_\theta\times\vec{r}_r\right|\,dA$$

$$= \int_0^{2\pi}\int_0^1 \frac{1}{\cancel{\sqrt{2}}}\cdot\left[\frac{-2xy^3}{\sqrt{x^2+y^2}}+x^2\right]\cdot\cancel{\sqrt{2}}\,r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left[\frac{-2(r\cos\theta)(r\sin\theta)^3}{\sqrt{r^2}}+(r\cos\theta)^2\right]r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left(-2r^4\cos\theta\cdot\sin^3\theta+r^3\cos^2\theta\right)dr\,d\theta$$

$$= \int_0^{2\pi}\left(-\frac{2}{5}r^5\cdot\cos\theta\sin^3\theta+\frac{1}{4}r^4\cdot\cos^2\theta\right)\Big|_{r=0}^{r=1}d\theta$$

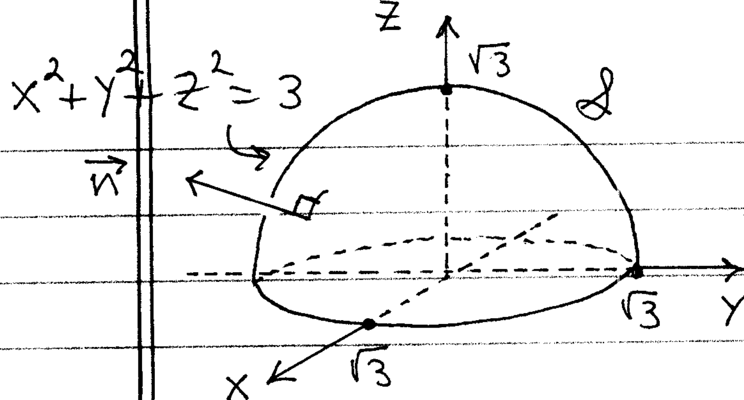
$$= \int_0^{2\pi}\left(-\frac{2}{5}\cos\theta\cdot\sin^3\theta+\frac{1}{4}\cdot\frac{1}{2}(1+\cos 2\theta)\right)d\theta$$

$$= \left(-\frac{2}{5}\cdot\frac{1}{4}\sin^4\theta+\frac{1}{8}\left(\theta+\frac{1}{2}\sin 2\theta\right)\right)\Big|_0^{2\pi}$$

$$= \left(-\frac{1}{10}\underbrace{\sin^4 2\pi}_{0}+\frac{1}{8}\left(2\pi+\frac{1}{2}\underbrace{\sin 4\pi}_{0}\right)\right)$$
$$-\left(-\frac{1}{10}\underbrace{\sin^4 0}_{0}+\frac{1}{8}\left(0+\frac{1}{2}\underbrace{\sin 0}_{0}\right)\right) = \frac{\pi}{4}$$

17. $\vec{F}(x,y,z) = (3y)\,\vec{i}+(5-2x)\,\vec{j}+(z^2-2)\,\vec{k}$

$$S:\begin{cases} x=\sqrt{3}\sin\phi\cos\theta \\ y=\sqrt{3}\sin\phi\sin\theta \\ z=\sqrt{3}\cos\phi \end{cases} \quad \text{for } 0\le\phi\le\pi/2,\; 0\le\theta\le 2\pi$$



$\vec{\nabla} f = (2x)\vec{i} + (2y)\vec{j} + (2z)\vec{k}$,

$|\vec{\nabla} f| = \sqrt{(2x)^2+(2y)^2+(2z)^2}$

$= \sqrt{4(x^2+y^2+z^2)}$

$= \sqrt{4(3)} = 2\sqrt{3}$;

$$\vec{n} = \frac{\vec{\nabla} f}{|\vec{\nabla} f|} = \left(\frac{x}{\sqrt{3}}\right)\vec{i} + \left(\frac{y}{\sqrt{3}}\right)\vec{j} + \left(\frac{z}{\sqrt{3}}\right)\vec{k} \ ;$$

$$\text{curl}\, \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3y & 5-2x & z^2-2 \end{vmatrix}$$

$$= (0-0)\vec{i} - (0-0)\vec{j} + (-2-3)\vec{k} = (-5)\vec{k} \ ;$$

$$\vec{r}_\phi \times \vec{r}_\theta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \sqrt{3}\cos\phi\cos\theta & \sqrt{3}\cos\phi\sin\theta & -\sqrt{3}\sin\phi \\ -\sqrt{3}\sin\phi\sin\theta & \sqrt{3}\sin\phi\cos\theta & 0 \end{vmatrix}$$

$$= (0-3\sin^2\phi\cos\theta)\vec{i} - (0-3\sin^2\phi\sin\theta)\vec{j}$$
$$+ (3\sin\phi\cos\phi\cos^2\theta + 3\sin\phi\cos\phi\sin^2\theta)\vec{k}$$

$$= (-3\sin^2\phi\cos\theta)\vec{i} + (3\sin^2\phi\sin\theta)\vec{j}$$
$$+ (3\sin\phi\cos\phi)\underbrace{(\cos^2\theta + \sin^2\theta)}_{1}\vec{k}$$

$$= (-3\sin^2\phi\cos\theta)\vec{i} + (3\sin^2\phi\sin\theta)\vec{j} + (3\sin\phi\cos\phi)\vec{k} \ ;$$

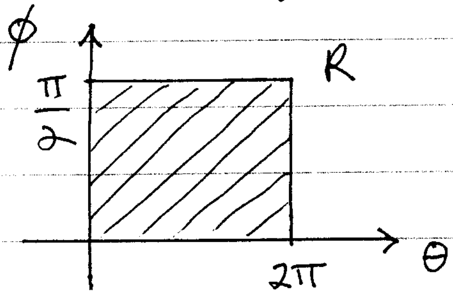
$$|\vec{r}_\phi \times \vec{r}_\theta| = \sqrt{(-3\sin^2\phi\cos\theta)^2 + (3\sin^2\phi\sin\theta)^2 + (3\sin\phi\cos\phi)^2}$$

$$= \sqrt{9\sin^4\phi\cos^2\theta + 9\sin^4\phi\sin^2\theta + 9\sin^2\phi\cos^2\phi}$$

$$= \sqrt{9\sin^4\phi\,\underbrace{(\cos^2\theta + \sin^2\theta)}_{1} + 9\sin^2\phi\cos^2\phi}$$

$$= \sqrt{9\sin^2\phi\,(\underbrace{\sin^2\phi+\cos^2\phi}_{1})} = 3\sin\phi \; ;$$

then



$$\iint_S \operatorname{curl}\vec{F}\cdot\vec{n}\,dS$$

$$= \iint_R \operatorname{curl}\vec{F}\cdot\vec{n}\cdot|\vec{r}_\phi\times\vec{r}_\theta|\,dA$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \frac{-5}{\sqrt{3}}\,z\cdot 3\sin\phi\;d\phi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \frac{-5}{\cancel{\sqrt{3}}}\cdot\cancel{\sqrt{3}}\cos\phi\cdot 3\sin\phi\;d\phi\,d\theta$$

$$= \int_0^{2\pi}\left(-15\cdot\frac{1}{2}\sin^2\phi\,\Big|_{\phi=0}^{\phi=\frac{\pi}{2}}\right)d\theta$$

$$= \int_0^{2\pi}\left(-\frac{15}{2}\overset{1}{\cancel{\sin^2}\frac{\pi}{2}} - -\frac{15}{2}\overset{0}{\cancel{\sin^2}0}\right)d\theta$$

$$= \int_0^{2\pi} -\frac{15}{2}\,d\theta = -\frac{15}{2}\theta\,\Big|_0^{2\pi} = -15\pi\,.$$